Water network sectorization based on a genetic algorithm and minimum dissipated power paths


ABSTRACT

Water Network Sectorization (WNS) consists of dividing a water system into sectors with an independent water supply. When each district is supplied by its only water source the districts can be defined as i-DMAs (isolated District Meter Areas) because they are completely cut off from the rest of the water network. This isolation of the i-DMAs may decrease hydraulic performance of the water system reducing its topologic (network loops) and hydraulic (diameter sections) redundancy. Traditionally the design of WNS is carried out by empirical or simulation assisted trial-and-error approaches that are difficult to apply to large water distribution systems. In this paper an original methodology for automatic sectorization of water networks is proposed. The methodology is based on Shortest Path techniques that allow defining a tree graph of the network with dissipated power used as weight of the pipes (or links). Once the districts are found, a swapping phase follows which is achieved by using a genetic algorithm (GA) that allows refining the choice of nodes that belong to each district. The objective function of the GA is based on network mean pressure. The methodology was tested, using different performance indices, on two real water supply systems.

Key words | dissipated power, genetic algorithm, shortest path, water network sectorization

INTRODUCTION

Two kinds of district metering design can be defined for water management and control, depending on project goals. The first one, named Water Network Partitioning (WNP) (Di Nardo et al. 2011) is achieved by defining some relatively small permanent districts, commonly called District Meter Areas (DMAs), through insertion of new boundary valves (or closing existing valves) and flow meters in order to have subsystems in which it is simpler to define water balance, identify water losses and carry out pressure management. The second one, applicable to multiple source networks, is named here Water Network Sectorization (WNS) and consists of dividing the water system into independent districts, each of them supplied by its water source (or water sources) with no connection to other districts, in order to achieve sectors with independent water supply (Tzatchkov et al. 2006a), or to improve network protection (Grayman et al. 2009; Di Nardo et al. 2012). In this case the districts are named independent-District Meter Areas or isolated-District Meter Areas (i-DMAs).

Although DMAs are normally designed to operate permanently, the closed boundary valves can be opened at any moment, manually or by remote control system, which allows ‘looping’ to be easily re-established, if it is necessary in a particular emergency situation.

i-DMAs enable actions to be taken to improve management and control of important aspects of water distribution. These actions may include but are not limited to: (a) better water quality control (there is no mixing of water from different sources); (b) water audit (hydraulic efficiency and non-revenue water); (c) daily demand curve...
characterization, especially night flow; (d) leak detection, by analysing the evolution of the minimum night flow; (e) investment planning directing supply to sectors with more non-revenue water; (f) district isolation in order to protect the rest of the water distribution network from accidental or malicious contamination; and (h) pressure management inside each district. This level of partitioning represents, however, a more difficult challenge because it can worsen hydraulic performance of water networks that are traditionally designed with many connections and loops, given that WNS is achieved by pipe closures that reduce pipe diameter availability with the consequent eventual decrease of network water pressure.

Traditional design of DMAs and i-DMAs has been based on empirical suggestions (limits on number of properties, length of pipes, etc.) (Water Authorities Association and Water Research Centre 1985; Wrc/WSA/WCA 1994; WIR Ltd 1999) combined with ‘trial and error’, which is very difficult to apply to large water supply systems, even when used with hydraulic network simulation software. Only recently, some techniques based on graph theory have been proposed to design DMAs (Tzatchkov et al. 2006a; Deuerlein 2008; Herrera et al. 2010; Di Nardo & Di Natale 2011, 2012; Di Nardo et al. 2011; Izquierdo et al. 2011).

In this paper an original approach for i-DMAs design, based on graph theory principles and a specially developed genetic algorithm (GA), is proposed. After hydraulic simulation, the water network is mapped onto a weighted oriented graph, using computed dissipated power in pipes as weights. Then the Minimum Dissipated Power Path (MDPP) for each water source is found and node swap between i-DMAs is achieved with the GA. WNS changes system layout, increasing head loss and decreasing topological and energy redundancy; this effect is due to the valve closures that reduce the number of network pipes and remove some network loops. Because of this each i-DMA increases internal power dissipation, energy indices, such as resilience (Todini 2000), can provide a good way to compare different i-DMA designs, and are used in this paper.

**METHODS**

The proposed methodology for i-DMA design is based essentially on two procedures: (a) search for the MDPPs related to each source; and (b) heuristic optimization achieved by swapping of nodes that belong to two different independent paths. The first procedure, based on graph theory, allows finding all nodes reached by a single source with minimum power dissipation. In fact water always chooses naturally the path with minimum dissipation to reach each node. In particular, in an oriented graph, water leaving a water source reaches only some nodes and not all network nodes. In this way, each group of nodes supplied by a source becomes i-DMA because it has the peculiarity of including all nodes with MDPP connected to that source.

A node can belong to several dissipated power paths, however, each of them stemming from a different source. In other terms, the areas supplied by different sources may have common nodes. Each of these common nodes must be assigned to only one particular i-DMA. The second procedure therefore consists in swapping nodes between the groups of nodes (or i-DMAs) that belong to each source, in such a way that a suitable objective function is optimized.

The two procedures were integrated in a tool for heuristic optimal design of i-DMAs in which each step (Figure 1) is reported as:

(a) do hydraulic simulation and compute the adjacency $A_{ij}$ and weight $W_{ij}$ matrices;
(b) compute the Shortest Paths $p_{ij}^{min}$ from each source $s$ to each network node;
(c) define the set of nodes $\{C_s\}$ reached by all sources;
(d) define the set of nodes $\{C_i\}$ reached by all sources;
(d) find and optimize the \( \{C_{i1}\} \) and \( \{C_{i2}\} \) sets by heuristic procedure based on a GA, where \( \{C_{i1}\} \) is the set of nodes supplied only by the given source and \( \{C_{i2}\} \) is the set of nodes supplied by other sources;

(e) insert \( N_{bf} \) boundary valves between the nodes that belong to \( \{C_{i1}\} \) and \( \{C_{i2}\} \);

(f) repeat the procedure from step (c) to step (f) for each network source \( s \).

The first objective, to find the ‘Shortest Path’, for the case of a Water Distribution System, can be solved by resorting to graph theory that allows determination of the minimum cost path between two nodes \( s \) (a source or reservoir) and \( i \) (a generic node) of a network represented by a directed graph \( G \) with \( n \) nodes, \( m \) links and weight function \( w_j \) that correlates to the respective cost, in this case the dissipated power \( q_i \Delta H_j \) (where \( q_i \) is flow and \( \Delta H_j \) head loss in \( j \)-th pipe), as reported by Di Nardo & Di Natale (2022).

Starting from the network model (with input data the given node water demand distribution \( Q_i \), node elevations \( z_i \), \( i=1..n \), source heads \( H_s \), \( s=1..r \), and pipe lengths \( L_j \), \( j=1..m \)), the pipe flows \( q_i \) and head loss \( \Delta H_j \) for each pipe, and the node heads \( H_i \) can be calculated by a Demand Driven approach (Rossmann 2000), and it is possible to define an adjacent matrix \( A \) and a weight matrix \( W \), both of them of order \( n \) (step a):

\[
A_{xy} = \begin{cases} 
1 & \text{if node } a_x \text{ is linked to node } a_y \text{ (for } x \neq y) \\
0 & \text{otherwise}
\end{cases} 
\]

\[
W_{xy} = \begin{cases} 
w_{xy} & \text{if } (A_{xy} = 1 \text{ and } w_{xy} > 0) \\
0 & \text{otherwise}
\end{cases}
\]

with the following weight function computed between two generic network nodes \( a_x \) and \( a_y \):

\[
w_{xy} = q_{xy}\Delta H_{xy} = q_{xy}(H_{a_x} - H_{a_y}) = w_i = q_i\Delta H_j
\]

Then, compatibly with the given network layout \( \forall s,i \) it is possible to identify all node paths \( \{p_i\}^{s,a_i} = \{a_{s1}, \ldots, a_{s}, \ldots, a_{j}\}^{s,a_i} \) belonging to the path set \( \{p\}^{s,a_i} = \{p_1^{s,a_i}, \ldots, p_{s}^{s,a_i}, \ldots, p_{p}^{s,a_i}\} \), that may be crossed by an infinitesimal flow \( dq \) from source \( s \) to the \( i \)-th node in the worst operating condition (peak water demand).

Among all possible paths of set \( \{p\}^{s,a_i} \) only one is the MDPP:

\[
p_{min}^{s,a_i} = \{a_{s1}, \ldots, a_{s}, \ldots, a_{j}\}^{s,a_i} \text{ with } \left( \min_{\beta \in \{p_{min}\}^{s,a_i}} \sum \omega_j \right)
\]

So, a new set \( \{p\}^{s}_{min} = \{p_{min}^{s,a_1}, \ldots, p_{min}^{s,a_s}, \ldots, p_{min}^{s,a_i}\} \), composed of all MDPPs \( n \) from each reservoir \( s \) to each node \( a_s \) can be defined.

Many Shortest Path resolution algorithms (Skiena 1990) can be found in the literature; in this paper the Dijkstra algorithm has been chosen (Dijkstra 1959) that allows all MDPPs \( \{p\}^{s}_{min} \) to be obtained easily if the weight matrix \( W \) is known (step b). Then, each subsystem \( \{p\}^{s}_{min} \) is a network subgraph \( G_s = (V_s, E_s) \), with \( V_s < V \), that can be associated to a specific category of graph called a tree graph because any two vertices in it are connected only by one path.

Let one node set be:

\[
\{C_s\} = \bigcup_{s=1}^{r} \{p\}^{s}_{min} - \{C_{s}\}
\]

composed of the nodes reached, with minimum energy paths, from source \( s \) and from the rest of \( r-1 \) sources (step c), and let another node set be:

\[
\{C_{s}\} = \{p\}^{s}_{min} - \{C_{s}\}
\]

composed of the nodes reached only from the \( s \)-th source and, finally, let a third node set be:

\[
\{U_s\} = \bigcup_{s=1}^{r} \{p\}^{s}_{min} - \{C_{s}\}
\]

reached only from the rest of \( r-1 \) sources.

Then it is possible to divide the set \( \{C_{s}\} \) into two subsets \( \{C_{s1}\} \) and \( \{C_{s2}\} \) (step d), with \( \{C_{s1}\} \cap \{C_{s2}\} = 0 \), in order to obtain a water subsystem supplied only by source \( s \), called isolated DMA, \( \{i-DMAs\} = \{C_{s1}\} \cup \{C_{s1}\} \), by inserting gate valves in the links (or pipes) between the nodes that belong to subsets \( \{C_{s1}\} \) and \( \{C_{s2}\} \) respectively (step e). This phase was achieved by a heuristic procedure carried out.
with a GA (Goldberg 1989) developed by the authors. The GA allows the best layout of the i-DMAs to be found by inserting gate valves in the corresponding pipes and swapping nodes, i.e. moving some nodes that belong to subsets \([C_{a1}]\) and \([C_{a2}]\) from one subset to the other, being compliant with the following objective function:

\[
O.F. = \min \left( \frac{1}{n} \sum_{i=1}^{n} h_i \right)
\]

where \(n\) is the number of nodes and \(h_i\) is the pressure value at \(i\)-th node.

The minimization of (8) was carried out with the help of the Genetic Toolbox of MATLAB\textsuperscript{®}. In particular, for every source \(s\), each GA individual is composed of a sequence of chromosomes whose length is equal to the number of nodes belonging to subset \([C_s]\). Each chromosome assumes value 0 if \(i\)-th node belongs to \([C_{a1}]\), or value 1 if it belongs to \([C_{a2}]\). A hundred generations were carried out with a population composed of 20 individuals and the crossover percentage was equal to \(P_{cross} = 0.8\). Then, repeating the procedure from step (c) to step (f) for each of the other \(r-1\) sources, the rest of the i-DMAs can be identified.

Finally, two categories of Performance Indices were used to test the results obtained with the proposed methodology: (a) energy indices, measured by the resilience index \(I_r\) proposed by Todini (2000), based on comparison between the actual dissipated power and the maximum dissipated power which is necessary to satisfy node demand constraints; and the resilience deviation index \(I_{rd}\) proposed by Di Nardo & Di Natale (2011), based on comparison between the resilience index of the original and of the sectorized network; (b) pressure indices, traditionally measured by mean node pressure \(h_{mean}\) and minimum node pressure \(h_{min}\).

**RESULTS AND DISCUSSION**

The methodology was tested on two real water supply systems: Parete (a small network in Italy) (Di Nardo & Di Natale 2012) and Matamoros (a large network in Mexico) (Tzatchkov et al. 2006b). Parete, with 10,800 inhabitants, is located in a densely populated area in the south of the Province of Caserta (Italy). Its water distribution network has low original resilience index \(I_r = 0.351\), mean node pressure \(h_{mean} = 31.05\), maximum node pressure \(h_{max} = 50.47\) and minimum node pressure \(h_{min} = 21.36\) m of head.

The kind of water consumption in Parete is exclusively residential with most houses, built in the 1970s and 1980s, with three to four floors, and the network is supplied by two sources. Each step of the proposed methodology is illustrated in detail in the case study of the Parete network. The network model considered 182 nodes and 282 links.

After the network hydraulic simulation of the water system of Parete an oriented graph was obtained and the matrices \(A_{xy}\) and \(W_{xy}\) were defined (step a). Then, by applying the Dijkstra algorithm, all MDPPs from sources \(s = 1\) and \(s = 2\) to each network node were found, obtaining two sets: \([p]_{min}^1\) and \([p]_{min}^2\) (step b). In Figure 2, the two tree graphs corresponding to the MDPPs, respectively for sources \(s = 1\) (Figure 2(a)) and \(s = 2\) (Figure 2(b)), are illustrated. As already noted, each source reached only a part of the \(n\) network nodes generating two subgraphs \(G_1 = (V_1,E_1)\) with \(V_1 = 142\) and \(G_2 = (V_2,E_2)\) with \(V_2 = 82\). It is worth noticing that the sum of \(V_1\) and \(V_2\) is higher than 182, i.e., higher than the total number of network nodes of Parete, because of the fact that some nodes can be reached from both sources (i.e., belong to both \(G_1\) and \(G_2\) (common nodes).

The next step (c) consists of identifying the common nodes that define the set \([C_1]\). In this case study \([C_1]\) is composed of 42 nodes, indicated by squares in Figure 2. The
nodes of sets $\{\mathcal{C}\}$ and $\{U_1\}$ are indicated by circles in the same figure.

Then, applying the heuristic procedure of step (d) the set $\{\mathcal{C}_1\}$ was subdivided into two subsystems: $\{\mathcal{C}_{11}\}$, composed of 16 nodes (illustrated by dashed boundary in Figure 3(a)), and $\{\mathcal{C}_{12}\}$, composed of 26 nodes (illustrated by dash-dot boundary in Figure 3(a)). In this way it was possible to define the first i-DMA (i-DMA1) $=\{\mathcal{C}\} \cup \{\mathcal{C}_{11}\}$, indicated by number 1 in Figure 3(b). In this case, with only two sources, it is obviously not necessary to come back to step (c) and the second isolated sector is simply i-DMA2 $=\{U_1\} \cup \{\mathcal{C}_{12}\}$ obtained by inserting boundary valves in all the links located between the two i-DMAs (step e). The procedure computed $N_{bv} = 9$ boundary valves to sectorize the network into two independent i-DMAs, as illustrated in Figure 3(b). The energy performance indices of the sectorized network are very good, with small alteration of the resilience index: $I_r = 0.322$, corresponding to $I_{rd} < 9\%$.

This good behaviour is confirmed also by the pressure indices reported in Table 1 for each i-DMA, comparing their values for the WNS and for the Original Water Network (OWN). In particular, the mean (29.74 m) and minimum (23.74 m) pressure heads of i-DMA1 after the sectorization increased compared with the OWN layout (27.64 and 21.56 m respectively); while i-DMA2 shows the inverse situation with more predictable decrease of pressure values (34.11 and 26.99 m respectively), although they are still higher than the design pressure head $h^* = 25$ m.

The city of Matamoros, illustrated in Figure 4, is located in the northeast part of the state of Tamaulipas, Mexico. The climate is semi-dry, with hot summers and cold winters (temperature range $-7$ to $40$ °C, annual precipitation 687.2 mm). The number of service connections to the city water distribution network is about 120,000.

The only water supply source is the Rio Grande River. Water is taken from the river at two points, and there are four water treatment plants. The distribution network is between 40 and 50 years old and interconnects the water sources and tanks. There are three large water tanks in the city (two of them with a capacity of 4,000 m³, and one with a capacity of 3,800 m³). The service provided to water users is supposed to be continuous (24/7), in the sense that no valves in the distribution network are opened and closed during the day. Water pressure in an important portion of the city is so low during some hours of the day, however, that the corresponding water users do not receive water during these hours and water supply is in fact intermittent for them. Implementing a model of the entire water distribution network was needed in order to

![Figure 3](https://example.com/figure3.png)

**Figure 3** | Different sets (a), and i-DMAs (b) of Parete network obtained by the proposed methodology.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Pressure indices of Parete network</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nodes</td>
<td>OWN $h_{\text{mean}}$ [m]</td>
</tr>
<tr>
<td>i-DMA1</td>
<td>116</td>
</tr>
<tr>
<td>i-DMA2</td>
<td>66</td>
</tr>
</tbody>
</table>

![Figure 4](https://example.com/figure4.png)

**Figure 4** | Matamoros Network with i-DMAs.
analyse the present situation and possible sectorization alternatives. The sectorization project considered the city’s population growth for 20 years from now.

The proposed procedure was also applied to this case study. The network model comprised 1,283 nodes, 1,651 links and nine source nodes represented in the Epanet program as reservoirs (fixed head nodes, three of them are water tanks and six are pumps represented by the head they provide).

The water distribution network has a low original resilience index $I_r = 0.439$ (with design pressure head of 12 m) and low mean node pressure head $h_{\text{mean}} = 17.46$ m and minimum node pressure head $h_{\text{min}} = 3.93$ m, but with a high value of maximum node pressure head $h_{\text{max}} = 51.34$ m. In Figure 4 all 9 i-DMAs are shown; each isolated district corresponds to one source.

The results are excellent, with $N_{\text{bw}} = 56$. The worsening of hydraulic performance is negligible, with $I_r = 0.429$, corresponding to $I_d = 2.3\%$. As reported in Table 2, the WNS slightly improved the low pressure values of i-DM2 (from 2.95 to 3.11) while pressure reduction can be observed in i-DM6 (from 4.96 to 3.11 m) and i-DM7 (from 13.25 to 9.15) which already had low pressure values (below the design pressure head) in the OWN. However this situation is local and refers only to one or two nodes in the districts. The mean pressure is also good after sectorization: i-DM2 (from 13.77 to 13.79 m), i-DM6 (from 11.90 to 10.52 m) and i-DM7 (from 17.51 to 15.70 m). It is evident that more actions were needed in order to achieve the design pressure head in all nodes of i-DMs 2, 6 and 7, such as increasing source heads – but their description is beyond the scope of this paper. The number of nodes for each i-DMA is reported in Table 2. It is worth noticing that this number ranges from 35 up to 352 nodes; this result depends essentially on the objective function used which has no constraints on the number of nodes but only on the network mean pressure. This simple objective function was a very useful and ‘smart’ one; indeed i-DMA 1, composed of a few nodes, corresponds to the whole industrial district of the city and the procedure was able to identify automatically the peculiarity of this district that had a larger water demand compared with the other districts that are mainly residential.

CONCLUSIONS

The methodology proposed, based on graph theory and heuristic procedure, is capable of identifying a ‘cost-effective’ configuration of i-DMs. Specifically MDPPs and a GA were used to define single i-DMs compatible with the level of service for the users. Two water networks were analysed using performance indices based on energy and pressure. The simulation results confirm the effectiveness of the proposed procedure with negligible alterations of hydraulic performance for both networks. The objective function used in the GA shows smart behaviour because nodes with different types of users were recognized and inserted in the corresponding i-DMA. The proposed procedure represents an effective tool for water utilities interested in sectorization of large water networks.

REFERENCES


Skiena, S. 1990 Implementing Discrete Mathematics: Combinatorics and Graph Theory with Mathematica. Addison-Wesley, Redwood City, CA.


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