Relativistic Effects in Transverse Flow in the Molecular Dynamics Framework

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(Received January 16, 1995)

In order to investigate relativistic effects we compare the transverse flow calculated using four versions of the QMD approaches with that of the full covariant RQMD approach. We conclude that the simplified RQMD (RQMD/S), which uses a common time coordinate for all particles, can be used instead of RQMD up to 6 GeV/u.

The Quantum Molecular Dynamics (QMD) approach is one of the most powerful models to describe heavy-ion\(^1\) and light-ion\(^2\) reactions in the several tens to the several hundreds of MeV/u energy region. In the high energy region above about 1 GeV/u, however, we must introduce the Lorentz covariant version of the QMD approach because relativistic effects become significant.

As an extension of the non-covariant QMD to the relativistic energy region, one may introduce relativistic kinematics and the Lorentz contracted phase-space distribution of the initial nuclei. However these immediately cause spurious excitation and the unphysical instability of the initial nuclei since the increase of the initial density due to the Lorentz contraction makes an additional repulsion through the density-dependent force. Furthermore, we cannot correctly evaluate the internal energies of fast-moving fragments at the end of the QMD calculation because the non-relativistic mean-field used in QMD is variant under the Lorentz transformation. In fact, these effects clearly appear in the transverse flow\(^3,4\) and in the multiplicity of alpha particles in heavy-ion collisions even at \(E_{\text{lab}} \approx 1\) GeV/u.\(^5\) At relativistic energy, therefore, the Lorentz covariant transport approach is desirable to make all nuclei and fragments possess consistent phase-space distributions under the Lorentz transformation.

The Relativistic QMD (RQMD) approach\(^6,7\) is the most useful theoretical model for this purpose; it is formulated to describe the interacting \(N\)-body system in a fully Lorentz covariant way based on the Poincaré-Invariant Constrained Hamiltonian Dynamics.\(^8\) This RQMD approach is now used by many researchers to analyze heavy-ion collisions at ultra-relativistic energies. There are, however, two kinds of difficulties in applying the RQMD approach to the analysis of experimental data in the several GeV energy region. One is that it requires too long a CPU time for a large system, and the other is that it is almost impossible in a practical sense to satisfy
energy conservation strictly. The latter problem is in particular very serious in studying the fragmentation process because fragment multiplicities are dominantly determined by the excitation energy of the residual nucleus in this energy region. In order to describe nuclear dynamics in a consistent way from the initial stage to the final time stage (except the statistical decay), where most of the time evolution of reactions proceed in the relatively low energy regime, we must solve these difficulties. In this paper, then, we discuss relativistic effects and propose a simplified version of RQMD (RQMD/S) to solve the above difficulties.

First we briefly explain the RQMD approach in the following. The position and momentum coordinates of the \( i \)-th particle, \( q_i \) and \( p_i \), are defined as four-dimensional dynamical variables and the functions of the time evolution parameter \( \tau \). The equations of motion are defined by the on-mass-shell constraints and the time-fixations.\(^{6,8}\) The actual forms of these two constraints are somewhat arbitrary, but they are restricted by the non-relativistic limit and cluster separability.\(^{6,8}\) The former implies that these constrains should be consistent in the non-relativistic framework in the non-relativistic limit, and the latter indicates that motion of a single particle in a cluster should not influence other particles in different clusters.

The on-mass-shell constraints are given\(^{6}\) by

\[
H_i = p_i^2 - m_i^2 - 2m_i U_i = 0,
\]

where \( m_i \) and \( U_i(q_i, p_i) \) are the mass and the Lorentz scalar quasi-potential for the \( i \)-th particle. The explicit form of the quasi-potential is determined by the requirement that it corresponds to the non-relativistic mean-field in the low energy limit.\(^{6,7}\) The argument of the potential is the square of the relative distance \( d_{ij} \) between two particles in the non-relativistic framework. In RQMD, on the other hand, it is taken as a square of the relative distance in the rest frame of their CM system as

\[
-q_{ij}^2 = -q_{ij}^2 + \frac{(q_{ij}, p_{ij})^2}{p_{ij}^0},
\]

with

\[
q_{ij} = q_i - q_j, \quad p_{ij} = p_i + p_j.
\]

The above expression includes some important effects of the Lorentz covariance as follows. First, the quasi-potential is given as the Lorentz scalar, while it is given as a time coordinate of the Lorentz vector in the usual QMD approach. Second, the change from \( r_{ij}^2 \) to \(-q_{ij}^2\) causes direction dependent forces. Then the attractive force in the fast moving nuclei is stronger in the moving direction than in the transverse direction. The above two effects keep the phase-space distribution stable after the Lorentz contraction in the fast moving matter. Without Lorentz covariance in the mean-field, however, we cannot correctly describe the intrinsic motion of nucleons, which is relatively low energy phenomena in the fast moving nuclei.

In the RQMD formulation, the time coordinate \( q_i^0 \) is distinguished from the time evolution parameter \( \tau \) and constrained by the time-fixation. In order to avoid the breaking of Lorentz covariance caused by two-body-collisions, Sorge et al. have taken the time-fixation as
Under this time-fixation the two colliding particles have equal values of their time coordinates in their CM frame in the dilute gas limit.

One of the reasons for the excessively long CPU time of RQMD calculations is that we must evaluate the inversion of the \((n \times n)\) matrix at every time-step to solve equations of motion under the above constraints. This process requires a CPU time almost ten times longer than that of normal QMD calculations. The other reason is related to the change of the particle number due to the meson production and absorption. In the above formulation, the change of the particle number breaks the time-fixation, so that we must again solve the values of \(q_i^0\) and \(p_i^0\) under the above two constraints. This is also a time-consuming process. Furthermore, this procedure leads to a discontinuity in the particle coordinates and then causes the change of the potential energy. Usually this energy deviation is neglected and the values of \(q_i^0\) and \(p_i^0\) are not resolved exactly at every time step when the particle number is changed. Therefore it is almost impossible in a practical sense to satisfy the energy conservation strictly, as mentioned before.

In order to avoid these difficulties, we propose the simplified version of RQMD (RQMD/S) for the several GeV energy region in the following way.

First, we introduce a simple form of the time-fixation instead of Eq. (4). The choice of the time-fixation is arbitrary except for the non-relativistic limit and the cluster separability. We then adopt the time-fixations to equalize the time evolution parameter \(\tau\) and all time coordinates of baryons and mesons as

\[
\chi_i = \bar{a} \cdot (q_i - q_N) = 0, \quad (i = 1 \sim N - 1)
\]

\[
\chi_N = \bar{a} \cdot q_N - \tau = 0,
\]

where \(\bar{a}\) is a four-dimensional unit-vector taken as \((1; 0)\) in a reference frame. For the transformation from one frame to the other, the unit-vector \(\bar{a}\) must be changed under the Lorentz transformation to maintain the Lorentz covariance. Thus these time-fixations are still defined Lorentz covariantly, though the 0-th component of the position coordinate \(q_i^0\) is not equivalent to the time evolution parameter \(\tau\), except in the reference frame. If \(\bar{a}\) is also taken as \((1; 0)\) in the other frame, the final result in this frame is, in principle, not equivalent to that in the previous one, because the calculations are performed under different time-fixations. However, it is pointed out\(^9\) that the choice of the time-fixations does not strongly affect the particle motions as long as the \(q_i^0\) are taken as arguments of the quasi-potential \(U_i\). Hence our formulation maintains the frame-independence approximately even if \(\bar{a}\) is taken as \((1; 0)\) in any frame.

By this simplified choice of the time-fixations, the change in particle number does not break the time-fixations. Therefore, the energy conservation after the change of particle number is always satisfied. Instead of this advantage, this choice results in a loss of Lorentz covariance in the two-body-collisions. We thus use Kodama's prescription\(^9\) in the two-body-collisions, which can approximately maintain the Lorentz covariance in the several GeV/u energy region\(^{10}\) under our scheme with a
common time coordinate for all particles.

Second, we replace the momentum coordinate \( p_i = \sqrt{p_i^2 + m_i^2} + 2m_i U_i \) in the argument of the quasi-potential \( U_i \) by the kinetic energy \( \varepsilon_i = \sqrt{p_i^2 + m_i^2} \). This approximation does not affect the final results a great deal because the quasi-potential is much smaller than the kinetic energy in the relativistic energy region. Using this approximation, in addition to the simplified choice of the time-fixations, we can analytically solve the matrix inversion mentioned above reducing the CPU time to the same order as in the normal QMD.

Now we examine the above simplified RQMD (RQMD/S) through discussions on relativistic effects. For this purpose we propose five kinds of methods as follows. The first is the standard QMD, so-called QMD/G, which does not include the Lorentz contraction of the initial distribution. The second is the QMD/L, which is QMD including the Lorentz contracted initial distribution. The third is the QMD/R, where the mean-field is treated as a time-component of the vector type, but the argument is varied along Eqs. (2) and (3). The fourth is the RQMD/S, which is the simplified RQMD explained above. The last one is the full RQMD.

Here we calculate the directed transverse momentum, which is most sensitive to the relativistic effects, defined as

\[
\langle P_{x, \text{dir}} \rangle = \frac{1}{A} \sum_{i=1}^{4} \text{sign}[Y_{CM}(i)] p_x(i),
\]

where \( Y_{CM}(i) \) and \( p_x(i) \) are the center-of-mass rapidity and the transverse momentum in the reaction plane of the \( i \)-th nucleon.

In the numerical calculations the predictor-corrector method is used to integrate the equation of motion. For the two-body effective interaction, we use a Skyrme-type interaction with HARD EOS (incompressibility \( K = 380 \text{ MeV} \)). The widths of the wave packets are taken from the values for Ca + Ca in Ref. 11. The initial nuclei are given with the cooling method. We omit the Coulomb force and two-body collision term for simplicity because our purpose is to examine the relativistic effects.

In fact the two-body collision is shown in Ref. 13) not to affect the transverse flow significantly in the high energy region.

In Fig. 1 we plot the energy dependence of \( \langle P_{x, \text{dir}} \rangle \) in the QMD/G, QMD/R and RQMD/S simulation as the difference from that of RQMD for \( {}^{40}\text{Ca} + {}^{40}\text{Ca} \) reactions at \( b = 2 \text{ fm} \), for the energy range from 150 MeV/u to 6 GeV/u.
As mentioned previously, the Lorentz contraction of the initial phase space distribution increases the flow, which is shown by the change from QMD/G (dot-dashed line) to QMD/L (dashed line). By the full covariant treatment of the interaction, however, this effect is counterbalanced with the Lorentz covariance of the mean-field, but it still remains an increased flow from the QMD/G value.\(^{41}\)

As seen in Fig. 1, the prescription of QMD/R does not deviate largely from the full covariant treatment up to 3 GeV/u. At much higher energy, however, the result of QMD/R decreases linearly from that of RQMD. On the other hand, RQMD/S gives results very similar to RQMD up to 6 GeV/u and beyond.

The difference between the RQMD (RQMD/S) and QMD/R comes from the different treatment of the potential, a Lorentz scalar type in the former, while a time-component of the vector type in the latter. This is understood qualitatively by considering single particle motion under a fixed external potential \(U\). In the Lorentz scalar treatment of the potential \(U\), the single particle energy \(p_i^0\) is expressed in this simple case as

\[
p_i^0 = \sqrt{p_i^2 + m_i^2 + 2m_iU}.
\]  

Accordingly, the equation of motion is

\[
\dot{p}_i = -\sum_j \frac{m_j}{p_j^0} \frac{\partial U_j}{\partial r_i}.
\]  

On the other hand, in QMD/R they are

\[
p_i^0 = \sqrt{p_i^2 + m_i^2 + U_i}
\]  

and

\[
\dot{p}_i = -\sum_j \frac{\partial U_j}{\partial r_i}.
\]

The form of \(\partial U_j/\partial r_i\), which is attractive at the early time stage of nucleus-nucleus collisions, is almost the same in the QMD/R and RQMD (RQMD/S). In the high energy region, the single particle energy \(p_i^0\) has approximately the same value for all nucleons in the early time stage. Thus the force of QMD/R becomes larger and deviates linearly from that of the RQMD (RQMD/S) as energy increases. Above 3 GeV/u, hence, the difference between the Lorentz scalar and the time component of the Lorentz vector becomes significant, and the full covariant prescription for the mean-field is necessary to describe the reactions, particularly the nucleus-nucleus collisions.

There is a visible difference between the two results of RQMD and RQMD/S. This difference must arise from the different definitions of the time-fixation Eqs. (4) and (5), but it is negligibly small in comparison with the results of the other approaches. From this we can confirm that the final results do not significantly depend on the time-fixation.

In the above discussion we have considered the relativistic effects only in the mean-field, not to that in the two-body collisions, because our simplification is not
believed to involve any significant problems around several GeV/u. In order to confirm this, finally, we calculate the rapidity distribution in the central Ca + Ca collisions at $E_{lab} = 5$ GeV/u using the full RQMD and RQMD/S approaches with the two-body collision term. The two-body collision is thought to be important for the stopping power. Here the Cugnon parametrization is adopted for the baryon-baryon cross-sections. The results are shown in Fig. 2. We can see that the two results are equivalent within the statistics.

In summary we have calculated the directed transverse momentum using five kinds of (R)QMD approaches and have discussed their relativistic effects. It has been shown that the Lorentz covariance of the mean-field is very important above approximately the 1 GeV energy region. The RQMD/S can give almost the same results as the full RQMD in the transverse flow, which is thought to be the most sensitive observables to the relativistic effect at present. In addition we have confirmed that the two treatments give the same results in the rapidity distribution if including the two-body collisions. Hence we can conclude that the RQMD/S approach can be used up to 6 GeV instead of the full RQMD approach.

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