# The extension of EPANET source code to simulate unsteady flow in water distribution networks with variable head tanks

Diego Avesani, Maurizio Righetti, Davide Righetti and Paolo Bertola

## ABSTRACT

This paper describes the modifications applied to EPANET, a public-domain water distribution system modelling software package, that does not correctly compute the hydraulics of a water distribution network (WDN) with variable tank heads in (slow) unsteady flow conditions. Firstly the methodology adopted to extend the Global Gradient Algorithm (GGA) implemented in the original EPANET source code to the Extended Period Simulation-GGA (EPS-GGA) is described. Then the convergence and stability conditions of the theta method, used for the discretisation in time of the set of differential equations describing the hydraulic behaviour of a WDN, are discussed. The reasons for EPS-GGA numerical stability are demonstrated and a fully implicit discretisation of differential equations (i.e. theta = 1) is suggested as the optimal choice as implicitly proposed in Giustolisi *et al*. but without theoretical justification. Both the modified and original versions of EPANET are applied to a particularly severe test case of a WDN. Moreover, the procedures for the correct numerical representation of the tanks' maximum and minimum level boundary conditions are developed and compared with previously proposed procedures. The modified version of EPANET source code does not show the significant instabilities which are evident in the original version, nor the lack of consistency due to the improper maximum and minimum level boundary condition schematisations formerly proposed in the scientific literature.

**Key words** | EPANET, tanks, unsteady flow, water distribution network

**Diego Avesani** (corresponding author)
**Maurizio Righetti**
**Davide Righetti**
**Paolo Bertola**
Department of Civil and Environmental Engineering,
via Mesiano 77,
I-38123 Trento,
Italy
E-mail: *diego.avesani@ing.unitn.it*

## NOTATION

| | |
|---|---|
| $\mathbf{A}_{11}$ | Diagonal matrix whose elements represent head losses |
| $\mathbf{A}_{12}$, $\mathbf{A}_{21}$ | Incidence matrix of internal nodes (non-reservoir nodes) |
| $\mathbf{A}_{10}$ | Incidence matrix of the reservoir nodes |
| $\Delta t$ | Simulation time step |
| $h_i$ | Water head in node $i$ |
| $\mathbf{H}_t$ | Vector of $n_n$ unknown nodal heads |
| $\mathbf{H}_{t-\Delta t}$ | Vector of node head computed at simulation time $t$-$\Delta t$ |
| $\mathbf{H}_{t0}$ | Vector of initial node heads |
| $n_i$ | Number of nodes connected to node $i$ |
| $n_s$ | Number of reservoirs in the WDN |
| $n_t$ | Number of tanks in the WDN |
| $n_j$ | Number of junctions in the WDN |
| $\bar{\Omega}_i$ | Average tank cross section in the $\Delta t$ interval |
| $\Omega_i$ | Tank cross section area |
| $p$ | Exponent of the chosen head losses formula |
| $t$ | Simulation time |
| $t_0$ | Simulation initial time |
| $\theta$ | Averaging weight of the implicit scheme |
| $\mathbf{Q}_t$ | Vector $n_p$ of unknown pipe discharges |
| $\mathbf{Q}_{ik}$ | Flow in the pipe connecting nodes $i$ and $k$ |
| $Q_{t-\Delta t}$ | Vector of pipe flows computed at simulation time $t$-$\Delta t$ |
| $\mathbf{Q}_{t0}$ | Vector of initial pipe flows |
| $i$, $j$, $k$ | Generic node indexes |

doi: 10.2166/hydro.2012.013

## Operators and acronyms

| | |
|---|---|
| EPS-GGA | Extended Period Simulation-Global Gradient Algorithm |
| GGA | Global Gradient Algorithm |
| G-GGA | Generalized-Global Gradient Algorithm |
| MLBC | Maximum Level Boundary Conditions |
| mLBC | Minimum Level Boundary Conditions |
| ODE | Ordinary Differential Equation |
| WDN | Water Distribution Network |

## INTRODUCTION

EPANET is a public-domain, water distribution system modelling software package developed by the EPA and freely distributed. It performs extended-period simulation of hydraulic behaviour in pressurised pipe network systems. Many commercial packages, for example WaterGems (Bentley System 2009) and MIKE-URBAN (URBAN 2006) use EPANET as an hydraulic engine, making EPANET a standard in water distribution network (WDN) modelling. EPANET correctly performs extended period simulation of a WDN as a sequence of steady flows under the hypothesis of a slow variation of the flow conditions in time such as, for example, diurnal changes in water demand.

However, oscillations and instabilities have been reported by several users of EPANET despite the hypothesis of a slow time variation in the flow conditions being satisfied when two or more tanks hydraulically adjacent are present in the network. Some manuals (URBAN 2006) recommend to model hydraulically adjacent tanks as a single composite tank with an equivalent total surface area and storage volume equal to the sum of the individual tanks so as to overcome the unwanted oscillations of the solution. This approach, however, proves only to be a deception and it does not resolve the problem of numerical instability. This approach is applicable only to adjacent tanks connected to each other by a single and relatively short pipe without any other control measures.

Recently the problem of spurious oscillations has been faced by Todini (2011), who has attributed the nature of this instability to the methodology used to numerically describe the time variations in the tank levels. Todini (2011) proposed to solve the steady state of a WDN (Global Gradient Algorithm (GGA), see Todini & Pilati (1998)) with a new numerical scheme: the EPS-GGA (Extended Period Simulation-GGA), based on the implicit theta method scheme for discretisation in time. The EPS-GGA method couples the steady-state energy balance equations for the pipes with the mass balance equations of the tanks, from which new unknowns appear, which are the tank water levels. These new unknown variables can be calculated simultaneously with the other unknown ones found in the problem (the junction heads). This approach is widely used in hydraulic problem solving. Le Coq *et al.* (1985), Malik *et al*. (1998) and Todini (2011) extended this approach to the problem of a water distribution network and tested it with some very simple test cases, although the approach in Todini (2011) has not yet been directly implemented into the EPANET software, and the numerical problems relating to the correct boundary conditions of the tanks (the maximum and minimum levels allowed) still have to be properly faced. This last point is particularly crucial because, as shown below, the correct representation of the boundary conditions can greatly affect the simulation results.

The rest of this paper is organised as follows. In the second section the strategy adopted in order to transform the GGA algorithm developed in the EPANET source code into the EPS-GGA algorithm is summarised, together with an explanation of the source code modifications and the description of the strategy adopted to correctly evaluate the initial conditions (ICs). Moreover the convergence of the numerical method used for discretisation in time of the mass balance equation at variable head tanks (the implicit weighted theta method) is discussed in the third section; the performances of the original EPANET source code and of the modified version, which will be refered to as EPANET-EPS from now on, are compared in the next section through its application in a real case. The fifth sction describes the procedures proposed to appropriately take into account the maximum and minimum boundary level conditions at the variable head tanks (MLBC and mLBC, respectively). These original procedures are applied to some test cases and the results are then compared with the results obtained applying the methodologies already implemented in EPANET and the one suggested in Todini (2011). At the end the concluding remarks are reported.

## SIMULATION MODEL AND SOURCE CODE MODIFICATIONS

The flow problem of a WDN with variable head tanks, under the hypothesis of negligible inertial and dynamical effects, can be described by the following set of mass and energy balance equations (see Todini 2011):

$$\frac{d\Omega_i h_i}{dt} = \sum_{k}^{n_i} Q_{ik} + q_i \tag{1a}$$

$$h_i - h_j = -K\left|Q_{ij}\right|^{p-1} Q_{ij} \tag{1b}$$

where $h_i$ is the water head in node $i$ at generic time $t$, $\Omega_i$ is the area of the tank in node $i$, $Q_{ik}$ is the flow in the pipe connecting nodes $i$ and $k$, $q_i$ is the external inflow to node $i$, $K$ is the headloss resistance coefficient, $p$ is the exponent of the chosen head loss formula and $n_i$ is the number of nodes connected to node $i$.

Todini (2011) introduced an extension of the GGA for the solution of the system of ordinary differential equations (1) (ODEs (1)) and used the implicit theta method for the discretisation in time of the equations.

Following Todini (2011) the matrix representation of system (1) is formulated as follows:

$$A_{11}^{t} Q_t + A_{12} H_t = -A_{10} H_0 \tag{2a}$$

$$A_{21} Q_t + A_{22} H_t = -q_t* \tag{2b}$$

where $\mathbf{A}_{11}$ is the diagonal matrix which its elements represent head losses (pipes, pumps and minor loss), $\mathbf{A}_{12}$ is the incidence matrix of internal nodes (in this case only non-reservoir nodes), where $\mathbf{A}_{12} = \mathbf{A}_{21}{}^{T}$, $\mathbf{A}_{10}$ is the incidence matrix of the reservoir nodes, $\mathbf{H}_t^T = [h_{1,t}, h_{2,t}, \ldots, h_{n_n,t}]$ is the vector of $n_n$ unknown nodal heads, which includes the variable unknown head tanks, $\mathbf{Q}_t^T = [Q_{1,t}, Q_{2,t}, \ldots, Q_{n_p,t}]$ is the vector $n_p$ of unknown pipes discharges. More details about matrices ($\mathbf{A}_{11}$, $\mathbf{A}_{12}$, $\mathbf{A}_{21}$ and $\mathbf{A}_{10}$) can be found in Todini & Pilati (1998), Salgado *et al.* (1994), Todini (2003, 2011) and Giustolisi (2010).

The system (2) is the formulation for the unsteady flow problem in looped WDNs, which is consistent with the formulation of the GGA given in Todini & Pilati (1998). The modification of GGA presented by Todini (2011) descends from the new formulation of $q_t$* and $\mathbf{A}_{22}$. According to Todini (2011) the term $\mathbf{A}_{22}$ is a $[n_n; n_n]$ diagonal submatrix, whose elements are defined as

$$A_{22}(i,i) = -\frac{\bar{\Omega}_i}{\theta \Delta t} \tag{3}$$

and the term $\mathbf{q}_t^{T}* = [q_{1,t}, q_{2,t}, \ldots, q_{n_n,t}]$ is the demand column vector of $n_n$ elements defined as

$$q*(i) = \begin{cases} q_{i,t} + \dfrac{1-\theta}{\theta}\left(\displaystyle\sum_{k}^{n_i} Q_{ik,t-\Delta t} + q_{i,t-\Delta t}\right) + \dfrac{\bar{\Omega}_i}{\theta \Delta t} h_{i,t-\Delta t} \text{ for tanks} \\ q_{i,t} \text{ for junction nodes} \end{cases} \tag{4}$$

where $t$ is the simulation time and $\Delta t$ is the simulation time step. Finally $\bar{\Omega}_i$ is the average tank cross section in the $\Delta t$ interval and $\theta$ is the time averaging weight of the implicit scheme in the EPS-GGA model (see Todini 2011). The system (2) can be solved following a Newton–Raphson iterative procedure, as described in Todini & Pilati (1998) and already implemented in EPANET. We can therefore introduce some low-level changes in the original EPANET-GGA source code in order to upgrade it into the new version EPANET-EPS, which contains the EPS-GGA numerical scheme. These low-level changes allow us to correctly manage the new submatrix $\mathbf{A}_{22}$ (Equation (3)) and the new demand vector $\mathbf{q}$* (Equation (4)) in the system (2). In particular these modifications concern the function *netsolve(int*, float*)*, contained in the file *hydraul.c* of the EPANET source code and can be summarised as:

- definition of the new function *coeffA22EPSGGA()* to calculate the elements of matrix $\mathbf{A}_{22}$ according to Equation (3);
- modification of the function *nodecoeffs()* to calculate the elements of $\mathbf{q}$* according to Equation (4).

For completeness the Listings (1) and (2) report the new function *coeffA22EPSGGA()* and the modified one *nodecoeffs()*.

**Listing 1** | Modified function nodecoeff( ) in EPANET 2 source code

```
void  nodecoeffs()
{
   int   i,n;
   float Dstar,Hstep_real;
   Hstep_real = (float)Hstep;
   for (i=1; i<=Njuncs; i++) //Junctions
    {
     X[i] -= D[i];
     F[Row[i]] += X[i]+((1-theta)/theta)*(Xp[i]-Dp[i]);
    }
   for (i=Njuncs-Ntanks; i<=Ntanks; i++)
   {
      n   = Tank[i].Node;
      F[Row[n]] +=(Tank[i].A * Hp[n]) /(theta*Hstep_real);
   }
}
```

**Listing 2** | New function coeffA22EPSGGA( ) introduced in EPANET 2 source code

```
void  coeffA22EPSGGA()
{
   int   i,n;
   float eta;
   float Hstep_real;
   Hstep_real = (float)Hstep;
      for (i=1; i<=Ntanks; i++)
       {
        n   = Tank[i].Node;
        eta = -Tank[i].A/(theta*Hstep_real);
        Aii[Row[n]] = Aii[Row[n]] - eta;
       }
}
```

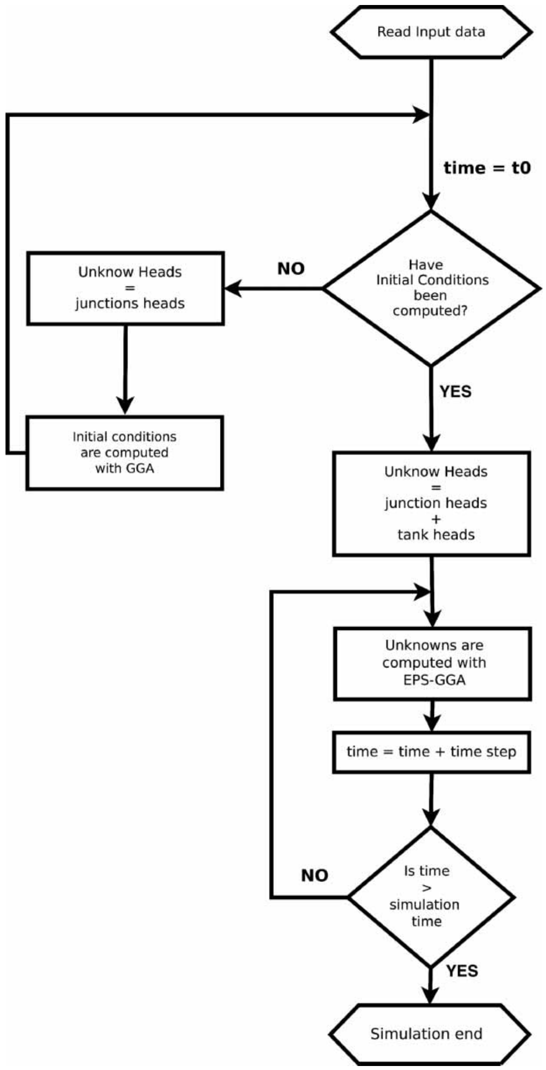


**Figure 1** | EPS-GGA flowchart for the new code EPANET-EPS.

These modifications imply the introduction of two new vectors $\mathbf{H}_{t-\Delta t}$ and $\mathbf{Q}_{t-\Delta t}$ into the original source code so as to store the hydraulic heads and flows in the WDN calculated at a previous time step $t$-$\Delta t$ (see Equation (4)). Moreover, due to the implicit scheme EPS-GGA, the exact ICs have to be calculated at time $t = t_0$ since the approximate solutions at the first iteration is not sufficient anymore as in the original version of EPANET (Rossman 2000). The strategy adopted and implemented into the EPANET source code applies the EPS-GGA only once the ICs have been calculated with the original GGA: more details are summarized in the flowchart (Figure 1).

From a computational point of view the implementation of the EPS-GGA scheme into the EPANET package entails some other important modifications due to the structure and form of the unknown hydraulic heads vector $\mathbf{H}_t$ (Equation (2)). In the GGA tanks there are nodes with fixed heads, like reservoirs, and as a consequence the only unknown node heads in the vector $\mathbf{H}_t$ are the junction heads (Equation (7)). In the EPS-GGA the vector $\mathbf{H}_t$ also includes the tank heads as unknown variables. This means that, once the IC are calculated at the time step $t_0$, the unknown vector $\mathbf{H}_t$ has to be reallocated to take into account the new unknown tank heads. To describe these at time $t_0$ the

ICs $\mathbf{H}_{t0}$ and $\mathbf{Q}_{t0}$ are calculated with GGA and $\mathbf{H}_{t0}$ has the structure

$$H_{t0}^{T} = \left[h_{1,t0}, h_{2,t0}, \ldots, h_{n_j,t0}\right] \tag{5}$$

with $n_j$ the number of unknown junction heads at initial simulation time $t_0$. Once the ICs are known the vector for unknowns becomes

$$H_{t}^{T} = \left[h_{1,t0}, h_{2,t0}, \ldots, h_{n_j,t0}, h_{n_{j+1},t0}, \ldots, h_{n_j+n_t,t0}\right] \tag{6}$$

with $n_t$ the number of tanks in the WDN.

From a computationl point of view, so as to easily extend the unknown vector the allocation order of junctions, tanks and reservoirs has to be changed. If you look at Equation (7) you can see the order in which the node heads are allocated into EPANET, where $n_s$ is the number of reservoirs in the WDN:

$$H_{\mathrm{epa}}^{T} = [\overbrace{\underbrace{h_{1,t0}, h_{2,t0}, \ldots, h_{n_j t0},}_{\text{unknowns}}}^{\text{junctionheads}} \overbrace{h_{n_{j+1,t0}}, \ldots, h_{n_{(j+1)}+n_s,t0},}^{\text{reservoirheads}} \overbrace{\underbrace{h_{n_{j+1}+n_s+1.t0}, \ldots, h_{n_j+n_y+n_s,t0}}_{\text{unknowns}}}^{\text{tankheads}}] \tag{7}$$

Equation (7) clearly shows that the unknown variables are allocated at the beginning or at the end of vector $\mathbf{H}_{\mathrm{epa}}$ at generic simulation time $t$. It was decided therefore to allocate the unknown tank heads just after the unknown junction heads to easily extend the vector of unknown $\mathbf{H}_t$ once the ICs have been computed (Equation (8)). In this way the subroutines in the EPANET source code can be used for any further modifications. In Equation (8) you can see the unknown junction heads and tank heads $n_j + n_t$ terms followed by the known reservoirs heads:

$$H_{\mathrm{epa}}^{T} = [\overbrace{\underbrace{h_{1,t0}, h_{2,t0}, \ldots, h_{n_j t0},}_{\text{unknowns}}}^{\text{junctionheads}} \overbrace{\underbrace{h_{n_{j+1}t0}, \ldots, h_{n_{j+1}+n_t,t0},}_{\text{unknowns}}}^{\text{tankheads}} \overbrace{h_{n_{j+1}+n_t+1,t0}, \ldots, h_{n_j+n_t+n_s,t0}}^{\text{reservoirheads}}] \tag{8}$$

## ANALYSIS ON THE TIME AVERAGING WEIGHT

The subset of differential equations (1a) can be written in the following more compact form:

$$\frac{\mathrm{d}h_i}{\mathrm{d}t} = h_i' = g(Q(h)) \tag{9}$$

and, when applied to Equation (9), the theta method takes the form

$$h_t = h_{t-\Delta t} + \Delta t(1-\theta)g(h_{t-\Delta t}) + \Delta t(\theta)g(h_t) \tag{10}$$

The conditions to impose the values of $\theta$ for the stability and convergence of a generic system of ODEs have been thoroughly analysed in the scientific literature (see, e.g., Stuart & Peplow (1991), Shampine (1994), Barclay *et al.* (2000) and Hairer & Wanner (2010)), and recently in Giustolisi *et al.* (2012) the GGA-EPS has been generalised in the Generalized-GGA (G-GGA) method. Nevertheless the application of the theta method to the solution of the flow problem in a WDN with the EPS-GGA method is not completely exhausted yet and deserves further analysis. For example, in Todini (2011) $\theta = 0.822$ is suggested as optimal: however, no further explanations are given. It is well known (see, for example, Hairer & Wanner (2010)) that, for the A-stability of the theta method, $\theta$ has to be $\geq 0.5$. However, this condition does not ensure the non-oscillatory monotonic local convergence of the method (Barclay *et al.* 2000), which can be guaranteed only if

$$\theta > 0.5 \tag{11}$$

and

$$\Delta t < \frac{2}{g'(\beta)(2\theta-1)} \text{ for } g'(\beta) > 0 \tag{12}$$

$$(1-\theta)\Delta t|(g'(\beta)) < 1 \text{ for } g'(\beta) < 0 \tag{13}$$

where $\beta$ represents a solution of ODE (9). In particular Equations (12) and (13) state that an optimal choice for $\theta$ cannot be independent of the choice of both the discretisation timestep $\Delta t$ and the value of $g'(\beta)$. Therefore a 'universal' optimal value for $\theta$ and $\Delta t$ cannot be determined because the $g'(h)$ function will ultimately depend on the features of the WDN under consideration.

Looking at Equations (12) and (13), it is worthwhile mentioning that, as $\theta$ is chosen closer to 1, the conditions on $\Delta t$ are less stringent. Alternatively, for a fixed $\Delta t$, conditions (12) and (13) are satisfied for $\theta$ values closer to 1 as long as $|g'(\beta)|$ increases.

These considerations call for a more in-depth analysis on the choice of the optimal values for the $\theta$ parameter. This analysis has been performed using the WDN scheme proposed by Todini (2011, see Figure A1 in the appendix). The only difference in this WDN scheme is the diameter of pipe 2, which has been changed to 400 mm instead of 200 mm in order to enhance the unsteadiness of the system. A constant time integration step $\Delta t = 5$ min has been used for all the simulations. In Figures 2(a) and (b) the pipe flows and the tank head patterns are reported respectively for a simulation using EPANET-EPS with $\theta = 0.51$. Anomalous oscillation in time are evident for both the tank heads and especially for pipe flows. For the simulation with $\theta = 0.822$ (the same value as used in Todini (2011)), these oscillations are not present anymore in the tank head patterns reported in Figure 2(d), but can still be noticed in the pipe flows pattern shown in Figure 2(c). It is worthwhile noting that the simulation of Todini (2011) did not present oscillations at all, but the system was 'less unsteady' in that case as noted earlier. Finally in Figures 2(f) and (e) the pipe flows and the tank head patterns are reported respectively for a simulation using EPANET-EPS with $\theta = 1$. In this case no oscillations are present at all in the tank head and pipe flow patterns. This analysis therefore confirms that a choice of $\theta$ close to unity is recommended, especially in cases presenting strong temporal variations in tank levels, which means high values of $|g'(\beta)|$ are expected.

These results coincide perfectly with what is suggested in Giustolisi et al. (2012) where the new Generalized-GGA in introduced. According to Giustolisi et al. (2012) the EPS-GGA is only a particular case of the G-GGA, showing that

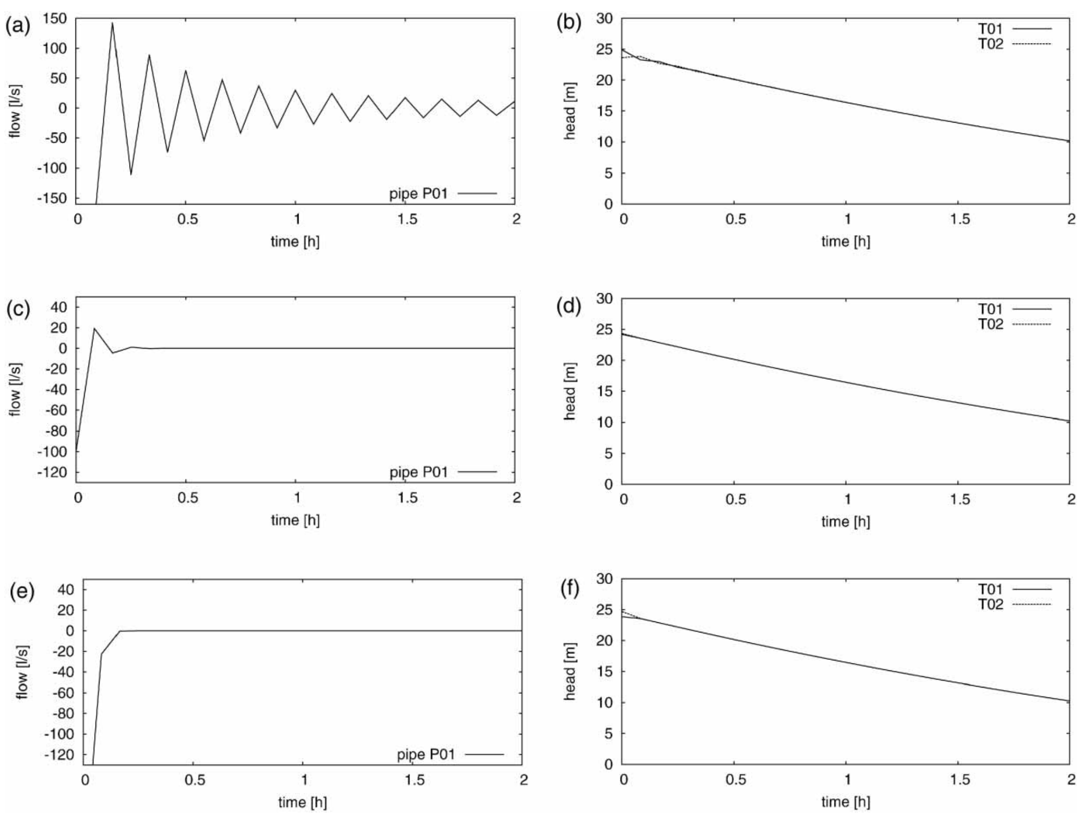


**Figure 2** | Performances of EPANET-EPS for different values of the parameter $\theta$. Integration time step $\Delta t = 5$ m in pipe P01 flow on the left and tanks T01 and T02 heads on the right. (a) and (b) with $\theta = 0.51$, (c) and (d) with $\theta = 0.822$, as suggested in Todini (2011), and (e) and (f) with $\theta = 1.00$, as implicitly suggested in Giustolisi et al. (2012).

the use of theta is not required by setting the value of theta equal to 1 in the EPS-GGA.

## APPLICATION TO A REAL CASE

The original EPANET and the modified version EPANET-EPS are applied to a real case in order to further investigate the performances of the new code. The test case carried out was that of the water supply system of Mataba in Rwanda, which supplies water to about 7,000 people. The scheme of the system is shown in Figure 3, while Table 1 reports its main features and the pattern function of the demand at the network nodes is reported in Figure 4. This network is characterised by four small tanks located close to each other. This could be a particularly critical configuration for the problems of instability and spurious oscillations in the solution of the system, even for the new version of EPANET proposed in this paper. A spring supplies tank T01 through a pump PP. The tank T01 in turn supplies the entire water distribution network. The water demand is concentrated just at the public fountains, represented by small circles in Figure 3. The role of these tanks is to temporarily store the excess water from the spring, conforming to the demand pattern represented in Figure 4.

Some of the results obtained by the numerical simulations are summarised in Figure 5, which refers to the hydraulic behaviour of tanks T02, T03 and of pipe P09. The solution obtained with EPANET clearly shows strong oscillations in the daily time history of the tank heads T02 and T03 (Figures 5(c) and (e), respectively). These oscillations are completely absent in the corresponding solutions obtained by implementing EPANET-EPS (Figures 5(d) and (f), respectively). The spurious oscillations are far more evident in the daily time history of the flow obtained using EPANET at pipe P09 (Figure 5(a)), but yet again are not present at all in the corresponding time history obtained with EPANET-EPS as shown in Figure 5(b).

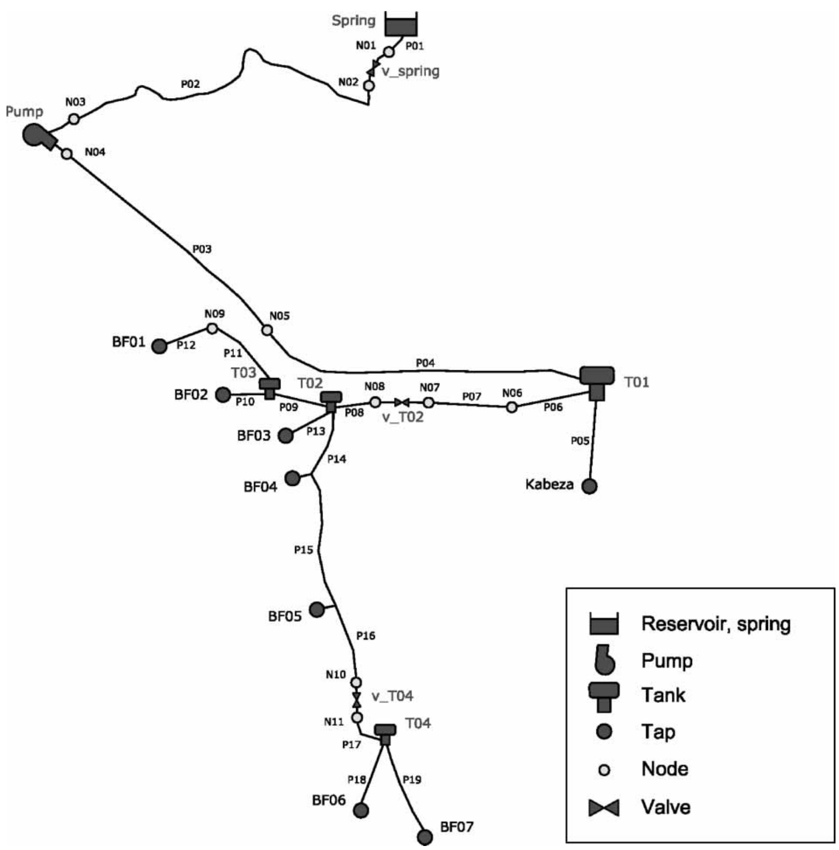


**Figure 3** | The water distribution network of Mataba (Rwanda).

**Table 1** | Tank heads, initial level, and maximum and minimum level in the Mataba water distribution network

| Tank | Head (m) | Init.Lev. (m) | Min.Lev. (m) | Max.Lev. (m) |
|---|---|---|---|---|
| Tank T01 | 2,193 | 1.5 | 1 | 2 |
| Tank T02 | 2,140 | 2 | 0.5 | 2 |
| Tank T03 | 2,109 | 2 | 0.5 | 2 |
| Tank T04 | 2,068 | 2 | 0.5 | 2 |

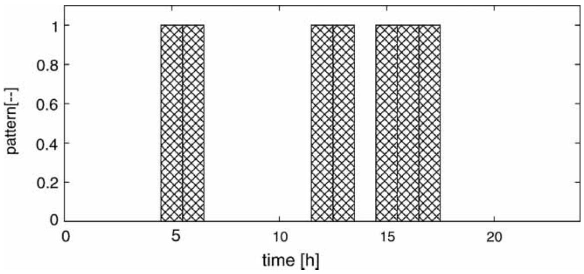


**Figure 4** | Water demand pattern at the taps in the Mataba WDN.

## BOUNDARY CONDITIONS: MAXIMUM AND MINIMUM TANK LEVELS

In this section we describe the methodologies adopted for taking into account the maximum and minimum values within which the tank levels have to operate. This is a key point, often inadequately analysed, which would deserve much more attention. In the following it will be shown that an improper operative implementation of the boundary limits can lead to unrealistic solutions of hydraulic problems. The subsequent paragraphs aim to describe and apply original procedures to fulfil the maximum and minimum level BC of the tanks: a comparison is done with the results obtained by applying the procedures in Todini (2011) and those implemented in EPANET.

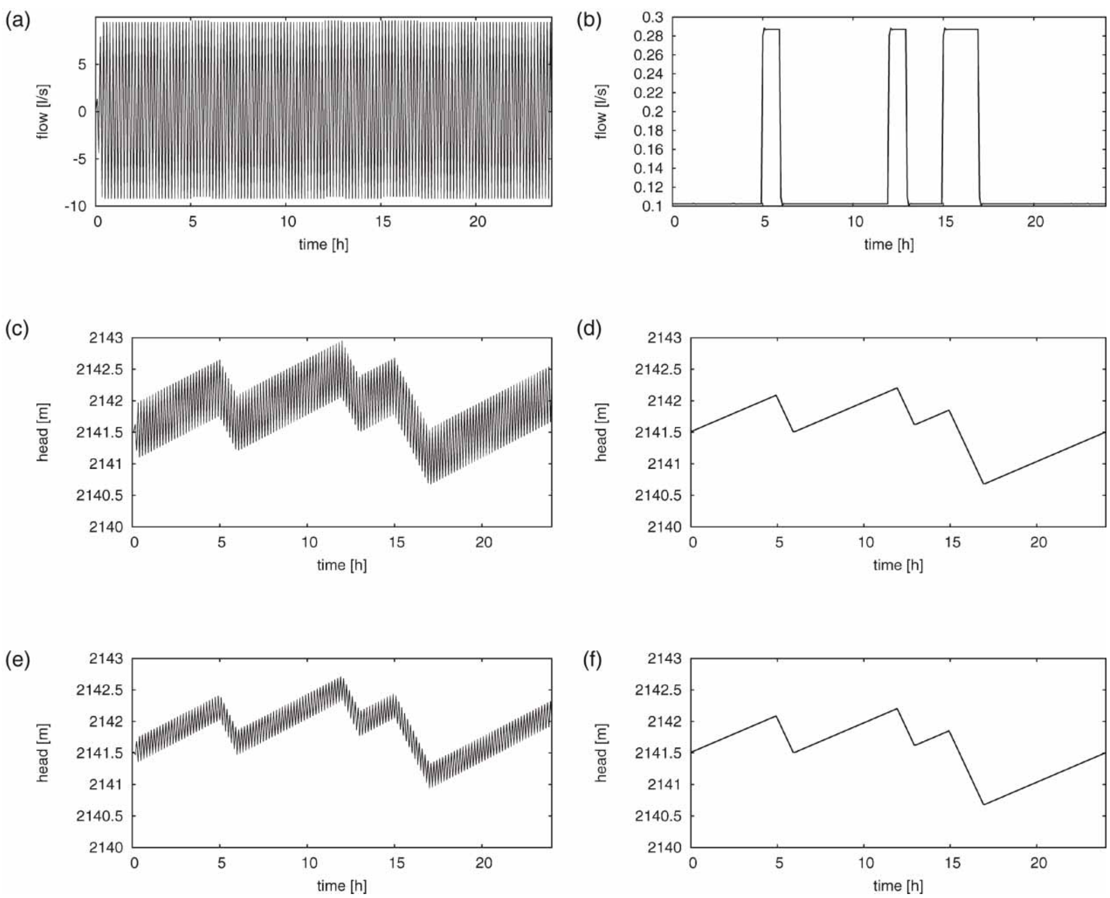


**Figure 5** | Mataba WDN simulation. Pipe P09 flow patterns and head patterns at tanks T02 and T03. (a) Flow pattern in pipe P09 simulated with EPANET; (b) flow pattern in pipe P09 simulated with EPANET-EPS; (c) head pattern at tank T02 simulated with EPANET; (d) head pattern at tank T02 simulated with EPANET-EPS; (e) head pattern at tank T03 simulated with EPANET; and (f) head pattern at tank T03 simulated with EPANET-EPS.

## The maximum level boundary condition (MLBC)

In order to correctly simulate tank behaviour, we have developed an accurate strategy to manage the MLBC for the tank levels, which does not involve much other than the code modifications already introduced in the previous paragraph.

The strategy consists in connecting the tank, through a pipe, to an emitter located right at the maximum level of the tank, $h_{\mathrm{Tmax}}$. The emitter and the connecting pipe (the so-called 'fictitious pipe' in the EPANET Manual (Rossman 2000)) have to be characterised by a discharge coefficient value and by a pipe diameter value respectively both large enough to allow for spillout without a significant increase of the water level of the tank. The advantages of this procedure is mainly that it does not require any modification of the EPANET source code, but just a proper set-up of the input file. Alternatively, Todini (2011) suggests changing the variable head tank node to a fixed head node once the tank head rises to the $h_{\mathrm{Tmax}}$ value. Instead, EPANET stops any inflow if a tank reaches its maximum level. All three procedures have been applied for comparison to the WDN represented in Figure 6(a). The system is composed of a reservoir connected to four tanks, each of them having maximum level limitations. This scheme has been chosen because it is particularly severe for the MLBC test. In Figure 6(b) the set of emitters linked to each tank is reported. The features of the WDN are reported in Table 2 and the graphical representation of the demand pattern function at the network nodes is reported in Figure 7.

In Figure 8(a) the tank head patterns for each tank are reported, as calculated by the original version of EPANET with a integration time step of 1 s, which can be considered as the reference solution.

Figure 8(b) shows the tank patterns obtained with the original version of EPANET using a time step of 5 min. The expected unrealistic oscillations of the heads at tanks T02 and T04 are evident. Figure 8(c) shows the tank head patterns obtained with EPANET-EPS using time step 5 min in which the methodology proposed by Todini (2011) for the MLBC has been implemented. The obtained solution differs significantly from the reference solution when the tank levels approach their maximum levels. In particular, using Todini's procedure tanks T01 and T02 reach their MLBC between 7 h and 10 h, roughly, despite the fact that they should not even reach it at all with the reference solution. The results obtained with EPANET-EPS using the same time step of 5 min and the proposed procedure for the MLBC are reported in Figure 8(d). The solution does not differ significantly from the reference solution. This is a confirmation of the accuracy of the proposed procedure for the MLBC problem. We would like to point out that, even though the proposed method changes the physical representation of a tank (from one with an implicit altitude control valve to one with an overflow pipe), this does not produce different network solutions regardless of whether there are oscillating tanks or not. A further development of the EPANET 2 source code is suggested in order to take into account different tank typologies.

## The minimum level boundary condition (mLBC)

In this subsection the procedure implemented to face the problem of mLBC is described and then compared to the procedure developed in the original EPANET source code and with that suggested by Todini (2011). As a test case the same water supply system network proposed in Todini (2011) shown in Figure 9 was chosen. The initial and boundary conditions are listed below:

(i) The tank T02 minimum level is set to 5 m (not 0 m as in Todini (2011)).
(ii) The reservoir water head is set to −10 m (not 0 m as in Todini (2011)) to allow for the tanks to empty faster (see Figure 9 and Table 3).
(iii) The initial levels of the tanks T01 and T02 are set equal to 20 m and 30 m, respectively.
(iv) The tank T01 minimum level is set to 0 m.
(v) The tanks have the same cross-sectional area.
(vi) The reservoir R02 (not present in Todini (2011)) water head is set to 100 m to allow for the filling of tank T01.
(vii) The initial conditions flow are equal to zero along each pipe.
(viii) Pipe P01 has a diameter of 200 mm (in Todini (2011) the P01 diameter was 100 mm), all the other pipes have a diameter of 100 mm.

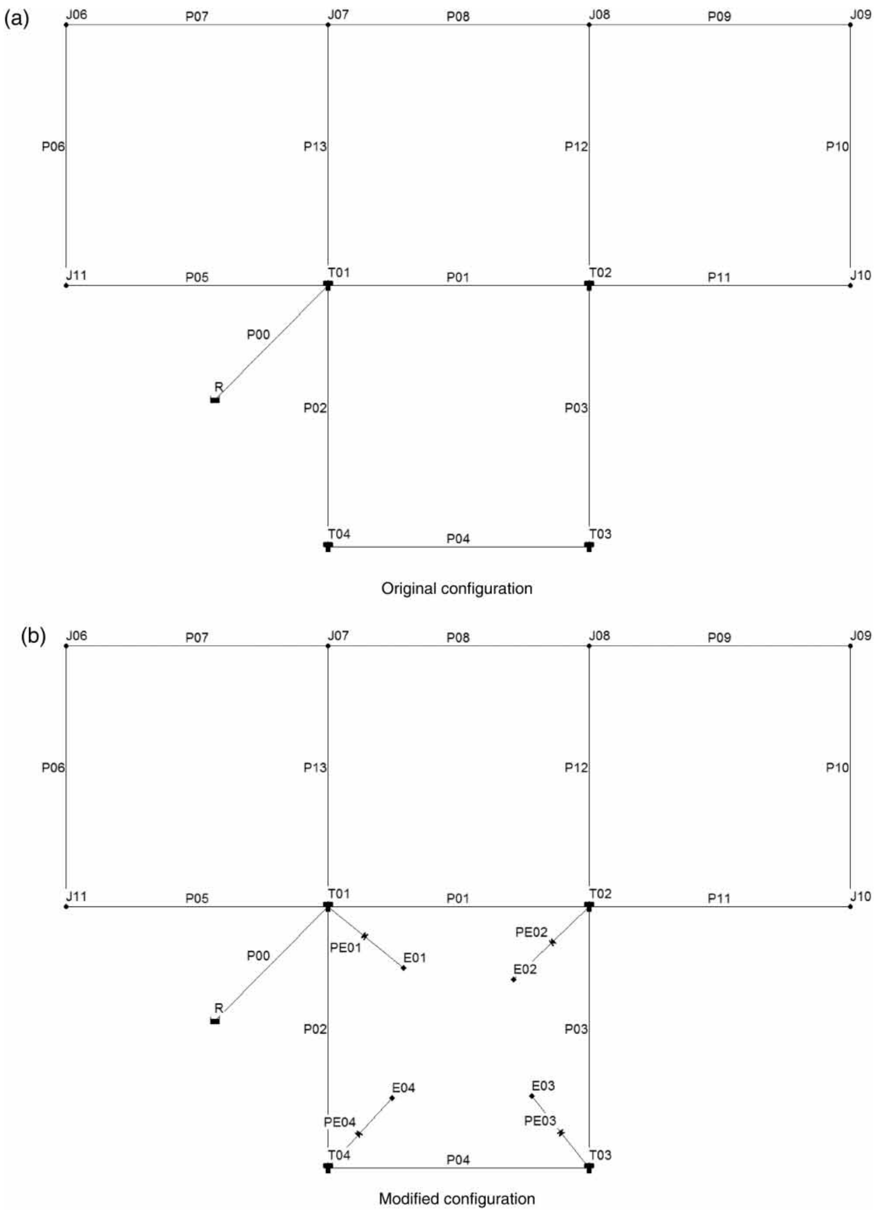


**Figure 6** | MLBC test case: schematic representation of the WDN.

(ix) The pipe P03 is open for simulation time $t = 3$ h and afterwards is closed.

(x) The pipe P05 is closed for simulation time $t = 3$ h and afterwards is open.

The introduction of reservoir R02 and conditions (ix) and (x) were not present in Todini (2011) and have now been introduced in order to analyse the behaviour of the system even during the phase of tank filling. In this configuration a flow from T02 to T01 through pipe P01 is expected, together with a flow from T01 and T02 to the reservoir R01, through pipes P02 and P03. Moreover the water levels in both tanks are expected to decrease until the minimum levels are reached in each tank (i.e. 0 m for T01 and 5 m for T02, respectively). The tank T02 should reach the minimum permitted level faster than

**Table 2** | Node topology and characteristic of the WDN used as test case for the MLBC

| | Elevation (m) | Max. Lev. (m) | Min. Lev. (m) | Demand (l/s) | Pattern |
|---|---|---|---|---|---|
| Junct 6 | 120 | – | – | 8 | 1 |
| Junct 7 | 90 | – | – | 8 | 1 |
| Junct 8 | 70 | – | – | 8 | 1 |
| Junct 10 | 90 | – | – | 8 | 1 |
| Junct 11 | 110 | – | – | 8 | 1 |
| Junct 12 | 130 | – | – | 8 | 1 |
| Reservoir R | 170 | – | – | – | – |

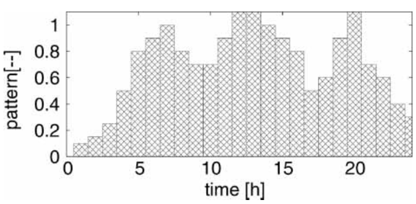


**Figure 7** | Maximum Level Boundary Condition test case: water demand pattern.

tank T01 and, as a consequence, as soon as T02 reaches its minimum level, the discharge in pipes P01 and P03 should go to zero. Then, after time $t = 3$ h, when pipe P05 is opened, we expect that T01 head starts to increase and then, when the head in T01 is higher than the T02 minimum level, the flow through P01 starts. Finally, when the discharge through P01 becomes higher than the discharge through P03 is when the head in T02 begins to rise up.

The EPANET original version was applied to this test case; the obtained results are reported in Figure 10(a) for the pipe flow patterns and in Figure 10(b) for the tanks T01 and T02 head patterns. One can observe that the head behaviour in tanks T01 and T02 shows some oscillations during the emptying phase and even the simulation of flow in pipe P01 shows unnatural oscillations. In particular, we note that tank T01 does not reach its minimum level. Indeed the water still continues to unrealistically flow from T02 to T01 through P01 and also from T01 and T02 to R01 through P02 and P03. The reason for this non-realistic behaviour is due to the fact that the original version of EPANET recognises the head

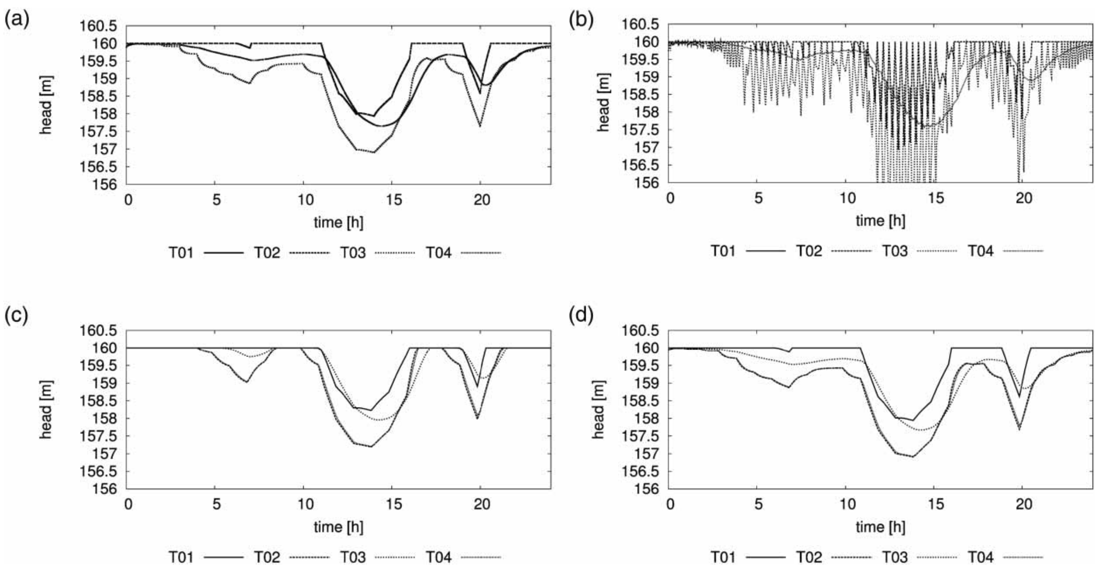


**Figure 8** | MLBC test case. Tank head, $\theta = 1$. (a) Reference solution obtained with EPANET and integration time $\Delta t = 1$ s; (b) simulation with EPANET and integration time $\Delta t = 5$ min; (c) simulation with EPANET-EPS and procedure of Todini (2011), integration time $\Delta t = 5$ min; and (d) simulation with EPANET-EPS and proposed procedure, integration time $\Delta t = 5$ min.

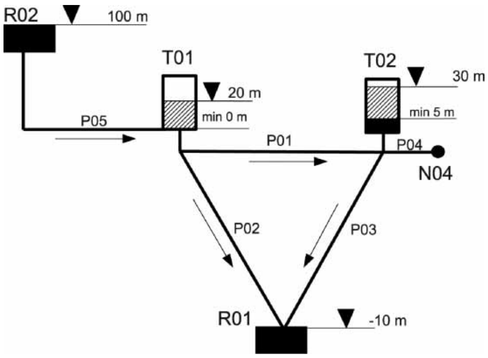


**Figure 9** | mLBC test case. Schematic representation of the WDN. Two emptying and filling interconnected tanks with different initial and minimum levels. The arrows indicate the positive sign of the water flow.

**Table 3** | Nodes, pipe topology and characteristic of the WDN used as a test case for the mLBC

| | Elevation (m) | Min. Lev. (m) | Diameter (mm) | Status |
|---|---|---|---|---|
| Tank T01 | 0 | 0 | – | – |
| Tank T02 | 0 | 5 | – | – |
| N04 | 0 | – | – | – |
| Reservoir R01 | −10 | – | – | – |
| Reservoir R02 | 100 | – | – | – |
| Pipe P01 | – | – | 200 | Open |
| Pipe P02 | – | – | 100 | Open |
| Pipe P03 | – | – | 100 | Closed till time $t = 3$h and after open |
| Pipe P04 | – | – | 100 | Open till time $t = 3$h and after closed |
| Pipe P05 | – | – | 100 | Close |

differences between the minimum levels in tanks T01 and T02 and, as a consequence, it continues to generate a discharge from T02 to T01 using the energy balance equation between the tanks. The wrong simulation of the mLBCs affects the hydraulic simulation of the system after $t = 3$ h when pipe P05 is opened and the tanks should fill up again.

The results obtained with EPANET-EPS and the procedure proposed by Todini (2011) for the mLBC are reported in Figure 10(c) for the pipe flow patterns and Figure 10(d) for the head patterns of tanks T01 and T02. In this case no oscillations of the flow in pipe P01 were observed. However, the results show some inconsistencies which deserve to be discussed in further detail. Looking at Figures 10(c) and (d) we can point out three instants:

(i) At time $t_A$ the tanks T01 and T02 assume the same water level (Figure 10(d)). At the same time, as expected, the flow in pipe P01 connecting T01 to T02 goes to value zero (Figure 10(c)).

(ii) At time $t_B$ the water level in tank T02 reaches its minimum permitted value (5 m, Figure 10(c)), nevertheless the level still continues to decrease even for time $t > t_B$. Correspondingly, the flow in pipe P01 abruptly assumes non-realistic positive values (Figure 10(c)), thus indicating that water unrealistically moves from T01 to T02; this flow continues until $t_C$.

(iii) At time $t_C$ the water level in tank T01 reaches its minimum allowed value (0 m), at the same time even T02 incorrectly reaches this level.

After this, the water levels of the tanks suddenly decrease to the minimum head of the WDN, which corresponds to R01 head (−10 m). For time $t = 3$ h, when pipe P05 gets open, an abrupt increase of the tank head is noticed. This takes place because the tanks have become regular junction nodes, which are instantaneously affected by head changes. This example clearly shows that the procedure proposed in Todini (2011) does not allow one to thoroughly take into account the minimum level condition.

In order to overcome the above-mentioned inconsistencies, the original EPANET 2 procedure (*tankstatus()*) that control the tank minimum level condition has been slightly modified to take into account adjacent tanks too: the pipe linking two adjacent tanks is disconnected when the tank water level of one of the two tanks reaches values lower than the mLBC levels and the water level of the other tank is lower than the water minimum level of the first.

The results obtained with EPANET-EPS and the new procedure are reported in Figure 10(e) for the pipe flow patterns and in Figure 10(f) for the head patterns for tanks T01 and T02. No oscillation of the tank heads and no inconsistency of flows in the pipes are noticed. Moreover the

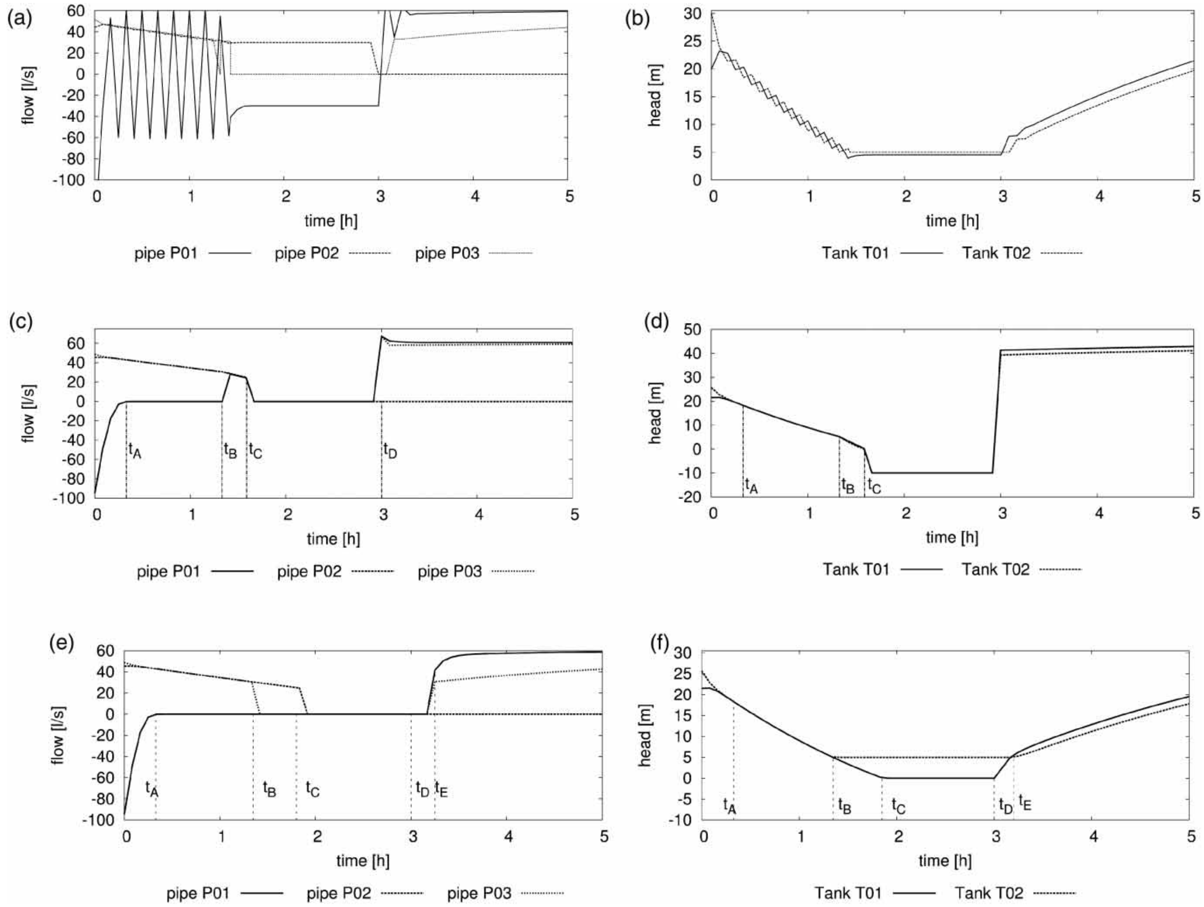


**Figure 10** | mLBC test case. Pipe flow and tank head, integration time step $\Delta t = 5$ min, $\theta = 1$. Pipe flow on the left and tank head on the right. (a) Pipe flow patterns obtained with EPANET; (b) tank head patterns obtained with EPANET; (c) pipe flow patterns, simulation with EPANET-EPS and procedure of Todini (2011); (d) tank head patterns, simulation with EPANET-EPS and procedure of Todini (2011); (e) pipe flow patterns, simulation with EPANET-EPS and proposed procedure; and (f) tank head patterns, simulation with EPANET-EPS and proposed procedure.

water levels in tanks T01 and T02 do not go below the minimum permitted conditions and the flow in pipe P01 maintains zero values when the water levels in T01 and T02 are the same or even when one of them reaches the minimum level. As expected at time $t = 3$ h tank T01 starts to fill and there is no flow through pipe P01 till time $t_E$ when the water level in T01 reaches the T02 minimum level. After time $t_E$ T02 also starts to fill.

## CONCLUSIONS

The original source code of EPANET has been extended to included the EPS-GGA method, with the aim of overcoming the problems of spurious water level oscillations at variable tank heads as sometimes reported by EPANET users. Such problems as oscillations are related to the explicit nature of the scheme adopted by the steady-state EPANET-GGA version for the solution of the unsteady problem of a WDN with variable tank heads. The developed implicit EPANET-EPS version has been successfully applied to different test cases, showing significant improvements with respect to the original EPANET-GGA scheme. EPANET-EPS also includes some procedures suitably developed and tested for the correct numerical representation of the boundary conditions of the tank levels, in particular:

(i) A simple and proper methodology has been presented for the schematisation of the maximum tank level conditions, which is particularly easy to be implemented with the tools recently included in EPANET.

(ii) The extension of the minimum tank level conditions has been implemented. This procedure seems to correctly impose the respect of the maximum and minimum permissible levels in a generic tank belonging to the WDN without producing the numerical inconsistencies which are typical of other schemes already implemented in EPANET original source code or proposed in the scientific literature (e.g. Todini 2011) and tested in this paper.

(iii) Finally, starting from the work of Hairer & Wanner (2010) and Barclay *et al.* (2000), the dynamics of the theta method for the EPS-GGA has been theoretically demonstrated. Illustrative examples show that a fully implicit discretisation of the mass and energy balance of the WDN governing equations ($\theta = 1$) can be considered an optimal choice in order to avoid spurious oscillations, as suggested implicitly in Giustolisi *et al.* (2012).

## ACKNOWLEDGEMENTS

The authors would like to acknowledge the stimulating role of Ezio Todini for his fruitful discussions which clarified many aspects of the water level oscillations in GGA-based water distribution analysis models. Particular thanks are due to Andrea Filippi who has contributed and worked on the project as well as the hydraulic simulation of the Mataba WDN. Part of the work has been funded by EU contract EU no. 9 ACP RPR 50 21, 'Interventions pour le renforcement du reseau hydraulique et la sensibilisation un usage responsable en collaboration avec le District de Gicumbi dans la Province du Nord Rwanda'.

The authors would also like to thank the three anonymous referees for their constructive comments and remarks.

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First received 26 January 2011; accepted in revised form 1 February 2012. Available online 13 June 2012