cantilevered tubes conveying essentially incompressible fluid was first observed experimentally at McGill University—quite accidentally—in 1971 [1]. It was studied analytically by means of linear theory, at McGill, for both cantilevered systems and systems supported at both ends [2], and elsewhere [3]. It was found that cantilevered systems first lose stability by flutter, while systems with both ends supported lose stability by buckling and then, almost immediately with increasing flow, by flutter. Moreover, this was found to occur in both the beam-type modes ($n = 1$) and the shell-type modes ($n \geqslant 2$).

So far as the cantilevered system was concerned, the experiments verified fully the existence of both beam and shell-type flutter; the former is predominant for relatively thick pipes [4], and the latter for thin pipes (cylindrical shells) [2].

In the case of pipes supported at both ends, however, there were two problems. First, beam-type flutter was predicted when the system was analyzed by means of shell theory, but apparently not when analyzed by means of beam theory! Second, beam-type flutter in this case was not observed experimentally. The first paradox was finally resolved [5,6] when it was realized that previous studies into the stability of this system, considered to be *conservative*, were conducted in terms of static stability methods, incapable of perceiving the existence of dynamic instabilities. Proper analysis showed that beam theory concurs with shell theory in predicting the existence of postbuckling beam-type flutter [5,6]; the system, of course, is not conventionally conservative but *gyroscopic conservative*.

However, the final proof of the existence of postbuckling beam-type flutter in systems with both ends supported rested with experimental evidence. Yet, experimenters were unable to reproduce it. Recourse was made to nonlinear theory, accordingly, to investigate whether linear stability theory is in fact valid in this respect or not. Recent studies [7,8] concluded that, indeed, postbuckling beam-type flutter of such systems is not possible.

However, the existence of shell-type flutter of thin elastic tubes conveying fluid with both ends supported was not questioned because it was actually *observed* [2]. With their present study, the authors[1] cast doubt on this, albeit for a physical model not very close to a round tube, but nevertheless in its essentials quite similar. We thus see the beginnings of yet another question, another paradox; perhaps the authors might like to comment on this.

The "rectangular tube" model of a circular one was actually used by Weaver and Paidoussis previously [9] in not too dissimilar manner (albeit using linear theory), in an attempt to throw some light into the stability of such systems in general. (Unfortunately, this and some of the other references are not quoted in the paper.) The discusser wonders if the same secondary interest that motivated his own and Weaver's work applied to Matsuzaki and Fung's work also: namely, the understanding of such phenomena as the collapse and flutter of pulmonary passages or, for that matter, the generation of Korotkoff sounds in haemodynamics?

References

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Authors' Closure

The authors would like to thank Professor Paidoussis for his interest in our paper.

Among others, the experimental work by Conrad [10] on steady and unsteady flow in a collapsible tube has stimulated us to conduct a series of research [11–13]. Therefore, we have no idea at all to "cast doubt on the existence of shell-type flutter of thin elastic tubes conveying fluid with both ends supported." The main objective of our paper and reference [13] is to show that no postdivergence flutter oscillation is predicted within the framework of nonlinear plate and linearized potential flow theories. We do not think this is a paradox or a contradiction to the experimental results. As pointed out in the discussions and concluding remarks of our paper, we need extreme care in comparing theoretical predictions with experimental results. In a laboratory test it is observed that the amplitude of oscillation of tubes conveying flow is large. Therefore, we wonder if the assumption of the linearized potential flow is valid in analyzing the flutter oscillation of the tube. We suspect that flow separation might play an important role like in stall flutter of the rotary blade of a helicopter. It does not seem that all the existing linear and nonlinear analyses take into full account physical situations of this unexpectedly complex phenomenon.

Finally, a few words will be given to stability analysis based on linear plate or shell equations. A number of such analyses have predicted occurrence of postdivergence flutter. According to our analytic, linear analysis on stability of a cylinder [12], however, one may conclude at most that a disturbance about the undeformed configuration grows in an oscillatory manner as long as the disturbance is infinitesimal.

References

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On Some General Properties of Combined Dynamical Systems[1]

J. P. Wright.[2] Rayleigh's results on added mass and stiffness have produced various statements and interpretations concerning the effects of combining or modifying dynamical systems. The purpose of this letter is to direct the reader's attention to references [1, 2], which provide a basis for further discussion of the results in the paper.

[1] By E. H. Dowell, and published in the March, 1979 issue of the ASME JOURNAL OF APPLIED MECHANICS, Vol. 46, No. 1, pp. 206–209.

[2] Weidlinger Associates, 110 East 59th Street, New York, N. Y. 10022.

As stated on page 720 of [1], "... if two systems are joined together the joining mechanism represents a constraint on the total variable set of the unjoined systems. Therefore the eigenvalues of the joined system are bracketed by the composite eigenvalue spectrum of the component systems." This statement follows from the bounding conditions [1, p. 719]

$$\omega^2{}_k \leq {}^m\omega^2{}_k \leq \omega^2{}_{k+m}$$

where $\{\omega^2{}_k\}$ for $k = 1, 2, \ldots, n$ is the set of frequencies of the unconnected systems with the frequencies ordered in a sequence of increasing values and $\{{}^m\omega^2{}_k\}$ for $k = 1, 2, \ldots, n - m$ is the similarly ordered set of frequencies of the combined system that is obtained by imposing m linear constraint conditions. (reference [2, p. 33], also contains essentially the same result.) The case $m > 1$ corresponds to component systems connected at more than one point, a topic that was mentioned in the paper's concluding remarks.

Basically the subject paper concentrates on the lower bounding conditions with $m = 1$. As stated in the Introduction, "Specifically it is shown that, if two component systems are connected at a point, each combined system frequency is increased from its component value or, in the exceptional case, is unchanged." It should be noted that, in this interpretation, the highest component mode "disappears" in the combined system (see page 207 near bottom of left column). An alternative interpretation can be obtained by concentrating on the upper bounding conditions. In this case the statement can be changed simply by replacing "increased" by "decreased" with the implicit understanding that the lowest component mode disappears rather than the highest.

Additional consideration is needed for components involving massless degrees of freedom. For example, the disconnected spring-mass, Fig. 4 (b) of the paper, has two degrees of freedom, z and z_0, connected by a spring; the mass at z is positive but the mass at z_0 is zero. As implied on page 208, the two frequencies of this component are zero and infinity, as can be seen by considering the limiting case of a mass m_0 at z_0 with m_0 tending to zero. Alternatively, a small but finite mass can be taken from the beam attachment point when the spring-mass is conceptually detached. By one technique or another, it is necessary to use component mass matrices which are strictly positive-definite since this property is fundamental to the derivation of the bounding conditions.

References

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Author's Closure

The author would like to thank the discusser for bringing the two references to his and the reader's attention. These references do offer an approach for establishing eigenvalue bounds using algebraic methods and mini-max properties of quadratic forms. Those readers who are familiar with this approach may find the graphical, constructive method of the author's paper an interesting alternative. All readers should find a comparison of the two approaches of interest.

The discusser's remarks on the interpretation of the paper's results are also useful and recommended to the reader.