Composite Weak Vector Bosons in a Left-Right Symmetric Preon Model

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We take the viewpoint that the standard model is a low energy effective theory among composite quarks, leptons and weak bosons in a left-right (LR) symmetric preon model with a hypercolor $SU(N)_{HC}$ gauge interaction. Starting from NJL-type interactions with global $SU(2)_h \times SU(2)_s$ symmetry, we construct the composite weak vector bosons from a pair of spinor preons and derive their effective interactions with quarks and leptons, which are essentially identical, at the tree-diagram level, to those in the LR symmetric gauge model. Through the process of this approach, some physical aspects of the LR gauge model are clarified.

§ 1. Introduction

The success of the standard model (SM) of $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge symmetry becomes more remarkable as experiments attain higher precision. However, it seems to us that SM is still unsatisfactory theoretically to be truly a fundamental theory. As is well known, there remain many problems to be solved: i) The Higgs sector is considered to be somewhat phenomenological and contains too many free parameters. ii) The physical origin of repetition of three generations is not known. iii) In SM there exists a maximal left-right asymmetry notwithstanding the fact that quarks and leptons have both left- and right-handed degrees of freedom (note that neutrinos should be massive, at least, to explain the oscillation phenomena).

In this work we shall follow the line of approach to these problems in which SM is considered as a low energy effective theory among composite particles (quarks and leptons, and possibly also weak and Higgs bosons) of preons confined by some fundamental (hypercolor) gauge interaction like QCD. Here, we should like to point out that there is a possibility to solve all the above problems along this line, although it seems to be an extremely difficult task to find such a realistic model as may pass through the recent high-precision experimental check. On the other hand, in most of the more conventional approaches, the super-symmetric generalizations of SM, the above problems seem to remain unsolved.

There are many pioneering works\cite{11-16} in the above line of preon models from various viewpoints. Particularly we prefer those models\cite{11-13} with symmetrical left- and right-freedom, considering problem iii) seriously. In a previous paper\cite{13} we adopted a fermion-boson-type preon model\cite{1,3} with massive Dirac spinor preons $F$, where the weak vector bosons, $W_L$ and $W_R$, in the respective worlds of left-hand ($h=L$) and of right-hand ($h=R$) are expected to be composites of a pair of $F_L$ and $F_R$, and investigated the possibility for the existence of extra iso-scalar vector bosons phenomenologically.

In this paper we shall further attempt to develop a realistic preon model from the
same standpoint theoretically, by treating the composition of weak bosons dynamically. In particular we shall investigate the physical background of the left-right (LR) symmetric gauge model, which are generally accepted as a natural extension of SM in relation to the above-mentioned problem iii). For this purpose we shall resort to a solvable model of the Nambu-Jona-Lasinio type (NJL) adapted for our relevant global symmetry of the confining gauge interaction. The composite model of the NJL type with the vector-vector interaction had been proposed in the cases of $SU(2)_L$ and of $SU(N)$, but in those cases without reference to LR symmetry.

§ 2. Basic set up of the preon model

2A. Preons and composites

In a previous paper, we presented a model scheme for quarks and leptons as bound states of two kinds of Dirac spinor preons $(F^u, F^d)$ and of four kinds of scalar preons $(C^{(i)}, S^{(i)})$ carrying the generation number $(i=1, 2, 3)$. These preons have the fundamental local gauge interaction, which is symmetric under the group $G_{loc} = SU(N)_{HC} \times SU(3)_c \times U(1)_{em}$, and are supposed to belong to the representation as shown in Table I. The preons are also supposed to belong to the representations of the group $G_{gl} = SU(2)_L \times SU(2)_R \times U(1)_B \times U(1)_Y$ of the global symmetry, as shown in Table II. Then the electric charge is given by the LR-symmetric formula, giving a clear physical meaning to the hypercharge as

$$Q = I_L^3 + I_R^3 + \frac{B - L}{2},$$

$$Y = I_R^3 + \frac{B - L}{2},$$

Table I. Local quantum numbers of the preons.

<table>
<thead>
<tr>
<th>Preon</th>
<th>$SU(N)_{HC} \times SU(3)<em>c \times U(1)</em>{em}$ representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F^u$</td>
<td>$(N_{HC}, 1, 1/2)$</td>
</tr>
<tr>
<td>$F^d$</td>
<td>$(N_{HC}, 1, -1/2)$</td>
</tr>
<tr>
<td>$C^{(i)}$</td>
<td>$(N_{HC}, 3^*, -1/6)$</td>
</tr>
<tr>
<td>$S^{(i)}$</td>
<td>$(N_{HC}, 1, 1/2)$</td>
</tr>
</tbody>
</table>

Table II. Global quantum numbers of the preons.

<table>
<thead>
<tr>
<th>Preon</th>
<th>$SU(2)_L \times SU(2)_R \times U(1)_B \times U(1)_Y$ representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(F^u)_L$</td>
<td>$(2, 1, 0, 0)$</td>
</tr>
<tr>
<td>$(F^u)_R$</td>
<td>$(1, 2, 0, 0)$</td>
</tr>
<tr>
<td>$C^{(i)}$</td>
<td>$(1, 1, -1/3, 0)$</td>
</tr>
<tr>
<td>$S^{(i)}$</td>
<td>$(1, 1, 0, -1)$</td>
</tr>
</tbody>
</table>
where $I_L^{(R)}$ is the 3rd component of the weak iso-spin $SU(2)_{L(R)}$, and $B(L)$ is the baryon (lepton) number. Here it may be worthwhile to note that only the scalar preons have non-zero $B$ and $L$.

The quarks and leptons are the hypercolor singlet composite states with preon configurations:

\[
\begin{pmatrix}
u_e \\
\end{pmatrix} = \begin{pmatrix}
F^{\mu} \bar{S}^{(1)}_\mu \\
F^d \bar{S}^{(1)}_d
\end{pmatrix}, \quad
\begin{pmatrix}
\nu_\mu \\
\mu^-
\end{pmatrix} = \begin{pmatrix}
F^{\mu} \bar{S}^{(2)}_\mu \\
F^d \bar{S}^{(2)}_d
\end{pmatrix}, \quad
\begin{pmatrix}
\bar{\tau}^-
\end{pmatrix} = \begin{pmatrix}
F^{\mu} \bar{S}^{(3)}_\mu \\
F^d \bar{S}^{(3)}_d
\end{pmatrix}. \tag{2.2b}
\]

The weak vector bosons are built from a pair of spinor preons $\psi$ and $\bar{\psi}$ as

\[
I_L = 1; W_{L\mu} \approx \bar{\psi}_L \gamma_\mu \psi_L, \quad I_L = 0; W_{L\mu}^0 \approx \bar{\psi}_L \gamma_\mu \psi_L \tag{2.3a}
\]

and

\[
I_R = 1; W_{R\mu} \approx \bar{\psi}_R \gamma_\mu \psi_R, \quad I_R = 0; W_{R\mu}^0 \approx \bar{\psi}_R \gamma_\mu \psi_R \tag{2.3b}
\]

where

\[
\psi = \begin{pmatrix}
F^{+} \\
F^{-}
\end{pmatrix}. \tag{2.4}
\]

Here it is to be noted that in our model the extra weak bosons (iso-vector $W_R$ and iso-scalar $W_L^0$ and $W_R^0$), in addition to the usual iso-vector bosons $W_L$, are naturally expected to exist. We have investigated the lower limit of the mass of iso-scalar $W_L^0$ permitted from low energy experiments in our previous paper.13)

2B. Fundamental gauge interactions for the preon system

From the above, we see that the Lagrangian density for our fundamental local $G_{HC}$ gauge interactions is given as

\[
L = -\frac{1}{4} F^\mu_\nu F^\nu_\mu - \frac{1}{4} g^{a}_{\mu\nu} a^{a}_{\mu\nu} - \frac{1}{4} G^{a}_{\mu\nu} G^{a}_{\mu\nu} + \bar{\psi} i \gamma^{\mu} \left( \partial_\mu + ie \frac{\lambda^{a}}{2} A^{a}_{\mu} + ig_{NC} \frac{\lambda^{a}}{2} G^{a}_{\mu} \right) \psi
\]

\[
+ \left| \left( \partial_\mu - e \frac{1}{6} A^{a}_{\mu} - g_{NC} \frac{\lambda^{a}}{2} G^{a}_{\mu} \right) C^{(i)} \right|^2
\]

\[
+ \left| \left( \partial_\mu + e \frac{1}{2} A^{a}_{\mu} - g_{NC} \frac{\lambda^{a}}{2} G^{a}_{\mu} \right) S^{(i)} \right|^2, \tag{2.5}
\]

where the electromagnetic $U(1)_{em}$, the color $SU(3)_c$ and the hypercolor $SU(N)_{HC}$ field strength are defined, respectively, from their vector fields as

\[
F^\mu_\nu = \partial_\mu A^0_\nu - \partial_\nu A^0_\mu, \tag{2.6a}
\]

\[
g^{a}_{\mu\nu} = \partial_\mu a^{a}_{\nu} - \partial_\nu a^{a}_{\mu} + gc^{abc} g^{b}_{\mu} g^{c}_{\nu}, \tag{2.6b}
\]

and
Here it is to be noted that this $L$, Eq. (2.5), has a symmetry under $G_{ct}$, and in particular under the global chiral $SU(2)_L \times SU(2)_R$ transformation.

§ 3. Derivation of effective interactions among composite quarks, leptons and weak bosons

3A. NJL type effective interactions

In this section we shall construct the effective interactions among composite quarks, leptons and weak vector bosons, assuming the situation that our relevant experimental energy $\sqrt{s}$ is in the region far below the composite scale $A_{cs}$ as

$$\sqrt{s} \ll A_{cs}.$$ (3.1)

For this purpose, we start from solvable NJL type interactions of spinor preons with our relevant global symmetry $G_{ct}$ of the fundamental gauge interaction. We consider only the non-differential contact interaction of preons. In this paper we shall focus on the composition of the weak vector bosons from a pair consisting of a spinor preon and an anti-preon, and on the derivation of their effective interaction with quarks and leptons.

Thus our relevant Lagrangian density is given by

$$L=\bar{\psi}i\gamma^\mu\left(\partial_\mu + ie\frac{r^3}{2}A^0_\mu\right)\psi - m_F\bar{\psi}\psi - \frac{1}{4}F^a_{\mu \nu}F^{a \mu \nu}$$

$$- \frac{G_L}{N_{hc}}\left(\bar{\psi}\gamma_\mu \frac{r}{2} - \gamma_5 \psi \right)^2 - \frac{G_R}{N_{hc}}\left(\bar{\psi}\gamma_\mu \frac{r}{2} + \gamma_5 \psi \right)^2,$$ (3.2)

where $m_F$ is the mass of spinor preons $\phi$, $G_L$ and $G_R$ are, respectively, the coupling constants among left- and right-handed preons, $A^0_\mu$ is the $U(1)_{em}$ gauge field, and $N_{hc}$ is the hypercolor number. This Lagrangian has the chiral symmetry of $SU(2)_L \times SU(2)_R$ in the limit $m_F \to 0$. The strong attractive force with positively large coupling constant $G_{L(R)}$ can generate bound states in the spin-one and "hypercolor"-singlet states. We shall attempt to identify them with the "gauge" bosons in the LR gauge model (without Higgs scalar bosons).

3B. Lagrangian of weak bosons interacting with preons

Introducing the auxiliary fields by following the conventional procedure, the Lagrangian density (3.2) is rewritten into convenient form for our purposes as

$$L'=\bar{\psi}i\gamma^\mu\left(\partial_\mu + ie\frac{r^3}{2}A^0_\mu + i\tau \cdot \vec{W}_L \frac{1}{2} - \gamma_5 + i\tau \cdot \vec{W}_R \frac{1}{2} + \gamma_5 \right)\psi - m_F\bar{\psi}\psi$$

$$- \frac{1}{4}F^a_{\mu \nu}F^{a \mu \nu} + \frac{N_{hc}}{G_L} \vec{W}_L \cdot \vec{W}_L + \frac{N_{hc}}{G_R} \vec{W}_R \cdot \vec{W}_R,$$ (3.3)

where $\vec{W}_L$($\vec{W}_R$) is an auxiliary field corresponding to the "left-handed(right-handed)" composite weak boson. This Lagrangian has the $SU(2)_L \times SU(2)_R$ local gauge sym-
metry in the limit $m_F \rightarrow 0$, $G_L \rightarrow \infty$ and $G_R \rightarrow \infty$.

The next step is to generate the kinetic and interaction terms of the auxiliary field, $\tilde{W}_L$ and $\tilde{W}_R$, through the quantum effects of spinor preons corresponding to the loop diagrams in Fig. 1. We consider the two cases$^\ast$ of regularization methods for the loop integral, [type I] the usual straight cut (including the quadratic divergence) and [type II] the dimensional regularization (retaining only the logarithmic divergence). The difference between type I and type II appears only in the mass terms of the composite weak bosons. The details of calculations are given in Appendix A. The result is to add to the Lagrangian (3·3) the following terms:

$$\Delta L' = -\frac{1}{4} Z_w \tilde{W}_{L \mu \nu} \cdot \tilde{W}_{L \mu \nu} + \frac{3}{2} m_F^2 Z_w \tilde{W}_{L \mu} \cdot \tilde{W}_{L \mu} - \frac{1}{4} Z_w \tilde{W}_{R \mu \nu} \cdot \tilde{W}_{R \mu \nu}$$

$$+ \frac{3}{2} m_F^2 Z_w \tilde{W}_{R \mu} \cdot \tilde{W}_{R \mu} - \frac{3}{2} m_F^2 Z_w (\tilde{W}_{L \mu} \cdot \tilde{W}_{R \mu} + \tilde{W}_{R \mu} \cdot \tilde{W}_{L \mu})$$

$$- \frac{1}{2} \lambda_L \sqrt{Z_w} \tilde{W}_{L \mu \nu} F_{\mu \nu} - \frac{1}{2} \lambda_R \sqrt{Z_w} \tilde{W}_{R \mu \nu} F_{\mu \nu}$$

$$+ \frac{e}{2} \varepsilon_{3ij} Z_w F_{\mu \nu} \tilde{W}_{L \mu} \tilde{W}_{L \nu} + \frac{e}{2} \varepsilon_{3ij} Z_w F_{\mu \nu} \tilde{W}_{R \mu} \tilde{W}_{R \nu},$$

(3·4)

where

$$\tilde{W}_{L \mu \nu} = \partial_\mu \tilde{W}_{L \nu} - \partial_\nu \tilde{W}_{L \mu} - 2 \tilde{W}_{L \mu} \times \tilde{W}_{L \nu},$$

(3·5a)

$$\tilde{W}_{R \mu \nu} = \partial_\mu \tilde{W}_{R \nu} - \partial_\nu \tilde{W}_{R \mu} - 2 \tilde{W}_{R \mu} \times \tilde{W}_{R \nu}.$$  

(3·5b)

The constant $Z_w$ is given by

$$Z_w = \frac{N_c}{12 \pi^2} \ln \frac{\Lambda^2}{m_F^2},$$

(3·6)

where $\Lambda$ is the cutoff of the preon loop integrals. In order to make the kinetic terms in Eq. (3·4) have the proper normalization, we rescale the boson fields as

$$W_{L \mu} = \sqrt{Z_w} \tilde{W}_{L \mu},$$

(3·7a)

$^\ast$ There seems to be no plausible criterion for choosing either in the present framework, where the NJL type interaction (3·2) is treated as an ideal one. Accordingly, our estimate of the weak boson mass is not reliable, depending upon the regularization methods (see Eqs. (3·11) and (3·12)). However, we take this only as a technical difficulty, which may be resolved in the case that the bound state problem is solved, starting directly from the fundamental Lagrangian (2·5).
Consequently we obtain, after adding $\Delta L' (3.4)$ to $L' (3.3)$ and rescaling (3.7), the effective Lagrangian $L_{\text{eff}}$ for the system of spinor preons and the composite vector bosons interacting with $U(1)_{em}$ field:

$$
L_{\text{eff}} = \bar{\psi} i\gamma^\mu D_\mu \psi - m_F \bar{\psi} \gamma^\nu \psi - \frac{1}{4} F^\mu_{\nu\rho\sigma} F^{\mu\nu\rho\sigma} - \frac{1}{4} \mathbf{W}_{L\mu\nu} \cdot \mathbf{W}_L^{\mu\nu} + \frac{1}{2} M_L^2 \mathbf{W}_{L\mu} \cdot \mathbf{W}_L^{\mu} 
$$

$$
- \frac{1}{2} M R^2 \mathbf{W}_{R\mu} \cdot \mathbf{W}_R^{\mu} - \frac{3}{2} m_F^2 (\mathbf{W}_{L\mu} \cdot \mathbf{W}_L^{\mu} + \mathbf{W}_{R\mu} \cdot \mathbf{W}_R^{\mu})
$$

$$
- \frac{1}{2} \frac{e}{g} F^\mu_{\nu\rho\sigma} (\partial^\nu W_L^{3\rho} - \partial^\nu W_L^{3\sigma}) - \frac{1}{2} \frac{e}{g} F^\mu_{\nu\rho\sigma} (\partial^\nu W_R^{3\rho} - \partial^\nu W_R^{3\sigma}),
$$

(3.8)

where the covariant derivative $D_\mu$ is defined by

$$
D_\mu = \partial_\mu + ie\frac{\tau^3}{2} A_\mu^0 + ig \frac{\tau_2}{2} W_L \frac{1 - \gamma_5}{2} + ig \frac{\tau_2}{2} W_R \frac{1 + \gamma_5}{2},
$$

(3.9)

and the "gauge" coupling constants of left-handed and right-handed weak bosons $g_L$ and $g_R$, respectively, are given by a common formula,

$$
g_L = g_R = g = \frac{1}{\sqrt{N_{HC}}} \frac{\Lambda^2}{4\pi^2 \ln \frac{\Lambda^2}{m_F^2}},
$$

(3.10)

independently of $G_L$ and $G_R$. The masses of $W_L$ and $W_R$ are given, respectively, in each case of regularization as

[type I]:

$$
M_L^2 = \frac{2 N_{HC}}{G_L Z_W} - \frac{N_{HC} Z_W \Lambda^2}{8 \pi^2} + 3 m_F^2,
$$

(3.11a)

$$
M_R^2 = \frac{2 N_{HC}}{G_R Z_W} - \frac{N_{HC} Z_W \Lambda^2}{8 \pi^2} + 3 m_F^2,
$$

(3.11b)

[type II]:

$$
M_L^2 = \frac{2 N_{HC}}{G_L Z_W} + 3 m_F^2,
$$

(3.12a)

$$
M_R^2 = \frac{2 N_{HC}}{G_R Z_W} + 3 m_F^2.
$$

(3.12b)

It is interesting that the mixing parameters between $A_\mu^0$ and $W_L^2 (W_R^2)$ are also given commonly as

$$
\lambda_L = \frac{e}{g}, \quad \lambda_R = \frac{e}{g}.
$$

(3.13)

The first relation, the so-called unification condition, was required in the current mixing scheme between $A_\mu^0$ and $W_L^2$, while it was derived in the NJL type subquark model (with only global $SU(2)_L$ symmetry), in order to derive similar
neutral current interactions as SM at low-energy, as will be shown later. In the LR gauge model,\textsuperscript{10} inspired by the preon model, relation (3.13) was also supposed. In deriving Eq. (3.8), following the conventional procedure, we kept only the most divergent terms, taking the limit of infinite $\Lambda$, when the finite term becomes zero. It is to be noted that in Eq. (3.8), the preon loop diagrams necessarily generate the current mixing terms between the $A^\mu_0$ and $W^\mu$, which were assumed in the previous works,\textsuperscript{13} and, in addition, the mass mixing terms between $W_{L\mu}$ and $W_{R\mu}$.

3C. Lagrangian of weak bosons interacting with composite quarks and leptons

By attaching the "spectator" scalar preons to the spinor preons (as is seen in Fig. 2), we may simply regard Eq. (3.8) as the Lagrangian of weak bosons interacting with the composite quarks and leptons. Since we are supposing the physical situation of Eq. (3.1), this attaching process seems not to produce any physical effects, such as form factor effects. Thus we obtain our relevant effective Lagrangian for the composite quark, lepton and weak boson system as

$$L_{eff} = \sum_{i=1}^{3} \bar{\psi}_i (i \gamma^\mu \partial_\mu - m_i) \psi_i - \frac{g}{\sqrt{2}} \sum_{i=1}^{3} (\bar{\psi}_{i,L}^\dagger \gamma^\mu \psi_{i,L}^\dagger W_{iL} + \bar{\psi}_{i,L}^\dagger \gamma^\mu \psi_{i,L}^\dagger W_{iL})$$

$$- \frac{g}{\sqrt{2}} \sum_{i=1}^{3} (\bar{\psi}_{i,R}^\dagger \gamma^\mu \psi_{i,R}^\dagger W_{iR} + \bar{\psi}_{i,R}^\dagger \gamma^\mu \psi_{i,R}^\dagger W_{iR}) - \frac{1}{2} W_{L\mu}^\dagger W_{L\mu}^\mu + M_{L}^2 W_{L\mu}^\mu W_{L\mu}^{-\mu}$$

$$- \frac{1}{2} W_{R\mu}^\dagger W_{R\mu}^\mu + M_{R}^2 W_{R\mu}^\mu W_{R\mu}^{-\mu} - 3m_e^2(W_{L\mu}^\dagger W_{L\mu}^{-\mu} + W_{R\mu}^\dagger W_{R\mu}^{-\mu})$$

$$- e \sum_{i=1}^{3} Q_i \bar{\psi}_{i,L}^\dagger \gamma^\mu \psi_{i,L}^0 - g \sum_{i=1}^{3} (\bar{\psi}_{i,R}^\dagger \gamma^\mu \frac{\epsilon^3}{2} \psi_{i,R}^\dagger) W_{R\mu}^\dagger - \frac{1}{4} F_{\mu\nu}^0 F^{0\mu\nu} - \frac{1}{4} W_{L\mu}^3 W_{L\mu}^{3\mu} + \frac{1}{2} M_{L}^2 W_{L\mu}^{3\mu} W_{L\mu}^{3\mu}$$

$$- \frac{1}{4} W_{R\mu}^3 W_{R\mu}^{3\mu} + \frac{1}{2} M_{R}^2 W_{R\mu}^{3\mu} W_{R\mu}^{3\mu} - \frac{3}{2} m_e^2(W_{L\mu}^3 W_{L\mu}^{3\mu} + W_{R\mu}^3 W_{R\mu}^{3\mu})$$

$$- \frac{1}{2} \frac{\epsilon}{g} F_{\mu\nu}^0 (\partial^\mu W_{L}^{3\nu} - \partial^\nu W_{L}^{3\mu}) - \frac{1}{2} \frac{\epsilon}{g} F_{\mu\nu}^0 (\partial^\mu W_{R}^{3\nu} - \partial^\nu W_{R}^{3\mu})$$

$$+ i \frac{\epsilon}{2} F_{\mu\nu}^0 (W_{L}^{-\mu} W_{L}^{-\nu} - W_{L}^{+\mu} W_{L}^{-\nu}) + i \frac{\epsilon}{2} F_{\mu\nu}^0 (W_{R}^{-\mu} W_{R}^{+\nu} - W_{R}^{+\mu} W_{R}^{-\nu}) ,$$

(3.14)

\begin{align*}
(F^\mu C) & \Rightarrow \bar{q} & (F^\mu S) & \Rightarrow I \\
(F^\mu C) & \Rightarrow q & (F^\mu S) & \Rightarrow l
\end{align*}

Fig. 2. Weak bosons interacting with composite quarks and leptons.
where the $m_i$ are the masses of $i$-th generation quarks and leptons generated by hand. Here the quantities are defined as follows: $W^\pm_{i\alpha} = (W^\pm_{i\alpha} + iW^\mp_{i\alpha})/\sqrt{2}$ and $W^0_{i\alpha} = (W^0_{i\alpha} + iW^\pm_{i\alpha})/\sqrt{2}$. $\psi_{i\alpha} = (\psi^L_{i\alpha}, \psi^R_{i\alpha})^T$, $((\nu_{i\alpha}, l_{i\alpha})^T)$ or $(u_{i\alpha}, d_{i\alpha})^T$ is the SU(2)$_L$ doublet of left-handed fermion fields of the $i$-th family, and similarly $\psi_{i\alpha} = (\psi^L_{i\alpha}, \psi^R_{i\alpha})^T$, $((\nu_{i\alpha}, l_{i\alpha})^T)$ or $(u_{i\alpha}, d_{i\alpha})^T$ is the SU(2)$_R$ doublet of right-handed fermion fields, where $d_{i\alpha} = \sum_j V^L_{i\alpha} d_{i\alpha}$ and $d_{i\alpha} = \sum_j V^R_{i\alpha} d_{i\alpha}$.

Here it may be worthwhile to note that the universality of "gauge" interactions through quarks and leptons in the three generations is understood, in our viewpoint, as they are actually the same interactions of common spinor preons inside quarks and leptons, which are discriminated only by their associating spectator scalar preons. $V^L$ is the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. $V^R$ is a similar mixing matrix concerning the right-handed world. This matrix $V^R$ is in our model identical to $V^L$, since the mixing mechanism is determined through "common" scalar preons having the generation number, while it is, in principle, independent of $V^L$ in the LR gauge model. Here it is to be noted that in our model the mixing matrix of the leptons is able to be treated on the same footing as the quark-mixing matrix.

3D. Low-energy effective Lagrangian for composite quarks, leptons and weak bosons

The Lagrangian (3.14) contains the off-diagonal mixing terms being bilinear in boson fields. We shall rewrite it in terms of the physical boson fields with the diagonal mass-matrices. The physical boson fields are obtained by the transformation as

$$
\begin{align}
(W^\pm_{\mu}) &= \left( \begin{array}{cc}
\cos \xi & \sin \xi \\
-\sin \xi & \cos \xi
\end{array} \right) (W^\pm_{\mu}), \\
(A_\mu) &= \left( \begin{array}{cc}
1 & \frac{e}{g} \\
0 & -\sqrt{1-(e/g)^2}
\end{array} \right) (A_\mu), \\
(Z_\mu) &= \left( \begin{array}{cc}
0 & \frac{(e/g)^2}{\sqrt{1-(e/g)^2}} \\
0 & \sqrt{1-(e/g)^2}
\end{array} \right) (Z_\mu),
\end{align}
$$

with $\xi$ defined by

$$
\tan 2\xi = \frac{12m_F^2}{g^2N_{hc}} \left( \frac{1}{G_L} - \frac{1}{G_R} \right),
$$

where we have neglected the quantities of order $O((m_L^2/M_R^2), (m_F^2/M_R^2))$ (see Appendix B). Here $A_\mu$, $W^\pm_{\mu}$, $Z_\mu$, $W^0_{i\alpha}$ and $Z_\alpha$ denote, respectively, physical photons, physical charged weak bosons, physical neutral weak bosons, extra charged weak bosons, and extra neutral weak bosons.

The charged quark and lepton currents are defined by
In a similar way, the electromagnetic and the neutral quark and lepton currents are defined by

\[ J_{\mu}^{\text{em}} = \sum_i Q_i \bar{\psi}_i \gamma_{\mu} \psi_i, \]  
\[ J_{\mu}^{3} = \sum_i \left( \bar{\psi}_i \gamma_{\mu} \frac{\tau^3}{2} \psi_i \right), \]  
\[ J_{\mu}^{3} = \sum_i \left( \bar{\psi}_i \gamma_{\mu} \frac{\tau^3}{2} \psi_i \right). \]

In the following, we shall omit the trilinear and quadrilinear terms in boson fields. Then our relevant effective Lagrangian is given by

\[
L_{\text{eff}} = \int \left( i \gamma^{\mu} \partial_{\mu} - m_F \right) \psi_i - \frac{1}{2} W_{\mu \nu} W^{\mu \nu} + M_w^2 W_{\mu}^+ W_{\mu}^- \\
- \frac{g}{\sqrt{2}} \left( J_L W_{\mu}^+ W_{\mu}^- + \frac{g}{\sqrt{2}} \left( J_R W_{\mu}^+ W_{\mu}^- \right) \right) \\
- \frac{1}{2} W_{\mu \nu} W^{\mu \nu} + M_{F}^2 W_{\mu}^+ W_{\mu}^- \\
- \frac{g}{\sqrt{2}} \left( J_L W_{\mu}^+ W_{\mu}^- + \frac{g}{\sqrt{2}} \left( J_R W_{\mu}^+ W_{\mu}^- \right) \right) \\
- \frac{1}{4} F_{\mu \nu} F^{\mu \nu} - e A_{\mu} J_{\mu}^{\text{em}} - \frac{1}{4} Z_{\mu \nu} Z^{\mu \nu} + \frac{1}{2} M_2^2 Z_{\mu} Z^{\mu} \\
- \frac{e}{\sin \theta_w \cos \theta_w} \left( J_{\mu}^{3} \right) - \frac{e}{\sin^2 \theta_w} \left( J_{\mu}^{3} \right) Z_{\mu} - \frac{1}{4} Z_{\mu \nu} Z^{\mu \nu} + \frac{1}{2} M_2^2 Z_{\mu} Z^{\mu},
\]

where

\[ W_{\mu}^+ = \partial_{\mu} W_{\mu}^+ - \partial_{w} W_{\mu}^+ , \]  
\[ W_{\mu}^+ = \partial_{\mu} W_{\mu}^+ - \partial_{w} W_{\mu}^+ , \]  
\[ Z_{\mu} = \partial_{\mu} Z_{\nu} - \partial_{w} Z_{\nu} , \]  
\[ Z_{\mu} = \partial_{\mu} Z_{\nu} - \partial_{w} Z_{\nu} , \]  
\[ F_{\mu \nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} , \]  
\[ M_w^2 = M_l^2 - 3 m_e^2 \tan 2 \xi , \]  
\[ M_w^2 = M_l^2 + 3 m_e^2 \tan 2 \xi , \]  
\[ M_2^2 = \frac{1}{1 - \sin^2 \theta_w} M_l^2 . \]
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\[ M^2_W = \frac{\cos^2 \theta_W - 2 \sin^2 \theta_W}{1 - 2 \sin^2 \theta_W} M_Z^2, \quad (3.20d) \]

and \( \sin \theta_W = e/g \) is the weak angle. Our Lagrangian (3.18) essentially has a structure similar to that of the LR gauge model without the Higgs bosons at the tree level. It is to be noted that although the charged sector is exactly the same as the LR symmetric model, the neutral sector is only approximately identical. In the limit \( M_w(M_z) \to \infty \), this Lagrangian is identical to that in the electroweak gauge sector of the SM.

§ 4. Concluding remarks

First we summarize our results. In this paper we take the standpoint that the SM is a low energy effective theory among composite quarks, leptons and weak bosons in an LR symmetric preon model with a hypercolor \( SU(N)_{HC} \) gauge interaction. Starting from the NJL-type interactions of massive Dirac preons (mass, \( m_F \)) with global \( SU(2)_L \times SU(2)_R \) symmetry, we have constructed the composite weak vector-bosons of a pair of spinor preons, and derived their effective interactions with quarks and leptons. The main results are as follows: i) The respective coupling constants \( g_L(g_R) \) of our “gauge-bosons” \( W_L(W_R) \) in the left-(right-) handed worlds become equal, \( g_L = g_R \), regardless of the values of the NJL-coupling constants. ii) The masses of \( W_L(W_R) \) become zero in the strong coupling limit \( G_L(G_R) \to \infty \), and the effective interactions realize the local \( SU(2)_L(SU(2)_R) \) gauge symmetry, when applying the dimensional regularization for loop-integrals. iii) The mixing of our neutral \( W \) bosons with photons becomes of the current-current type, and the unification condition \( (3.13), \lambda_L = \lambda_R = e/g \), on the “gauge” coupling constant \( g_L(g_R) \) and \( e \) is necessarily derived. Accordingly, as is well known, our effective interactions become identical to those of the LR gauge model of the tree-diagram level. iv) The KM-mixing matrices of charged currents become trivially equal in the left- and right-hand worlds.

We now give several additional remarks on our approach.

i) Tentative estimate of model parameters, \( \Lambda \) and \( N_{HC} \). Supposing the case of \( M_w \text{ and } M_Z \) becoming infinite (which corresponds to the weak coupling limit \( G_R \to 0 \) in the right-handed world), our effective interactions Eq. (3.18) become identical to those of SM in the case of massive \( W \) bosons without Higgs particles. Our model in this limit coincides with that of Ref. 22). Furthermore, taking the strong coupling limit \( G_L \to \infty \) (when \( M_w = M_Z \to 0 \) with \( m_F = 0 \) and with the type II of dimensional regularization) in Eq. (3.12a), we are led to a maximal value of the preon-mass \( m_F^{\text{max}} \),

\[ m_F^{\text{max}} = \frac{1}{\sqrt{3}} M_w = 46 \text{ GeV} \quad (4.1) \]

with the experimental value\(^{27} \) of \( M_w = 80.22(\pm 0.26) \text{ GeV} \). Concerning the number of hypercolor dimension, the following is derived from Eqs. (3.10) and (3.13):

\[ N_{HC} = \frac{6 \pi \sin^2 \theta_W}{a \ln(\Lambda/m_F)}. \quad (4.2) \]
In Table III we have given the values of \( A \) for tentative values of \( m_F = 46 \text{ GeV} \) \((m_F^{\text{max}})\) and 1 GeV, which are determined from Eq. (4·2), taking the experimental value\(^{27}\) \( \sin^2 \theta_W = 0.2318(\pm 0.0005) \), and using the value\(^{27}\) \( \alpha(M_Z) = e^2 / 4\pi = 1/128 \). From this table we see that it is in our model necessary to suppose a large \( N_{HC} \), which may guarantee the 1/\( N_{HC} \) expansion.

Table III. Tentative values of our model parameters \( A, N_{HC} \) and \( m_F \).

<table>
<thead>
<tr>
<th>( m_F = 46 \text{ GeV} )</th>
<th>( A ) (GeV)</th>
<th>( N_{HC} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10⁴</td>
<td>73</td>
<td>46</td>
</tr>
<tr>
<td>10⁵</td>
<td>13</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( m_F = 1 \text{ GeV} )</th>
<th>( A ) (GeV)</th>
<th>( N_{HC} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10⁶</td>
<td>49</td>
<td>35</td>
</tr>
<tr>
<td>10⁷</td>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>

\( \text{Mass of quarks and leptons.}\) In our approach we have introduced a "small" (bare or induced) spinor preon mass \( m_F \) in Eq. (3·2), which violates explicitly the chiral symmetry included in the original Lagrangian Eq. (2·5). Nonetheless, our values of model parameters seem to still guarantee the smallness of composite quark and/or lepton masses compared to the composite scale \( A \), as follows. Following the reasoning given in our previous work\(^{29}\), the mass of quarks and leptons is estimated as

\[
M_q(l) = \sqrt{\mathbf{p}^2 + m_F^2} + \sqrt{\mathbf{b}^2 + m_{C(s)}^2} + V(\mathbf{p}^2) \approx (m_F^2 + m_{C(s)}^2)/2\sqrt{\mathbf{p}^2},
\]

where \( m_{C(s)} \) is the scalar preon mass, and the average value of the internal momentum of preons may be identified as \( |\mathbf{p}| = A \). In deriving the right-hand side of Eq. (4·3), we have supposed chiral symmetry in the limit \( m_F = 0 \) and \( 2\sqrt{\mathbf{p}^2 + V} = 0 \). Then putting into Eq. (4·3) the values in Table III, for example \( m_F = 46 \text{ GeV}, m_{C(s)} = 0, A = 10^{4}(10^6) \) GeV in the case \( N_{HC} = 73(46) \), this yields a quark and lepton mass of \( M_q(l) \approx 100 \text{ MeV} \) (1 MeV).

\( \text{Physical effects from right-handed freedom.}\) We now consider whether any physical effect, concerning the right-handed world, may be seen or not. Our tentative value of \( A \) is of the order >10 TeV, which implies that the internal structure of composite \( W \)-bosons may not be recognizable presently, as was supposed at the beginning of §1. Accordingly, our model, as it is, is effectively an LR-symmetric massive Yang-Mills theory, and our effective Lagrangian Eq. (3·18) is similar to that of the LR-symmetric gauge model in the tree-diagram level: The charged-current sectors in the two cases are identical, and the results of analyses\(^{26}\) in the latter on the lower limit of \( M_w \) (the mass of \( W' \)) and the upper limit of \( \zeta \) (the LR-mixing parameter) are also applied in our case to give

\[
M_w > 1.4 \text{ TeV},
\]

\[
\zeta < 0.003.
\]

Our neutral-current sectors given in Eq. (3·18) are also equivalent to those in the LR-gauge model, where quantities of order \( M_L^2/M_R^2 \) and/or \( m_F^2/M_R^2 \) have been neglected. The corrections to this order of the Lagrangian is described in Appendix B. This change may affect the estimate of the axial vector coupling constant \( g_A \) in SM. By constraining the effect due to this correction to be within the uncertainty of \( g_A = 0.5008 \pm 0.0008 \) from the LEP and SLC experiments,\(^{27}\) we obtain the lower limit of the \( W' \)
mass as

\[ M_w \approx M_R > 3 \text{ TeV} \quad (4.5) \]

(See Appendix B for the details of this estimation.)

Almost the same results are obtained from a similar consideration on the vector coupling.

iv) Origin of parity violation. We have applied the effective NJL-type Lagrangian Eq. (3·2) with a chiral structure, while we have assumed the vector-like fundamental one, Eq. (2·5). Generally it is believed that the parity is not spontaneously broken in vector like theories. Accordingly, we may add a certain term with any chiral structure, which is not clear to us presently, to Eq. (3·2). However, we expect that our qualitative results summarized above remain valid without being affected by this added term, since the order of scale of parity violation in our approach is given by

\[ \Delta M_R \equiv M_R^{\uparrow} - M_R^{\downarrow} \approx 2 \text{ TeV} \quad \text{(see, Eq. (4·4))} \]

which is negligibly small compared to the composite scale \( \Lambda \geq 10^2 \text{ TeV} \).

Finally, we mention the prospects of our approach in connection to the recent high-precision experimental check, which may be extremely difficult to pass for any approaches over SM from the viewpoint of composite model.

A possible way for this may be just to follow the scenario in the LR symmetric gauge model, investigating its physical background from the viewpoint of our preon model. For this purpose it is necessary to treat dynamically the composition of Higgs scalars, required to exist there, and to apply the mechanism of dynamical breaking of the global \( SU(2)_L \times SU(2)_R \) symmetry. This is now under investigation, and it will be given in a separate paper.

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Appendix A

—— Derivation of Quantum Effects ——

In this appendix we derive the divergent parts \( \Delta L' \) given in § B, quantum effects of spinor preons, through the loop diagrams (Fig. 1). The generating functional of our model Lagrangian is given by

\[ Z = \frac{1}{Z_0} \int D\psi D\bar{\psi} D\bar{\psi}^{\dagger} D\bar{\psi}^{\dagger} D\psi^{\dagger} D\bar{\psi}^{\dagger} D\psi^{\dagger} D\bar{\psi}^{\dagger} \exp i \int d^4x L', \quad (A·1) \]

where \( Z_0 \) is the constant of normalization. Carrying out the path-integrals over \( \psi \) and \( \bar{\psi} \), we obtain \( \Delta L \) representing the quantum corrections to the Lagrangian \( L' \).
The correction owing to the preon loop diagrams is given by

\[ -iN_{HC} \text{Tr} \left[ \ln \left( i \gamma^\mu \partial_\mu - m_F - e \frac{e^3}{2} \gamma^\mu A_\mu^0 + \gamma^\mu \tau \cdot W_{L\mu} \frac{1 - \gamma_5}{2} + \gamma^\mu \tau \cdot W_{R\mu} \frac{1 + \gamma_5}{2} \right) \right] \]

\[ = iN_{HC} \sum_{n=1}^\infty \frac{1}{n!} \left( \frac{1}{i \gamma^\mu \left( \partial_\mu + \frac{e^3}{2} A_\mu^0 \right) - m_F} \right)^n \gamma^\mu \left( \tau \cdot W_{L\mu} \frac{1 - \gamma_5}{2} + \tau \cdot W_{R\mu} \frac{1 + \gamma_5}{2} \right) \]  

(A·2)

We expand each term on the right-hand side of Eq. (A·2) in terms of the fields \( W_{L\mu} \) and \( W_{R\mu} \). Equation (A·2) is represented by a sum of Feynman diagrams. We retain only divergent terms. After some trace calculations we have

\[ \Delta L' = - \frac{1}{4} Z_w \bar{W}_{L\mu} \cdot \bar{W}_L^{\mu\nu} + \frac{3}{2} m_F^2 Z_w \bar{W}_{L\mu} \cdot \bar{W}_L^{\mu\nu} - \frac{1}{4} Z_w \bar{W}_{R\mu} \cdot \bar{W}_R^{\mu\nu} \]

\[ + \frac{3}{2} m_F^2 Z_w \bar{W}_{R\nu} \cdot \bar{W}_R^{\nu\rho} - \frac{3}{2} m_F^2 Z_w (\bar{W}_{L\nu} \cdot \bar{W}_R^{\nu\sigma} + \bar{W}_{R\nu} \cdot \bar{W}_L^{\nu\sigma}) \]

\[ + 2 \frac{e}{g} \sqrt{Z_w} (\bar{W}_{L\mu}^3 F^{0\mu\nu} + \bar{W}_{R\mu}^3 F^{0\nu\mu}) \]

\[ + \frac{1}{2} Z_w \epsilon_{3ij} (F_{\mu\nu}^0 \bar{W}_L^{i\mu} \bar{W}_L^{j\nu} + F_{\mu\nu}^0 \bar{W}_R^{i\nu} \bar{W}_R^{j\nu}) \]

\[ - Z_w \frac{1}{4} e^{\mu\nu\rho\sigma} \left[ (\partial_\rho \bar{W}_{L\mu}) \cdot (\partial_\sigma \bar{W}_{L\nu}) - (\partial_\rho \bar{W}_{R\mu}) \cdot (\partial_\sigma \bar{W}_{R\nu}) \right] \]

(A·3)

The last term is an anomalous surface term in our case.

**Appendix B**

--- Neutral Current Interactions in the Order of \( M_R^2/M_L^2 \) and \( m_F^2/M_R^2 \) ---

In this appendix, we describe the next order of the neutral current interaction Lagrangian in our model, which was neglected in § 3D. The physical boson fields are obtained by the transformation as

\[
\begin{pmatrix}
A_\mu \\
Z_\mu \\
Z'_\mu
\end{pmatrix} =
\begin{pmatrix}
1 & \sin \theta_w & \sin \theta_w \\
0 & -\cos \theta_w + O_{22} & \sin \theta_w \tan \theta_w + O_{23} \\
0 & O_{32} & \sqrt{\cos^2 \theta_w} + O_{33}
\end{pmatrix}
\begin{pmatrix}
A_\mu^0 \\
W_{L\mu}^0 \\
W_{R\mu}^0
\end{pmatrix}
\]  

(B·1)

with the \( O_{ij} \) defined by

\[ O_{22} = -\frac{\cos^2 \theta_w (1 - 3 \sin^2 \theta_w + \sin^4 \theta_w)}{2 \cos^3 \theta_w} \frac{M_R^2}{M_L^2}, \]  

(B·2a)

\[ O_{23} = -\frac{\cos^2 \theta_w (1 - 5 \sin^2 \theta_w + \sin^4 \theta_w)}{2 \cos^3 \theta_w} \left( \frac{M_L^2}{M_R^2} + \frac{6 m_F^2}{M_R^2} \right), \]  

(B·2b)

\[ O_{32} = -\frac{\cos^2 \theta_w (1 - 3 \sin^2 \theta_w + \sin^4 \theta_w)}{2 \cos^3 \theta_w} \frac{M_L^2}{M_R^2}, \]  

(B·2c)
\[ O_{33} = -\frac{\tan^2 \theta_W \sqrt{\cos 2 \theta_W}}{\cos \theta_W} \left( \tan^2 \theta_W \frac{M_L^2}{M_R^2} - \frac{3m_F^2}{M_R^2} \right). \] (B.2d)

The interaction Lagrangian is given as

\[
L_{\text{int}}^\text{NC} = (A_{\mu} Z_{\mu} Z_{\rho}) \begin{bmatrix}
1 & 0 & 0 \\
\tan \theta_W + \varepsilon_{21} & -\frac{1}{\cos \theta_W} + \varepsilon_{22} & \varepsilon_{23} \\
-\frac{\tan \theta_W}{\sqrt{\cos 2 \theta_W}} + \varepsilon_{31} & \frac{\sin \theta_W \tan \theta_W}{\sqrt{\cos 2 \theta_W}} + \varepsilon_{32} & \frac{\cos \theta_W}{\sqrt{\cos 2 \theta_W}}
\end{bmatrix} \begin{bmatrix}
\varepsilon_{1m} \\
\varepsilon_{L3\mu} \\
\varepsilon_{R3\mu}
\end{bmatrix} \] (B.3)

with \( \varepsilon_{ij} \) defined by

\[
\varepsilon_{21} = -\frac{\tan \theta_W}{\cos^2 \theta_W} \left( \tan^2 \theta_W \frac{M_L^2}{M_R^2} - \frac{3m_F^2}{M_R^2} \right), \] (B.4a)

\[
\varepsilon_{22} = \frac{\tan^2 \theta_W}{\cos \theta_W} \left( \tan^2 \theta_W \frac{M_L^2}{M_R^2} - \frac{3m_F^2}{M_R^2} \right), \] (B.4b)

\[
\varepsilon_{23} = -\frac{1}{2 \cos \theta_W} \left[ \frac{\tan^2 \theta_W (3 - 2 \sin^2 \theta_W)}{\cos^2 \theta_W} \frac{M_L^2}{M_R^2} - \frac{6m_F^2}{M_R^2} \right], \] (B.4c)

\[
\varepsilon_{31} = -\frac{\tan \theta_W \sqrt{\cos 2 \theta_W}}{\cos^2 \theta_W} \left( \frac{\tan^2 \theta_W M_L^2}{M_R^2} - \frac{3m_F^2}{M_R^2} \right), \] (B.4d)

\[
\varepsilon_{32} = \sqrt{\cos 2 \theta_W} \frac{\tan \theta_W}{\cos^2 \theta_W} \left( \tan^2 \theta_W \frac{M_L^2}{M_R^2} - \frac{3m_F^2}{M_R^2} \right). \] (B.4e)

Hence, the axial vector coupling constant of the physical Z boson to charged leptons is given by

\[
g_A' = \frac{1}{2} + \frac{3 \sin^2 \theta_W - 4 \sin^2 \theta_W}{4 \cos^2 \theta_W} \frac{M_L^2}{M_R^2} - \frac{3m_F^2}{2M_R^2} \left( 1 - \frac{\sin^2 \theta_W}{\cos^2 \theta_W} \right). \] (B.5)

In the estimation of Eq. (4.4), the following values of parameters were used:

\[ m_F = 46 \text{ GeV}, \] (B.6a)

\[ M_L = M_W = 80.22 \text{ GeV}, \] (B.6b)

\[ \sin^2 \theta_W = 0.2318. \] (B.6c)

References