Chapter 5

Collapse and Fragmentation of Isothermal Clouds

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We review and summarize the features of collapse and fragmentation of isothermal interstellar clouds. When a cloud collapses by its self-gravity, fragmentation occurs at a stage which is early or late, depending on the cloud mass. Fragmentation occurs at an early stage of collapse if a cloud is massive compared with the initial Jeans mass. On the other hand, a small cloud does not fragment at least until the central core becomes opaque and ceases to be collapsing.

A sheet-like or a filamentary cloud is likely to fragment. The typical size of a fragment is several times the thickness of a sheet or the radius of a filament. Most fragments of a sheet are elongated, and massive enough to collapse if the external pressure is negligible. Accordingly they collapse and become filamentary. After a filament becomes opaque and stops collapsing, it fragments to form blobs with small masses of the order of $10^{-1}M_\odot -10^{-3}M_\odot$.

If there is some rotation, a collapsing cloud forms not filaments but spiral arms. It is possible that the central part of these spiral arms eventually fragments to blobs with the masses of the order of $10^{-1}M_\odot -10^{-3}M_\odot$.

§ 1. Introduction

We expect that the study of the characteristics of collapsing interstellar clouds may disclose some conditions and constraints necessary for the formation of the solar nebula. Many simulations for the cloud collapse show that a collapsing isothermal cloud can fragment easily. Then, an idea may occur that in the case of the solar nebula its formation was not a result of fragmentation of a collapsing cloud, because the present solar system is isolated and not a member of any multiple stellar system. However, caution is necessary for this idea since a small multiple stellar system except a binary is usually dynamically unstable and disrupts afterwards through three-body encounters. We cannot hence take it for granted that the solar nebula was never a member of a multiple stellar system at its forming stage.

The main problem of the formation of the solar nebula is what was the principal mechanism of the angular momentum transport in the primordial contracting cloud.

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The Sun, which has most of the mass of the solar system, has extremely small specific angular momenta compared with the planets. What condition of the parental cloud has brought this angular momentum distribution has long been one of the main concerns of theorists and is still quite uncertain. The bulk of the angular momentum of the contracting cloud was presumably transferred to the very outermost region by magnetic braking, turbulent viscosity (Tscharnuter, 1985; Morfill et al., 1985; Cabot et al., 1987), and/or gravitational torque (Larson, 1984), and this transfer may have occurred in the slowly-contracting quasi-static stage after dynamical collapse.

For the investigation of the initial condition of this slowly evolving stage, it is necessary to disclose the features of the early dynamical stages. In the following sections we shall briefly review the characteristics of dynamical collapse and fragmentation of pre-stellar isothermal clouds which represent the cores of molecular clouds now observed.

§ 2. Collapse condition for isothermal clouds

In this section we describe the collapse condition for two cases of isothermal clouds, i.e., an isolated rotating cloud and a fragment of a sheet-like cloud.

2.1. Isolated rotating cloud

The close investigation by Kiguchi et al. (1987) of hydrostatic equilibrium of a rotating isothermal cloud embedded in a tenuous medium, as is described in Chapter 4, solved the condition of the global collapse of such a cloud. Their result can be expressed by the following empirical formula that a cloud with the mass $M$ and the angular momentum $J$ collapses gravitationally if the mass is greater than the critical mass $M_c$, i.e.,

$$M > M_c = \left[ (1.18)^{5/2} + \left( \frac{J/J_*}{0.191} \right)^{4/5} \right]^{12/5} M_* , \quad (5\cdot2)$$

where $M_*$ and $J_*$ are the units given in terms of the gravitational constant $G$, the sound speed in the cloud $c_s$ and the pressure of the surrounding medium $P_{\text{ext}}$ by

$$M_* = c_s^4 G^{-3/2} P_{\text{ext}}^{-1/2} \quad \text{and} \quad J_* = c_s^7 G^{-2} P_{\text{ext}}^{-1} . \quad (5\cdot2\cdot1)$$

The above relation $(5\cdot2\cdot1)$ between $M_c$ and $J$ is shown by the dashed curve (the collapse curve) in Fig. 2 of Chapter 4.

When $J$ tends to zero, $M_c$ approaches the critical mass of the Bonnor-Ebert sphere, i.e., $1.18 M_*$ (Ebert, 1955). When $J \gtrsim J_*$, collapse condition $(5\cdot2\cdot1)$ reduces to

$$\frac{c_s J}{G M_*^2} < 0.191 . \quad (5\cdot2)$$

This collapse condition nearly holds for a cloud not only in equilibrium but also in dynamical nonequilibrium: Miyama, Hayashi and Narita (1984) performed simulations of dynamical evolution of rotating isothermal clouds which are initially uniform spheres, and found that a cloud with $c_s J / G M_*^2 \lesssim 0.20$ begins to collapse; otherwise oscillates or expands.
Further, Miyama et al. (1988) investigated the stability of the rotating equilibria constructed by Kiguchi et al., and found that a cloud with the mass larger than $4M_\ast$ is dynamically unstable to nonaxisymmetric perturbations. This means that rotation can increase the mass of an equilibrium cloud by a factor of about 4. The flatness of a $4M_\ast$ cloud, i.e., the ratio of the equatorial diameter to the maximum thickness of the cloud is about 3.5. In other words, a cloud with the flatness larger than 3.5 gives rise to dynamical instability and contracts. It is to be noticed that the Jeans mass $M_J$ is defined by

$$M_J = \frac{\pi^{3/2} c_s^3}{G^{3/2} \rho^{1/2}} ,$$  \hspace{1cm} (5.2.4)$$

where $\rho$ is the mean density of a cloud. Then we have $M_J \approx 4M_\ast$ using Eq. (5.2.2), since the mean density of a rotating equilibrium is about $2P_{\text{ext}}/c_s^2$. The agreement of this value of $M_J$ with the minimum mass found by Miyama et al. (1988) for the dynamical instability is partly accidental because $M_J$ is the minimum mass for the growth of small perturbation in an infinite uniform medium as was found by Jeans (1929).

2.2. Infinite sheet cloud

One of expected initial states of a collapsing cloud is a fragment of a parental sheet cloud because a thin sheet is always unstable to fragmentation and because a sheet is likely to be formed by various dynamical mechanisms in interstellar clouds such as a cloud-cloud collision (e.g., Nagasawa and Miyama, 1987), and compression through shock waves generated by supernovae or OB stars (Lada, 1987). Further, a cloud which is, initially, even nearly spherical contracts to form a sheet gravitationally (Lin, Mestel and Shu, 1965; Zel'dovich, 1970). Also if there are magnetic fields or a cloud is rotating, the formation of a sheet is a natural result of contraction. Therefore, now we consider whether or not a fragment of an infinite sheet collapses.

Defining the density in the midplane of a self-gravitating infinite stationary sheet by $\rho_0$ and the external pressure acting on it by $P_{\text{ext}}$, we obtain the column density $\sigma$ as

$$\sigma = c_s \sqrt{\frac{2\rho_0 \eta}{\pi G}} ,$$  \hspace{1cm} (5.2.5)$$

where $\eta$ is given by

$$\eta = \sqrt{1 - \frac{P_{\text{ext}}}{\rho_0 c_s^2}} .$$  \hspace{1cm} (5.2.6)$$

The linear perturbation theory for the sheet shows that the minimum wavelength $\lambda_c$ for fragmentation is given by (Elmegreen and Elmegreen, 1978)

$$\lambda_c = \frac{\pi \sigma}{\rho_0 f} ,$$  \hspace{1cm} (5.2.7)$$

where $f$ is a factor of order unity, which is written as

$$f \approx 0.639 - 0.129 \eta + 0.490 \eta^2 .$$  \hspace{1cm} (5.2.8)$$
Now if the minimum mass $M_{\text{min}}$ of a fragment is estimated by considering a square sheet with the side-length $\lambda_c$, then we have

$$M_{\text{min}} = \lambda_c^2 \sigma = \left( \frac{8 \pi}{f^2} \frac{\eta}{G} \right)^{3/2} \frac{c_s^3}{\rho_0^{1/2}}.$$ \hfill (5·2·9)

When the external pressure $P_{\text{ext}}$ is so strong that it is comparable with $\rho_0 c_s^2$ as may be the case for a shock-compressed sheet, $\eta$ is considerably smaller than unity and a fragment with mass $M_{\text{min}}$ is stable against collapse as was found by Elmegreen and Elmegreen (1978), because in this case $M_{\text{min}} < 1.18 M_*$ (see Eq. (5·2·1)). If these stable fragments, however, cool down through radiation by dusts or if they coalesce through mutual collisions, they may begin to collapse (see § 5 in Chapter 4 for the collapse induced by cooling).

Now we consider the case where $P_{\text{ext}} \ll \rho_0 c_s^2$, and hence $\eta \approx 1$ and $f \approx 1$. Then, expecting that the mean density of a formed fragment is comparable with $\rho_0$, we find that the mass $M_{\text{min}}$ is approximately equal to the Jeans mass given by Eq. (5·2·4). Accordingly a fragment with mass $M_{\text{min}}$ collapses if it is not rotating rapidly. Since the most unstable wavelength is about $2\lambda_c$, most fragments of a sheet have masses much larger than $M_{\text{min}}$ and collapse necessarily.

If a fragment with mass $M_{\text{min}}$ were to collapse only in one direction parallel to a side of the initial square and forms a filament, the line mass per unit length of the filament would become $(\eta/f)m$, where

$$m = \frac{2c_s^2}{G},$$ \hfill (5·2·10)

which is just the minimum line mass of an isothermal infinite filament which can collapse infinitely (Ostriker, 1964). This simple analysis also indicates that when the external pressure $P_{\text{ext}}$ is negligible, a fragment with $M_{\text{min}}$ can collapse or at least is marginally stable. In reality, a somewhat elongated fragment contracts more or less in the two directions, and hence, the line mass of a filament formed is larger than $(\eta/f)m$.

§ 3. Features of gravitational collapse and fragmentation

Now we review briefly the features of collapse and fragmentation of isothermal clouds, which have so far been found by numerical simulations. The cases of four different initial conditions are described here: rotating equilibria, infinite sheets, rotating disks and rotating spheres.

3.1. A rotating cloud in equilibrium

As is mentioned in § 2.1, rotating equilibria with masses larger than $4M_*$ are dynamically unstable to nonaxisymmetric perturbations. Miyama et al. (1988) simulated the evolution of various equilibria and found that in these massive clouds nonaxisymmetric fluctuations develop to form spiral arms and the outward transport of angular momentum through the gravitational torque induced by the spiral arms gives rise to the contraction of the inner parts (see Fig. 8 in Chapter 4). As the mass
and, hence, the flatness of a cloud increase, the number of formed spiral arms increases. They also found that when the mass of a cloud is much larger than $4M_\star$, the collapsed central part of the cloud fragments at a late stage of collapse.

3.2. An infinite sheet

Recently Miyama, Narita and Hayashi (1987a, 1987b) investigated the nonlinear effect on the growth of perturbations in an infinite stationary sheet to obtain full understanding of its fragmentation processes. They simulated the growth of fragments through random velocity fluctuations in the sheet where the external pressure is negligible (i.e., $P_{\text{ext}}=0.125\rho_0c_s^2$), and found that filaments and blobs are formed as shown in Fig. 1. They also found that if a fragment is initially elongated, that is, if the ratio of the major sidelength to the minor one of a rectangular fragment is greater than 1.2, a fragment becomes slenderer and slenderer as it contracts, and eventually forms a thin filament. Otherwise, if a fragment is nearly square, a fragment evolves to form a centrally condensed blob. They showed that 90% of fragments of a sheet collapse not to blobs but to filaments in the sense of probability.

![Image](https://academic.oup.com/ptps/article-abstract/doi/10.1143/PTPS.96.63/1919291) Fig. 1. Filaments and blobs are formed in an isothermal infinite sheet through the growth of random velocity fluctuations (by Miyama, Narita and Hayashi, 1987b).
According to their results, the subsequent fragmentation does not occur in a filament during the isothermal stages of contraction. That is, for a filament to fragment, it must be opaque to radiation. This is because the fragmentation time is comparable to the collapse time in the collapsing isothermal stage. Now let us estimate the mass of a fragment of a filament. We define the line mass of a filament $m$ by $\pi r^2 \rho$ and the optical depth $\tau$ by $\kappa \rho r$ where $\kappa$, $\rho$ and $r$ are the opacity, the mean density and the radius of a filament, respectively. Here, for the length of a fragment we adopt $2\pi r$, i.e., the most unstable wavelength of perturbations for a filament (Hayashi, 1987; Nagasawa, 1987). Then we obtain the mass of a fragmented blob as $M = 2\kappa m^2/\tau$. If we assume that a filament stops collapsing and the fragments when the optical depth becomes several, the mass $M$ is evaluated with the aid of Eq. (5·2·10) as

$$M \sim \frac{\kappa c_s^4}{G^2}. \quad (5·3·1)$$

Putting simply $\kappa \sim 1$ cm$^2$/g (c.f., Pollack et al., 1985) and $c_s \sim 2 \times 10^4$ cm/s (note that it is the sound speed in the isothermal stage), we obtain the mass of the order of $10^{-2} M_\odot$, although it may have some uncertainty of an order of magnitude.

3.3. A rotating disk

Previously, Narita, Hayashi and Miyama (1984) simulated the axisymmetric collapse of a rotating disk-like cloud which is initially force-balanced in the direction parallel to the rotation axis. They found that a massive disk with the mass $M \gg M_1$ (i.e., the case $a=0.05$ in their paper) gives rise to fragmentation at an early collapse stage, although the initial perturbation is very small. This implies that a cloud with the initial mass $M \gg M_1$ fragments during isothermal collapse, as far as its initial shape is not spherical but very flattened.

Now, we consider the collapse of the central part. In general, the central part of a rotating disk undergoes runaway collapse and does not fragment before the core becomes opaque. Under the assumption that the equation of state changes to be adiabatic at a certain density, they found that in the case of a slow rotator one or two rings are formed in the inner region after the formation of a thin disk by mass accretion to the adiabatic core. These rings are dynamically unstable to fragmentation. It is to be noticed that the above is the result of axisymmetric simulations. Since the growth of nonaxisymmetric instabilities is faster than that of axisymmetric ones in a thin disk, many blobs with small masses will, in reality, be formed through the formation and the subsequent fragmentation of spiral arms in place of those of rings.

3.4. A rotating sphere

Many collapse simulations have been concerned with rotating clouds which are initially spherical (e.g., Miyama, Hayashi and Narita, 1984; Narita, Hayashi and Miyama, 1984; Boss, 1986). The initial states of these clouds are far from hydrostatic equilibrium, and any axisymmetric or nonaxisymmetric perturbations cannot have time to grow before the central density increases by many orders of magnitude. The
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Fig. 2. Fragmentation occurs after the core bounce of a rotating cloud with the mass $7.8M_\odot$. The fluid elements are projected onto the equatorial plane where $R_0$ is the initial radius of the cloud (by Miyama, Hayashi and Narita, 1984).

above is the case when the initial density is homogeneous. When a cloud is initially not homogeneous but centrally condensed, fragmentation during its collapse is harder to occur than in the homogeneous case as was found by Boss (1987).

The inner part of a rotating and collapsing isothermal cloud has a well-known $r^{-2}$ functional dependence of the equatorial density on the radius $r$ (Norman, Wilson and Barton, 1980) before an opaque core appears at the center. This density profile indicates that the isothermal gas pressure plays an important role during the collapse while the centrifugal force does not (Narita et al., 1984). The inner disklike part is not very flat because of the rather intense pressure force and hence it does not fragment during the collapse.

Miyama et al. (1984) investigated the collapse of clouds which are initially rigidly-rotating homogeneous spheres, and showed that if a cloud is massive or slowly rotating satisfying the condition

$$c_s J / GM^2 \leq 0.15,$$

it leads eventually to the formation and the subsequent fragmentation of a disklike core after the core bounces at the center (Fig. 2). Three cases of their simulation yielded 3, 6 and 8 fragments, respectively. We assume here that this bounce occurred when the central part becomes opaque, and then evaluate the masses of the fragments. If we assume that the density in the central region at the bounce time is $10^{-12} \text{g/cm}^3$, which implies that the initial uniform density is $10^{-15} \text{g/cm}^3$, and adopt $2 \times 10^4 \text{cm/s}$ for $c_s$, then the mass of the fragments lies in the range $(4-8) \times 10^{-3} M_\odot$. 

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§ 4. Effects of rotation

The mechanism of angular momentum transport has long been an important and hard problem for the formation of a star and that of the solar system. Since a star or the solar system has the specific angular momentum many magnitudes smaller than that of a typical interstellar cloud, most angular momentum of a cloud which begins contraction must be transferred outward during some stages of star formation.

A dimensional analysis or a second-order virial analysis, which assumes the simplified homologous contraction of a cloud, tells that the collapse of a rotating isothermal cloud is always bound to be stopped by the increased centrifugal force and results in a flat disk (Weber, 1976; Tohline, 1985a, 1985b). The flat disk may fragment into pieces directly in a dynamical time scale. If most of the spin angular momentum of a parent cloud is transformed into the orbital angular momenta of its fragments, then, each fragment collapses and forms a flat disk again. Bodenheimer (1978) proposed the hypothesis of a successive fragmentation as one possible mechanism of angular momentum transport under the assumption similar to the above scenario, although he assumed the formation of a ring in place of a disk.

In reality, however, the collapse of an isothermal cloud is extremely non-homologous (see § 3.4), and hence a second-order virial analysis is not sufficient to draw conclusion on the collapse of interstellar clouds. Also it is still uncertain whether or not an interstellar cloud does repeat fragmentation many times during its contraction stages before the mass of a fragment becomes that of an ordinary star.

One of prominent effects of rotation is to bend growing nonaxisymmetric density perturbations in a disklike cloud because of differential rotation (Tohline, Durisen and McCollough, 1985; Williams and Tohline, 1987). The resultant spiral arms give rise to the outward transport of the angular momentum through a mechanism of gravitational torque (Durisen and Tohline, 1985; Durisen et al., 1986; Miyama et al., 1988) and lead to further contraction of the inner part of a collapsing cloud.

§ 5. Summary and discussion

We summarize in the following the features of collapse and fragmentation of molecular clouds nearly in the isothermal phase, that is clouds with the low densities, $\rho \leq 10^{-13}$ g/cm$^3$.

1) Fragmentation of a collapsing massive cloud In the case of a cloud which is much larger than the initial Jeans mass $M_J$ and is very flattened, it fragments at a relatively early stage of collapse.

2) Condition of fragmentation (a) A sheet-like or disklike cloud can fragment when its flatness, i.e., the ratio of the equatorial length to the thickness (height) of the cloud is larger than several. The shape of fragments of an infinite sheet is likely to be oblong rather than it is square, i.e., it is more or less elongated. (b) A filament-like cloud can fragment when the ratio of its length to the radius is larger than several.

3) Condition of collapse If the external pressure is negligible, most fragments of a
sheet are massive and collapse by their own gravity. Continuing to collapse, they become slenderer and form filaments.

4) **Fragmentation of a collapsing filament** A collapsing filament does not fragment until it becomes opaque. When it becomes opaque and the temperature is increased, it ceases to collapse and then fragments to some blobs with mass of the order of \(10^{-1}M_\odot - 10^{-3}M_\odot\).

5) **Effect of rotation** When a collapsing cloud becomes bar-like or filamentary, differential rotation bends it to form spiral arms. The outward transport of angular momentum through the gravitational torque induced by the spiral arms gives rise to further contraction of the inner part of the cloud.

6) **Fragmentation of the central part of a rotating cloud** The central part of a collapsing rotating cloud is not so flat as to fragment until it becomes opaque. After it becomes opaque, continuous mass accretion leads to the formation of a thin disklike core which fragments subsequently, if the angular momentum of the cloud is small (see Eq. (5·3·2)). The masses of the fragments are of the order of \(10^{-1}M_\odot - 10^{-3}M_\odot\).

In the present review we have simplified or neglected some important points:

1) The assumption of the constant external pressure in space and time. The actual external pressure generated in various processes is probably neither homogeneous nor constant during the cloud collapse, and may seriously distort the configuration of a cloud from that considered above.

2) The assumption of the isothermality. Temperature variation in a cloud gives rise to some change in the feature of a collapsing cloud.

3) Neglect of magnetic fields. Magnetic fields may play an important role at the collapse stage of a cloud as well as for the equilibrium configuration of a cloud.

We will discuss here some future work briefly. Important problems are how much mass of a contracting envelope accretes onto several small blobs formed in the central region, and how many blobs survive eventually in the system after coalescence of each other as well as escape from the system through multibody encounters. To resolve these questions we must deal with (1) the slow evolution of a contracting envelope through the loss of angular momentum and through heating by compression and radiation energy transport, and at the same time (2) the dynamical evolution of a multibody system in a rather dense gaseous medium where a blob is growing in mass as well as contracting by itself. This investigation may elucidate the initial mass function of stars.

References


Tohline, J. E., 1985a: Icarus 61, 10.


