**First-Passage Time in a Random Vibrational System**

M. SHINOZUKA. The author ought to be congratulated on developing a new approach of solving one of the most difficult problems in stochastic mechanics.

The writer has also been interested in the first-passage time problem and derived [1] in an entirely different approach upper and lower bounds for the probability \( P(\tau; t) = \lambda_1 \lambda_2 > 0 \), that a stochastic process \( y(t) \) exits from the strip \( -\lambda_1 \leq y(t) \leq \lambda_2 \) at least once in the interval \( 0 \leq t \leq T \). Later, the method is extended [2] to include the case where \( \lambda_1 \) and \( \lambda_2 \) are functions of time.

The method can apply to a wide class of random processes; the usual assumptions that \( y(t) \) is Gaussian or stationary and that \( n(t) \) in (1) is white or almost white are not needed. Admittedly, the universality of the method is achieved in the expense that the bounds are not necessarily close, in particular, when \( y(t) \) is nonstationary.

However, when the input \( n(t) \), and therefore usually the response, are sharply nonstationary, as may be seen in the ground acceleration due to earthquake and the structural response to it, the method can be conveniently employed.

In the hope that it is of some interest to those who are concerned with this type of problem, the writer wishes to present the result of his recent computation concerning earthquakes. Since the ground velocity eventually has to approach zero, the ground acceleration \( -n(t) \) is assumed to be \( \xi(t) \) where \( \xi(t) \) is the output of the following differential equation that might characterize the mechanical behavior of the ground.

\[
\xi(t) + 2\mu \xi(t) + \omega_n^2 \xi(t) = \psi(t) N(t)
\]

where \( \psi(t) = (e^{-\beta t} - e^{-\beta t}) / \beta > 0 \) for \( t > 0 \) and \( \psi(t) = 0 \) for \( t < 0 \) and \( N(t) \) is a Gaussian white noise with \( \langle N(t)N(t + \tau) \rangle = D \beta(\tau) \). Without difficulty, it can be shown that \( \xi(t) \to 0 \) as \( t \to \infty \). The choice of the parameters \( \alpha = 0.25/\text{sec}, \beta = 0.5/\text{sec}, D = 3.14 \times 10^4 \text{ft}^2/\text{sec}^3, \omega_n = 12.3/\text{sec}, \mu = 3.86/\text{sec}, w = 10/\text{sec}, \) and \( b = 0.05 \) results in: (a) The duration of significant ground disturbance is about 13 sec, (b) the maximum of the standard deviation of the ground acceleration is 11 percent of the acceleration due to gravity occurring at \( t = 2.8 \) sec, and (c) the (apparently) stationary part of the assumed ground acceleration process reproduces the autocovariance function from the observed record of the acceleration in a good agreement.

The upper and lower bounds of the probability of failure \( P(\tau; t) = \lambda_1 \lambda_2 > 0 \) are shown in Fig. 1 together with the plot of some of the maximum response based on fifty numerically simulated response processes \( y(t) \).

**References**


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**On Certain Approximations in Sandwich Plate Analysis**

C. E. S. UENG. The writer wishes to commend the author on his reported results concerning the five different approximations in sandwich plate analysis. This gives certain guidance for designers who face such a layered plate problem which involves so many parameters. It is interesting to note that approximations II and III, namely, the core resists only the shearing strains \( \tau_{xy} \), and the facings are treated as membranes, respectively, are good for most ordinary sandwich plates. This quantitative information not only verifies the prediction made by many previous investigators, but also makes it possible for such a problem to be manageable by engineers. As pointed out by the author, it is instructive to know that approximation IV errs most greatly for values of \( G_s/E \) that may be of practical interest. It seems that only very few were aware of this fact. The writer recently found that an upward deflection at the center of a uniformly loaded, simply supported rectangular sandwich plate would result from a similar approximation as IV in the paper. The ratios of \( G_s/E \) and \( t/h \) were not uncommon and the result is...

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**Authors’ Closure**

The authors appreciate the comments made by Professor Wallis, which point out the analogy between tube flow with suction and with blowing. In this connection, it may be worth mentioning that nearly all of the pressure drop along a porous tube with fluid injection and zero entrance velocity is due to a momentum increase. Wall friction contributes only very little.
certainly inconsistent with the physical viewpoint. Hence, special caution has to be exercised.

In addition, if the load intensity is kept constant but the ratio $G_c/E$ decreases, i.e., the core becomes softer and softer, the linear behavior between the deflection and load reduces significantly to the order of $t$. A finite deflection theory is therefore needed to give a better solution.

There are a few misprints in the paper, namely, the first term in equation (5) should be $T_i$ instead of $T_i$; the boundary conditions given by equation (8) are obvious for a simply supported case but not mentioned in the paper—the displacement functions given by equation (9) are for such a case; the first term in the first condition in (8) should be $u_i$ instead of $u_i$; and in Fig. 4 the arrowheads for the upper two curves should be interchanged.

**Author's Closure**

The author thanks Professor Ueng for his comments and the interest he has shown in the paper. He is correct about the misprints in equations (5) and (8); the author regrets having let them slip by. However, Fig. 4 (and its companion, Fig. 5) correctly represents the calculated results: no interchange of arrowheads should be made. Equations (8) are perhaps 'obvious' for a simply supported sandwich plate, but it seemed worthwhile to mention the discussion in reference [9] concerning how these equations relate to practical construction. (Another misprint has been noted: In equation (14), $E/E_e$ should read $E/E$.)

The upward center deflection found by Professor Ueng is surprising. It would be interesting to hear more about this problem.