Violation of Universal Yukawa Coupling and Quark Masses

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We analyze the quark mass hierarchy and the CKM matrix precisely using the universal Yukawa coupling framework with small violations. We estimate the ranges of the values of 8 violation parameters (\( \delta_2^u, \delta_2^d, \delta_3^u, \delta_3^d, \delta_4^u, \phi_2, \phi_3 \)) in our quark mass matrices satisfying quark mass ratios and the CKM matrix, where \( \phi_2 \) and \( \phi_3 \) are phases. Without these phases, the solution satisfying the quark mass ratios and the CKM matrix cannot be obtained. These parameters obtained can explain the CP violation effects seen in experiment. From the estimated parameters, we hypothesize a form for quark mass matrices containing only 5 parameters.

§1. Introduction

The origin of the mass hierarchy of quarks and leptons has been investigated using various theories beyond the standard model (SM) by many authors. However, many attempts beyond SM constructing mass matrix patterns at the GUT scale in SUSY theories or at the string scale in string models, although quite successful, cannot produce results in complete agreement with precise low energy data. With the present situation, in which there are precise analyses of \( B^0 - \bar{B}^0 \) mixing, the CP-violating parameter \( \varepsilon \) of the \( K^0 - \bar{K}^0 \) system, and the determination of the top-quark mass, one should analyze the mass hierarchy of quarks and the Cabibbo-Kobayashi-Maskawa (CKM) matrix for model building beyond SM using only properties independent of the model assumed.

For quark mass matrix patterns at low energy, there are the Fritzsch-type, the Stech-type model, the democratic-type model, and the universal Yukawa coupling-type model. As discussed in §3, a quark mass matrix like the universal Yukawa coupling-type model is translated into a Fritzsch-type model and other models beyond the SM by a unitary matrix. Thus, we adopt a quark mass matrix like the democratic and the universal Yukawa coupling-type models with small violations from the universality which cause the mass hierarchy. Our model does not make any assumptions on the violations, and treats violation parameters as free. First, we study the mass hierarchy mechanism in the limit of the universal Yukawa coupling. The \((u, c, t)\) and \((d, s, b)\) quark mass matrices are expressed, under the universality of the Yukawa coupling strength, as

\[
M^q = \Gamma^q \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad (q = u, d) \tag{1}
\]

where \( \Gamma^u \) and \( \Gamma^d \) are real, and are not assumed universal. The nonuniversality of \( \Gamma^u \) and \( \Gamma^d \) is guaranteed by, e.g., a minimal supersymmetric gauge model, in which
the up and down quarks acquire their masses through couplings to two different Higgs multiplets. It is well known that this type of the mass matrix is diagonalized as \( \text{diag}[0, 0, 3\Gamma^q] \) by the orthogonal matrix \( T_0 \); \( \text{diag}[0, 0, 3\Gamma^q] = T_0 M^q T_0^{-1} \), where \( T_0 \) is

\[
T_0 = \begin{pmatrix}
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\
\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}
\end{pmatrix}.
\]

Thus Eq. (1) gives a large mass gap between the heaviest quark and the other two quarks.

Next, we introduce small violations of the Yukawa coupling strength into universal coupling (Eq. (1)). We assume for the violation parameters that these violations are very small and are caused by the coupling between different quarks. In general, these small violations contain the phases. Under these assumptions, the quark mass matrices are parametrized as

\[
M^q = \Gamma^q \begin{pmatrix}
1 & (1 - \delta^q_1)e^{i\phi^q_1} & (1 - \delta^q_3)e^{i\phi^q_3} \\
(1 - \delta^q_1)e^{-i\phi^q_1} & 1 & (1 - \delta^q_3)e^{i\phi^q_3} \\
(1 - \delta^q_2)e^{-i\phi^q_2} & (1 - \delta^q_3)e^{-i\phi^q_3} & 1
\end{pmatrix}, \quad (q = u, d)
\]

where \( \delta^q_i \) and \( \phi^q_i \) are small real parameters:

\[
\delta^u,d_i \ll 1, \quad \phi^u,d_i \ll 1. \quad (i = 1, 2, 3)
\]

Here it should be stressed that the large mass differences are produced by the universal coupling and that what distinguish the quarks are not the masses but other characters. Branco, Silva-Marcos and Rebelo\(^4\) studied mass matrices of the type (3), but they equated the type of this mass matrix with square of the quark mass.

Under the assumption of small violation of universal coupling, we can obtain the second mass gap between two degenerate zero mass states which are taken from the universal mass matrix (1). This is shown from the mass matrix (3) neglecting phases:

\[
M^q = \Gamma^q \begin{pmatrix}
1 & 1 - \delta^q_1 & 1 - \delta^q_2 \\
1 - \delta^q_1 & 1 & 1 - \delta^q_3 \\
1 - \delta^q_2 & 1 - \delta^q_3 & 1
\end{pmatrix}, \quad (q = u, d)
\]

This mass matrix is transformed by \( T_0 \) to

\[
T_0 M^q T_0^{-1} = \Gamma^q \begin{pmatrix}
\delta^q_1 \\
(\delta^q_2 - \delta^q_3)/\sqrt{3} \\
(-\delta^q_2 + \delta^q_3)/\sqrt{6}
\end{pmatrix}
\begin{pmatrix}
(\delta^q_2 - \delta^q_3)/\sqrt{3} & (\delta^q_1 + \delta^q_3)/\sqrt{6} \\
(\delta^q_1 + 2\delta^q_2 + \delta^q_3)/3 & (9 - 2\delta^q_1 - 2\delta^q_2 - 2\delta^q_3)/3 \sqrt{2}
\end{pmatrix},
\]

and then if \( \delta^q_1 \ll \delta^q_2 \approx \delta^q_3 \ll 1 \), the three eigenvalues become approximately \( 0, 4\delta^q_3 \Gamma^q/3, (3 - 4\delta^q_2/3)\Gamma^q \). This tendency is confirmed, and the allowed ranges of
(δ_1^q, δ_2^q, δ_3^q) are determined precisely from the analysis of quark mass ratios and the CKM matrix in the following numerical study.

In §3, we comment on the difference between the results of our model and those of others. 1) - 4)

§2. Numerical analysis

The mass matrices (3) contain 6 violation parameters for the (u, c, t) and (d, s, b) sector in addition to the \( I^q \), respectively. First, we consider the 3 parameter case (Eq. (5)) neglecting the phases for simplicity, and later we consider the case including phases. We diagonalize the mass matrices (5) to the diag\([m_u, m_c, m_t]\) and diag\([m_d, m_s, m_b]\) for \( q = u \) and \( d \) by the unitary matrices \( T(\delta_1^q, \delta_2^q, \delta_3^q) \) and \( T(\delta_1^d, \delta_2^d, \delta_3^d) \), respectively:

\[
T(\delta_1^q, \delta_2^q, \delta_3^q)M_qT^{-1}(\delta_1^q, \delta_2^q, \delta_3^q) = M_D^q , \quad (q = u, d)
\]

\[
M_D^u = \text{diag}[m_u, m_c, m_t], \quad M_D^d = \text{diag}[m_d, m_s, m_b]. \tag{7}
\]

The eigenvalues of the mass matrices are not the physical masses but the parameters in the Lagrangian. These quark masses (eigenvalues) are running masses which should all be taken on a single energy scale. In order to estimate the parameters \((\delta_1^q, \delta_2^q, \delta_3^q)\), we use the quark mass ratios. These mass ratios are, to a good approximation, independent of the energy scale, so the scale can be arbitrarily chosen. 6)

For the values of the light and medium heavy quark \( u, d, s \) and \( c \) masses, we use the world average cited in Ref. 7),

\[
m_u = 2 - 8 \text{ MeV}, \quad m_c = 1.0 - 1.6 \text{ GeV},
\]

\[
m_d = 5 - 15 \text{ MeV}, \quad m_s = 100 - 300 \text{ MeV}, \tag{8}
\]

because these values are close to the mass values at the scale \( \mu = 1 \text{ GeV} \), 6) \( m_u = 5.1 \pm 1.5 \text{ MeV}, m_d = 8.9 \pm 2.6 \text{ MeV}, m_s = 175 \pm 55 \text{ MeV}, m_c = 1.35 \pm 0.05 \text{ GeV} \).

For the heavy quark \( b \) and \( t \) masses, we estimate the running mass \( m_q(\mu = 1 \text{ GeV}) \) related to the physical mass \( m_q^{\text{phys}} \) in the first order QCD as

\[
m_q^{\text{phys}} = m_q(\mu = m_q) \left[ 1 + \frac{4}{3\pi} \alpha_s(\mu = m_q) \right], \tag{9}
\]

where \( \alpha_s(\mu) \) is the running coupling constant and \( m_q(\mu) \) the running mass. Using the Gasser and Leutwyler procedure, 6) we can estimate the \( m_b(\mu = 1 \text{ GeV}) \) and \( m_t(\mu = 1 \text{ GeV}) \) from the physical mass \( m_b^{\text{phys}} = 4.3 \pm 0.2 \text{ GeV} \) and \( m_t^{\text{phys}} = 174 \pm 22 \text{ GeV} \) 7) as

\[
\Lambda = 0.1 \text{ GeV} \quad \quad \Lambda = 0.2 \text{ GeV}
\]

\[
m_b(\mu = 1 \text{ GeV}) = 5.08 \pm 0.28 \text{ GeV}, \quad 5.49 \pm 0.29 \text{ GeV},
\]

\[
m_t(\mu = 1 \text{ GeV}) = 289 \pm 41 \text{ GeV}, \quad 327 \pm 48 \text{ GeV}, \tag{10}
\]

for the flavor number \( N_f = 3 \). We write the values of \( m_b(\mu = 1 \text{ GeV}) \) and \( m_t(\mu = 1 \text{ GeV}) \) for the renormalization group invariant scale \( \Lambda = 0.1 \text{ GeV} \) and \( \Lambda \)
= 0.2 GeV cases, because these mass values are sensitive to the values of \( \Lambda \). Here-
after, we write the mass \( m_q(\mu = 1 \text{ GeV}) \) as \( m_q \). From these mass values, we obtain
the quark mass ratios,

\[
\frac{m_u}{m_c} = 0.0038 \pm 0.0025, \quad \frac{m_d}{m_s} = 0.050 \pm 0.035, \\
\frac{m_c}{m_t} = 0.0042 \pm 0.0013, \quad \frac{m_s}{m_b} = 0.038 \pm 0.019,
\]

(11)

where we have used the average values of \( m_q(\mu = 1 \text{ GeV}) \) and \( m_t(\mu = 1 \text{ GeV}) \) for
\( \Lambda = 0.1 \text{ GeV} \) and \( \Lambda = 0.2 \text{ GeV} \), and included the deviation from the average value
in errors.

We estimated the allowed regions of \((\delta_1^q, \delta_2^q, \delta_3^q)\) numerically satisfying the con­
straint in which the ratios of the eigenvalues \((m_u, m_c, m_t)\) and \((m_d, m_s, m_b)\) of the
mass matrices (5) are included in the experimental ranges of quark mass ratios (11).
We show the allowed regions for the \((m_u, m_c, m_t)\) sector in Figs. 1(a) and (b), and
for the \((m_d, m_s, m_b)\) sector in Figs. 1(c) and (d). Figures 1(a) and (b) display the
allowed regions of the \((\delta_1^u, \delta_2^u)\) plane corresponding to \( \delta_1^u = 0.00005, 0.0001 \) for the
\((u, c, t)\) sector, respectively, and Figs. 1(c) and (d) the allowed regions of the \((\delta_1^d, \delta_2^d)\)
plane corresponding to \( \delta_1^d = 0.005, 0.01 \) for the \((d, s, b)\) sector, respectively.

The analytic and approximate expressions for the eigenvalues of the mass matrix
(5) are

\[
m_1^q \approx \left[ \frac{1}{3} (\delta_1^q + \delta_2^q + \delta_3^q) - \frac{1}{3} \xi^q \right] \Gamma^q,
\]

\[
m_2^q \approx \left[ \frac{1}{3} (\delta_1^q + \delta_2^q + \delta_3^q) + \frac{1}{3} \xi^q \right] \Gamma^q,
\]

\[
m_3^q \approx \left[ 3 - 2 \left( \delta_1^q + \delta_2^q + \delta_3^q \right) \right] \Gamma^q,
\]

(12)

where

\[
\xi^q = \left[ (2\delta_1^q - \delta_2^q - \delta_3^q)^2 + 3(\delta_2^q - \delta_3^q)^2 \right]^{1/2},
\]

(13)

and the corresponding eigenvectors are \( U_1^q, U_2^q, U_3^q \):

\[
T^\dagger(\delta_1^q, \delta_2^q, \delta_3^q) = [(U_1^q), (U_2^q), (U_3^q)],
\]

\[
T(\delta_1^q, \delta_2^q, \delta_3^q) \approx \begin{pmatrix}
\cos \theta^q & \sin \theta^q & \lambda^q \cos \theta^q + \mu^q \sin \theta^q \\
-\sin \theta^q & \cos \theta^q & -\lambda^q \sin \theta^q + \mu^q \cos \theta^q \\
-\lambda^q & -\mu^q & 1
\end{pmatrix} T_0,
\]

(14)

where

\[
\lambda^q = \frac{1}{3\sqrt{6}} (\delta_2^q - \delta_3^q), \quad \mu^q = \frac{1}{9\sqrt{2}} (2\delta_1^q - \delta_2^q - \delta_3^q),
\]

\[
\theta^q = \frac{1}{2} \tan^{-1} \frac{\sqrt{3}(\delta_2^q - \delta_3^q)}{2\delta_1^q - \delta_2^q - \delta_3^q}.
\]

(15)

Next we consider the CKM matrix \( V \),

\[
V = T(\delta_1^u, \delta_2^u, \delta_3^u) T^\dagger(\delta_1^d, \delta_2^d, \delta_3^d).
\]

(16)
The matrix elements of $V$ are determined from various experimental results: nuclear beta decay, $K_{e3}$ decay, neutrino and antineutrino production of charm off valence $d$ quarks, neutrino production of charm, and semileptonic decay of $B$ mesons produced on the $Y(4S)$ $b\bar{b}$ resonance. The absolute values for these matrix elements are tabulated as\(^7\)

$$
V^{\text{exp}} = \begin{pmatrix}
0.9747 - 0.9759 & 0.218 - 0.224 & 0.002 - 0.005 \\
0.218 - 0.224 & 0.9738 - 0.9752 & 0.032 - 0.048 \\
0.004 - 0.015 & 0.030 - 0.048 & 0.9988 - 0.9995
\end{pmatrix}.
$$

\(17\)
We calculated numerically the allowed regions of \((\delta_1^u, \delta_3^u)\) and \((\delta_2^d, \delta_3^d)\) satisfying the restriction that the absolute values of matrix elements of \(V\) in Eq. (16) are included in the experimental range of matrix elements (Eq. (17)). First, we estimated the allowed regions of \((\delta_1^u, \delta_3^u, \delta_3^d)\) independent of the experimental constraint of the ranges of the mass ratio (11). We display the allowed region of \((\delta_2^u, \delta_3^u)\) for the \((u, c, t)\) sector and of \((\delta_2^d, \delta_3^d)\) for the \((d, s, b)\) sector in Fig. 2, fixing \(\delta_1^u = 0.0001\) and \(\delta_1^d = 0.01\). For \(\delta_1^u = 0.00005\) and \(0.0004\) and \(\delta_1^d = 0.005\) and \(0.02\), the allowed regions for \((\delta_2^u, \delta_3^u)\) and \((\delta_2^d, \delta_3^d)\) are similar to those of the case \(\delta_1^u = 0.0001\) and \(\delta_1^d = 0.01\) shown in Fig. 2.

As shown in Figs. 1 and 2, there are no common regions satisfying both constraints of the mass ratios and the CKM matrix in the \((\delta_2^d, \delta_3^d)\) plane of the \((d, s, b)\) sector. This fact is easily understood from the analytic expressions for \(V\). Using the approximate expression (14) for \(T(\delta_1^u, \delta_2^u, \delta_3^u)\), we obtain the approximate expression for \(V\)

\[
V \approx \begin{pmatrix}
\cos(\theta^u - \theta^d) \\
-\sin(\theta^u - \theta^d) \\
-(\lambda^u - \lambda^d) \cos \theta^d - (\mu^u - \mu^d) \sin \theta^d \\
(\lambda^u - \lambda^d) \cos \theta^u + (\mu^u - \mu^d) \sin \theta^u \\
-(\lambda^u - \lambda^d) \sin \theta^u + (\mu^u - \mu^d) \cos \theta^u \\
1
\end{pmatrix}.
\]

(18)
From this expression, for $\delta_2^d \approx \delta_3^d \gg \delta_1^d$, we obtain the ratio $|V_{cb}/V_{tb}| \approx | - (\lambda^u - \lambda^d) \sin \theta^u + (\mu^u - \mu^d) \cos \theta^u | \approx | \mu^d |$ and then $\delta_2^d \approx \delta_3^d > 0.20$ from the experimental ratio $|V_{cb}^{\text{exp}}/V_{tb}^{\text{exp}}| > 0.032$. On the other hand, from the ratio of the mass eigenvalues (12), for $\delta_2^d \approx \delta_3^d \gg \delta_1^d$, we obtain $m_2^d/m_3^d \approx 2(\delta_2^d + \delta_3^d)/9$ and then $\delta_2^d \approx \delta_3^d < 0.13$ from the experimental range $m_s/m_b = 0.038 \pm 0.019$.

Due to the fact that there is no common region in the $(\delta_2^d, \delta_3^d)$ plane satisfying the quark mass ratios and the CKM matrix, we consider the case containing the phases $\phi_i^d$ in the quark mass matrix as Eq. (3). Although there are 6 degrees of freedom for the phases $\phi_i^d$, only two phases, $\phi_2^d$ and $\phi_3^d$, are considered in our analysis, because the values of $\delta_2^d$ and $\delta_3^d$ in the violation parameters are approximately 0.1 and other parameters are extremely small ($\delta_1^d \sim 0.01, \delta_1^u \sim 0.0001, \delta_2^u \sim 0.01$), and therefore the phases with these other parameters contribute negligibly to the CKM matrix in contrast to two phases $\phi_2^d$ and $\phi_3^d$. Thus, we parametrize the $(d, s, b)$ sector quark mass matrix neglecting the $\phi_1^d$ in Eq. (3) as

$$M^d = \Gamma^d \begin{pmatrix} 1 & 1 - \delta_1^d & (1 - \delta_2^d)e^{i\phi_2} \\ 1 - \delta_1^d & 1 & (1 - \delta_3^d)e^{i\phi_3} \\ (1 - \delta_2^d)e^{-i\phi_2} & (1 - \delta_3^d)e^{-i\phi_3} & 1 \end{pmatrix}, \quad \phi_i^d \ll 1, \quad (i = 2, 3)$$

(19)

and for $M^u$ we use a matrix of the type represented by Eq. (5) with no phase. The approximate expressions for the eigenvalues of the mass matrix (19) are the same expressions $m_1^d$, $m_2^d$ and $m_3^d$ as in the case of Eq. (12), but the expression for $\xi^d$ is changed to one containing the phases $\phi_2$ and $\phi_3$ as follows:

$$\xi^d = \left[ (2\delta_1^d - \delta_2^d - \delta_3^d)^2 + 3(\delta_2^d - \delta_3^d)^2 + 3(\phi_2 - \phi_3)^2 \right]^{1/2}.$$  

(20)

The expression for the CKM matrix is given in this approximation as

$$V = T(\delta_1^u, \delta_2^u, \delta_3^u)T^d(\delta_1^d, \delta_2^d, \delta_3^d, \phi_2, \phi_3)$$

$$\approx \begin{pmatrix} \cos \theta^u c^d + \sin \theta^u s^d* & -\cos \theta^u s^d + \sin \theta^u c^d \\ -\sin \theta^u c^d + \cos \theta^u s^d* & \sin \theta^u s^d + \cos \theta^u c^d \\ (\lambda^u - \lambda^d*)c^d - (\mu^u - \mu^d*)s^d* & (\lambda^u - \lambda^d*)s^d - (\mu^u - \mu^d*)c^d \end{pmatrix},$$

(21)

where

$$c^d = \frac{\sqrt{\xi^d - (2\delta_1^d - \delta_2^d - \delta_3^d)}}{\sqrt{2\xi^d}},$$

$$s^d = \frac{-\sqrt{3} \left\{ (\delta_2^d - \delta_3^d) - i(\phi_2 - \phi_3) \right\}}{\sqrt{2\xi^d \xi^d - (2\delta_1^d - \delta_2^d - \delta_3^d)}},$$

$$\lambda^d = \frac{1}{3\sqrt{6}} \left\{ (\delta_2^d - \delta_3^d) - i(\phi_2 - \phi_3) \right\},$$

(21)
\[
\mu^d = \frac{1}{9\sqrt{2}} \left\{ 2\delta_1^d - (\delta_2^d + \delta_3^d) - 3i(\phi_2 + \phi_3) \right\}, \tag{22}
\]

and \(\theta^u, \lambda^u\) and \(\mu^u\) are given in Eq. (15). We calculated the allowed regions of \((\delta_1^d, \delta_2^d, \delta_3^d)\) and \((\delta_1^d, \delta_2^d, \delta_3^d, \phi_2, \phi_3)\) numerically which satisfy the two constraints of the experimental values of ranges of the quark mass ratios (Eq. (11)) and the CKM matrix \(V^\text{exp}\) (Eq. (17)), and show these regions in Fig. 3. We show the case \((\delta_1^d, \delta_2^d, \phi_2, \phi_3) = (0.0001, 0.01, -4^\circ, -4^\circ)\). Solutions corresponding to the cases other than \((\phi_2, \phi_3) \approx (-4^\circ, -3^\circ \sim -4^\circ)\) could not be obtained.

In order to see the effects of CP violation, we rephase the CKM matrix \(V = T(\delta_1^u, \delta_2^u, \delta_3^u)T^\dagger(\delta_1^d, \delta_2^d, \delta_3^d, \phi_2, \phi_3)\) to the standard parametrized CKM matrix \(V^R\), where the matrix elements \(V_{ud}^R, V_{us}^R, V_{cb}^R\) and \(V_{tb}^R\) are real numbers, by using the rephasing matrix \(P_u\) and \(P_d\) as

\[
V^R = P_u V P_d^\dagger, \quad P_u = \text{diag}[e^{i\alpha}, 1, e^{i\beta}], \quad P_d = \text{diag}[e^{i\alpha'}, 1, e^{i\beta'}]. \tag{23}
\]

The parameters \(\alpha, \alpha', \beta\) and \(\beta'\) are determined as

\[
\alpha = \tan^{-1} \frac{\text{Im}V_{ud}}{\text{Re}V_{ud}}, \quad \alpha' = -\tan^{-1} \frac{\text{Im}V_{us}}{\text{Re}V_{us}}, \quad \\
\beta = \tan^{-1} \frac{\text{Im}V_{cb}}{\text{Re}V_{cb}}, \quad \beta' = \tan^{-1} \frac{\text{Im}V_{tb}}{\text{Re}V_{tb}}. \tag{24}
\]

Fig. 3. The allowed regions of the \((\delta_2^u, \delta_3^u)\) and \((\delta_2^d, \delta_3^d)\) planes for \(\delta_1^u = 0.0001, \delta_1^d = 0.01, \phi_2 = -4^\circ\) and \(\phi_3 = -4^\circ\) satisfying the mass ratios (Eq. (11)) and the experimental CKM matrix elements (Eq. (17)).
Table I. The ranges of parameters \((\delta_1^u, \delta_2^u, \delta_3^u, \delta_1^d, \delta_2^d, \delta_3^d, \phi_+, \phi_-)\) satisfying the mass ratios (Eq. (11)) and the CKM matrix (Eq. (17)).

<table>
<thead>
<tr>
<th>Sector</th>
<th>Parameters</th>
<th>Ranges</th>
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<td>uct sector</td>
<td>(\delta_1^u)</td>
<td>(0.00001 \sim 0.0004)</td>
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<tr>
<td></td>
<td>(\delta_2^u)</td>
<td>(0.0064 \sim 0.0125)</td>
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<tr>
<td></td>
<td>(\delta_3^u)</td>
<td>(\pm(0.0 \sim 0.0043))</td>
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<td></td>
<td>(\phi_+)</td>
<td>(0.001 \sim 0.015)</td>
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<td></td>
<td>(\phi_-)</td>
<td>(0.040 \sim 0.129)</td>
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<tr>
<td></td>
<td>(\phi_\text{+-})</td>
<td>(\pm(-0.038 \sim -0.006))</td>
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<td></td>
<td>(\phi_\text{-})</td>
<td>(4^\circ \sim -3^\circ)</td>
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</tr>
</tbody>
</table>

In the standard parametrized CKM matrix \(V_R\), in which the element \(V_{cd}\) is almost real as recognized in the Wolfenstein parametrization, the parameters \(\rho\) and \(\eta\) characterizing the \(CP\) violation which are the vertex coordinate of unitarity triangle are expressed as

\[
\rho = \frac{\text{Re}(V_{ub}^R V_{ud}^R)}{|V_{ub}^R V_{ud}^R|}, \quad \eta = -\frac{\text{Im}(V_{ub}^R V_{ud}^R)}{|V_{ub}^R V_{ud}^R|}.
\]

The phenomenological constraints for parameters \(\rho\) and \(\eta\) have been examined by Pich and Prades using recent information on the non-perturbative hadronic inputs needed in the analysis of \(B^0 - \bar{B}^0\) mixing and the \(CP\)-violating parameter \(\varepsilon\) of the \(K^0 - \bar{K}^0\) system. Their results for \(\rho\) and \(\eta\) are \(\rho \approx [-0.1, 0.2]\) and \(\eta \approx [0.3, 0.5]\). Our results for \(\rho\) and \(\eta\) obtained from the parameters \((\delta_1^u, \delta_2^u, \delta_3^u, \delta_1^d, \delta_2^d, \delta_3^d, \phi_2, \phi_3)\) satisfying the quark mass ratios (Eq. (11)) and the CKM matrix (Eq. (17)) are in close agreement with the above phenomenological \(CP\) violation results.

We summarized the ranges of parameters \((\delta_1^u, \delta_2^u, \delta_3^u, \delta_1^d, \delta_2^d, \delta_3^d, \phi_2, \phi_3)\) and \((\delta_1^d, \delta_2^d, \delta_3^d, \phi_+\) \(= (\phi_2 + \phi_3)/2\), \(\phi_-\) \(= \phi_2 - \phi_3)\) in Table I, which were obtained in a previous analysis. Here \(\pm\) appears for \(\phi_\text{-}\), corresponding values are those of same sign.

A typical solution, the mass ratios, the CKM matrix elements and \((\rho, \eta)\) corresponding to this solution are as follows:

\[
\begin{align*}
\delta_1^u &= 0.0001, \quad \delta_2^u = 0.01, \quad \delta_3^u = 0.009, \\
\delta_1^d &= 0.01, \quad \delta_2^d = 0.07, \quad \delta_3^d = 0.102, \quad \phi_2 = -4^\circ, \quad \phi_3 = -4^\circ, \\
m_u &= 0.0058, \quad m_c = 0.0042, \quad m_d = 0.058, \quad m_s = 0.040, \\
m_t &= 0.9753, \quad m_b = 0.2210, \quad m_c = 0.0043, \\
V &= \begin{pmatrix} 0.9753 & 0.2210 & 0.0043 \\ 0.2210 & 0.9747 & 0.0347 \\ 0.0087 & 0.0339 & 0.9994 \end{pmatrix}, \\
\rho &= 0.022, \quad \eta = 0.56.
\end{align*}
\]

§3. Ansatz for quark mass matrix

As shown in the results of Table I and Eq. (26) analyzed phenomenologically, the small parameters \((\delta_1^u, \delta_2^u, \delta_3^u, \delta_1^d, \delta_2^d, \delta_3^d, \phi_+, \phi_-)\) satisfy
\[ \begin{align*}
\delta_1^u &\approx 0.0001, \quad \delta_0^u \approx 0.001, \quad \delta_\pm^u \approx 0.01, \\
\delta_1^d &\approx 0.01, \quad \delta_0^d \approx 0.03, \quad \delta_\pm^d \approx 0.09, \\
\phi_+ &\approx -4^\circ, \quad \phi_- \approx 0^\circ.
\end{align*} \]  

These are characterized as 'power rules' and parametrized by only 3 parameters, \( \lambda_u, \lambda_d \) and \( \phi \), as

\[ \begin{align*}
\delta_1^u &= \lambda_u^4, \quad \delta_0^u = \lambda_u^3, \quad \delta_\pm^u = \lambda_u^2, \quad \lambda_u \approx 0.1, \\
\delta_1^d &= \lambda_d^4, \quad \delta_0^d = \lambda_d^3, \quad \delta_\pm^d = \lambda_d^2, \quad \lambda_d \approx 0.3, \\
\phi_+ &= \phi \approx -4^\circ.
\end{align*} \]  

From these results, we can hypothesize a form for quark mass matrices as

\[
M^u = \Gamma^u \begin{pmatrix}
1 & 1 - \lambda_u^4 & 1 - \lambda_u^2 - \lambda_u^3 \\
1 - \lambda_u^4 & 1 & 1 - \lambda_u^2 + \lambda_u^3 \\
1 - \lambda_u^2 - \lambda_u^3 & 1 - \lambda_u^2 + \lambda_u^3 & 1
\end{pmatrix}, \\
M^d = \Gamma^d \begin{pmatrix}
1 & 1 - \lambda_d^4 & 1 - \lambda_d^2 - \lambda_d^3 + i\phi \\
1 - \lambda_d^4 & 1 & 1 - \lambda_d^2 + \lambda_d^3 + i\phi \\
1 - \lambda_d^2 - \lambda_d^3 - i\phi & 1 - \lambda_d^2 + \lambda_d^3 - i\phi & 1
\end{pmatrix},
\]

\[ \lambda_u \approx 0.1, \quad \lambda_d \approx 0.3, \quad \phi \approx -4^\circ. \]  

These mass matrices contain only 5 parameters, \( \Gamma^u, \Gamma^d, \lambda_u, \lambda_d \) and \( \phi \), which describe precisely the mass hierarchy of quarks and the CKM matrix at low energy, as found in the above numerical analysis.

We now comment on the differences between our model and others. \(^1,3\) The mass matrices and the CKM matrix are connected through the unitary matrices \( T_{u,d} \) as follows:

\[
T_u M^u T_{u}^{-1} = M^u_F = \text{diag}[m_u, m_c, m_t], \\
T_d M^d T_{d}^{-1} = M^d_F = \text{diag}[m_d, m_s, m_b], \\
V = T_u T_d^\dagger.
\]  

From \( V = T_u T_d^\dagger \) and \( V^{\exp} \approx 1 \), many models \(^1,3\) adopt the unitary matrices \( T_{u,d} \approx 1 \), \( M^u \approx \text{diag}[m_u, m_c, m_t] \), and \( M^d \approx \text{diag}[m_d, m_s, m_b] \). For example, the Fritzsch-type model \(^1\) adopts the following ansatz:

\[
M^u = \begin{pmatrix}
0 & a_u & 0 \\
a_u & 0 & b_u \\
0 & b'_u & c_u
\end{pmatrix}, \quad M^d = \begin{pmatrix}
0 & a_d & 0 \\
a_d & 0 & b_d \\
0 & b'_d & c_d
\end{pmatrix}, \quad a \ll b \sim b' \ll c. \]  

\[ (31) \]
Other types of models\textsuperscript{3} use the horizontal charge difference between the $SU(2)_L$ doublet and $SU(2)_R$ singlet. For example, Leurer, Nir and Seiberg\textsuperscript{3} consider the mass matrices for quarks as

$$
M_u \sim <\phi_u> \begin{pmatrix}
\varepsilon_2^2 & 0 & \varepsilon_2 \\
\varepsilon_1 \varepsilon_2 & \varepsilon_2 & \varepsilon_1 \\
\varepsilon_2 & 0 & 1
\end{pmatrix}, \quad M_d \sim <\phi_d> \begin{pmatrix}
\varepsilon_1^2 & \varepsilon_1 \varepsilon_2 & \varepsilon_1 \varepsilon_2 \\
0 & \varepsilon_1^2 & \varepsilon_1^2 \\
0 & \varepsilon_1 & \varepsilon_1
\end{pmatrix},
$$

$$
\varepsilon_1 \sim 0.04, \quad \varepsilon_2 \sim 0.008.
$$

If we transform as $T_u \rightarrow T_u S$ and $T_d \rightarrow T_d S$, where $S$ is some arbitrary unitary matrix, the CKM matrix $V$ remains unchanged. Thus our quark matrices, Eq. (29), can also be translated into the type $M^u \sim \text{diag}[m_u, m_c, m_t]$ and $M^d \sim \text{diag}[m_d, m_s, m_b]$ by the unitary matrix $S = T_0$ (Eq. (2)) as

$$
M^u \approx 3 \Gamma^u \begin{pmatrix}
\lambda_u^4 / 3 & \lambda_u^3 / 3 \sqrt{3} & -\lambda_u^3 / 3 \sqrt{3} \\
\lambda_u^3 / 3 \sqrt{3} & 4 \lambda_u^2 / 9 & \sqrt{2} \lambda_u^2 / 9 \\
-\lambda_u^3 / 3 \sqrt{3} & \sqrt{2} \lambda_u^2 / 9 & 1
\end{pmatrix},
$$

$$
M^d \approx 3 \Gamma^d \begin{pmatrix}
\lambda_d^4 / 3 & \lambda_d^3 / 3 \sqrt{3} & -\lambda_d^3 / 3 \sqrt{3} \\
\lambda_d^3 / 3 \sqrt{3} & 4 \lambda_d^2 / 9 & \sqrt{2} \lambda_d^2 / 9 + \sqrt{2} i \phi / 3 \\
-\lambda_d^3 / 3 \sqrt{3} & \sqrt{2} \lambda_d^2 / 9 - \sqrt{2} i \phi / 3 & 1
\end{pmatrix},
$$

$$
\lambda_u \approx 0.1, \quad \lambda_d \approx 0.3, \quad \phi \approx -4^\circ.
$$

The original Fritzsch-type model cannot explain the observed large top quark mass, while a modified model (see Dutta and Nandi of Ref. 1) like Eq. (31) can explain the large top quark mass but it must use the up quark mass matrix in which the (2,3) and the (3,2) elements are not equal. Other types of models\textsuperscript{3} using mass matrices like Eq. (32) consider the connection between the mass matrices like Eq. (32) and the mass matrix patterns at the GUT scale in SUSY theories or at the string scale in string models. Although these models are able to predict data with order-of-magnitude precision, they cannot produce results in complete agreement with precise low energy data.

In model building beyond the SM, the form Eq. (29) or Eq. (33) must be considered. We adopted the universal Yukawa coupling mass matrices (Eq. (3)) as a starting point which were parametrized as Eq. (29), but these matrices are translated into the mass matrices as $M^u \sim \text{diag}[m_u, m_c, m_t]$ and $M^d \sim \text{diag}[m_d, m_s, m_b]$, which are expressed as Eq. (33) in our case. More reasonable type of mass matrix in the former and the latter cases depends on which model for two cases can produce the form given by Eqs. (29) and (33). Furthermore, in the analysis of neutrino mixing, the weak interaction eigenstates depend on the unitary matrices $T_{u,d}$ adopted, so the analysis of mass matrix involving the lepton sector as in the case of the neutrino mixing problem will give information to check the validity of the former and the latter.
§4. Conclusion

We analyzed precisely the mass hierarchy of quarks and the CKM matrix at low energy using the universal Yukawa coupling framework with small violations (Eq. (3)). Eight violation parameters, (δ₁^u, δ₂^u, δ₃^u, δ₁^d, δ₂^d, δ₃^d, φ₂, φ₃), are estimated. The estimated values of these are tabulated in Table I and typical values are given in Eq. (26). Phases other than φ₂ and φ₃ do not contribute to our present analysis because of extremely small values of (δ₁^u, δ₂^u, δ₃^u, δ₁^d). We fitted 8 violation parameters to 8 experimental data: 4 quark mass ratios (Eq. (11)), 3 mixing angles determined by CKM matrix elements (Eq. (17)), and 1 phase determined by CP violation, which is related to ρ and η. From the estimated parameters, we can hypothesize a form for quark mass matrices which contain only 5 parameters, Γ^u, Γ^d, λ_u, λ_d and φ.

References

   G. C. Branco and J. I. Silva-Marcos, FISIST/12-95/CFIF.