A Set of Conditions for Teleportation without Resort to von Neumann's Projection Postulate

Akio Motoyoshi, Koichi Yamaguchi, Tetsuya Ogura and Tetsuya Yoneda

Department of Physics, Kumamoto University, Kumamoto 860

(Received November 20, 1996)

Although a quantum teleportation scheme of the type proposed by Bennett et al. is of great interest, it is insufficient from the measurement theoretical point of view. This insufficiency and the conditions for a correct teleportation are discussed. A set of four conditions is proposed for correct teleportation.

Since the pioneering works of Feynman\textsuperscript{1} and Deutsch\textsuperscript{2} using elements following the laws of quantum mechanics, interest has been growing in the area of so-called quantum computation. Among other things, a quantum teleportation scheme of the type proposed by Bennett et al.\textsuperscript{3} is of great interest from the measurement theoretical point of view. The original teleportation scheme of an unknown state $|\psi\rangle$, which consists of the "story of Alice and Bob", however, is insufficient due to the unclear treatment of the EPR\textsuperscript{4} problem and the assumption of von Neumann's projection postulate. At the same time, the former unclear treatment follows from the latter assumption. This teleportation scheme, based on the existence of non-local long range correlations between EPR pairs of particles relevant to the notion of wave function collapse, necessarily leads to epistemological discussion on the nature of physical reality. Unfortunately, tentative solutions to this problem always depend on what the authors decide to define as realism.\textsuperscript{5} Therefore, such philosophical arguments concerning the conceptual aspects of EPR phenomenon are beyond the realm of physics. We shall show that a quantum teleportation can be achieved within the framework of present-day quantum mechanics without resorting to von Neumann's projection postulate.

Today, the naive Copenhagen interpretation is criticized for various reasons. For example, in Machida and Namiki's theory of measurement,\textsuperscript{5,6} which does not rely upon von Neumann's projection postulate, making use of the statistical operator as the description of the quantum state and introducing the state of the apparatus system, they showed that the resolution of the difficulty concerning the wave function collapse does not consist of a demonstration of the disappearance of other branch waves, but follows from the concept of decoherence. Their theory is widely applicable, even to incomplete measurements.\textsuperscript{5} According to these lines of thought, one of the present authors (A. M.) attempted to analyze the process of decoherence.\textsuperscript{7}

In this paper, reviewing what is a correct understanding of the EPR problem, we intend to show how we can obtain a correct teleportation scheme without resorting to von Neumann's projection postulate.

A reasonable explanation of the EPR problem was given by Machida.\textsuperscript{8} In order
to make clearer the essential features of this problem, let us start with his explanation. To do so, one may consider a system consisting of only two spin 1/2 particles prepared in an EPR singlet state,

$$|\psi(\cdot\cdot)\rangle_{12} = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2 \right).$$

(1)

If one follows von Neumann’s projection postulate, the EPR paradox stems from this description of the quantum state by the state vector as the results of measurement in one subsystem, and one encounters a “spooky” action-at-a-distance. One may call this effect a non-local long range correlation. However, since the measurement yields a change of the pure state into a mixture of spin states, we cannot describe this change without the statistical operators. The statistical operator for this entire pure state is

$$\hat{\rho}_{12} = |\psi(\cdot\cdot)\rangle_{12} \langle \psi(\cdot\cdot)|.$$

(2)

In the case of the measurement of spin $S_2^{(1)}$ in the subsystem 1, the corresponding operator should be given by $\hat{S}_z^{(1)} \otimes \hat{1}^{(2)}$, since the object system of the measurement is the entire system, even if the quantity of measurement is $S_2^{(1)}$. The verification of the violation of Bell’s inequalities also supports this notion. This is a reflection of the nonseparability of the quantum systems in Bohr’s sense. To deny the existence of this effect implies that one must abandon the greater part of the successes of quantum mechanics. We can obtain the reasonable conclusions from this idea. In fact, the expectation value of the measurement of $S_2^{(1)}$ in the subsystem 1 becomes

$$\langle \hat{S}_z^{(1)} \otimes \hat{1}^{(2)} \rangle = \text{Tr}^{(2)}\{\hat{S}_z^{(1)} \otimes \hat{1}^{(2)}\} = \text{Tr}^{(1)}\{\hat{S}_z^{(1)}\} = \text{Tr}^{(1)}(\hat{\rho}_1 \cdot \hat{S}_z^{(1)}),$$

(3)

where

$$\hat{\rho}_1 = \text{Tr}^{(2)}(\hat{\rho}_{12}), \quad \hat{\rho}_2 = \text{Tr}^{(1)}(\hat{\rho}_{12}).$$

(4)

Since the expectation value (3) exactly coincides with that obtained in the ordinary formulation of quantum mechanics, the states of subsystems can be obtained by the partial tracing from $\hat{\rho}_{12}$, and since all information concerning a subsystem is given by the statistical operator, $\hat{\rho}_1$ and $\hat{\rho}_2$ in (4) are the statistical operators which specify the quantum mechanical states of subsystems themselves. Assuming a complete measurement, after the measurement of $S_2^{(1)}$, the state of the entire system is converted into

$$\hat{\rho}_{12} \rightarrow \hat{\rho}_{12,\infty} = \frac{1}{2} \left( |\uparrow\rangle_1 \langle | \otimes |\downarrow\rangle_2 \langle \downarrow| + |\downarrow\rangle_1 \langle \downarrow| \otimes |\uparrow\rangle_2 \langle \uparrow| \right).$$

(5)

This change of the statistical operator represents the disentanglement, and indicates the process which occurs as a result of the decoherence produced by the interaction of subsystem 1 with the detector of $S_2^{(1)}$. After the measurement of $S_2^{(1)}$, the states of subsystems become

$$\begin{cases}
\hat{\rho}_1 \rightarrow \hat{\rho}_{1,\infty} = \text{Tr}^{(2)}(\hat{\rho}_{12,\infty}) = \hat{\rho}_1, \\
\hat{\rho}_2 \rightarrow \hat{\rho}_{2,\infty} = \text{Tr}^{(1)}(\hat{\rho}_{12,\infty}) = \hat{\rho}_2.
\end{cases}$$

(6)
It should be noted that the measurement is represented by the process (5), and the partial tracing is not the measurement itself, even if (6) is satisfied in this special singlet case. Making use of the statistical operators, Cantrell and Scully\textsuperscript{10} have shown that the states of subsystem 2 are the same after the measurements of $S_{x}^{(1)}$ or $S_{x}^{(1)}$ in the subsystem 1. Even in more complicated cases, in which $\hat{\rho}_{1}$ has off-diagonal parts and $\hat{\rho}_{1,\infty} \neq \hat{\rho}_{1}$, we can show that $\hat{\rho}_{2,\infty} = \hat{\rho}_{2}$\textsuperscript{11}. In the EPR problem, the subsystems are generally in a mixture state, even if the entire system is in a pure state, and the state of subsystem 2 is determined by the law of conservation of the spin angular momentum according to the outcome of the measurement in subsystem 1. In this interpretation of EPR problem, contrary to the projection postulate, the other branch wave need not disappear, and we have no “spooky” action-at-a-distance. Therefore, if we use the statistical operators to describe the states, and if we confirm the important role of the conservation law for the identification of the partners of measurement outcomes in one subsystem, the formal inconsistency of the EPR problem can be removed within the framework of present-day quantum mechanics. Now, let us examine the original teleportation scheme, in which

$$|\Psi\rangle_{123} = |\phi\rangle_{1}|\Psi^{(-)}\rangle_{23}$$

$$= \frac{1}{2} \left\{ -|\Psi^{(-)}\rangle_{12}|\phi_{3}\rangle + |\Psi^{(+)}\rangle_{12} \left( -a|\uparrow\rangle_{3} + b|\downarrow\rangle_{3} \right) 
+ |\Phi^{(-)}\rangle_{12} \left( a|\downarrow\rangle_{3} + b|\uparrow\rangle_{3} \right) 
+ |\Phi^{(+)}\rangle_{12} \left( a|\downarrow\rangle_{3} - b|\uparrow\rangle_{3} \right) \right\}, \quad (7)$$

where

$$|\phi\rangle = a|\uparrow\rangle + b|\downarrow\rangle, \quad |a|^{2} + |b|^{2} = 1, \quad$$

$$|\Psi^{(+)}\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle \right), \quad (8)$$

$$|\Phi^{(\pm)}\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle|\uparrow\rangle \pm |\downarrow\rangle|\downarrow\rangle \right) \equiv \frac{1}{\sqrt{2}} \left( |1\rangle \pm |1\rangle \right),$$

$|\Psi^{(\pm)}\rangle$ and $|\Phi^{(\pm)}\rangle$ construct the Bell operator basis. In this scheme, the teleported states, which are the partners of measurement outcomes in subsystem 12, are decided by the projection postulate. Therefore, there is no discussion with regard to the law of conservation of the physical quantities subjected to the measurements. Their treatments are completely different from ours. Futhermore, if we rely upon the idea of the disappearance of other branch waves, it is difficult to understand the negative-results measurement and the results of some of the neutron interferometry experiments.\textsuperscript{5} The state $|\phi\rangle$ is a spinor, and since the measurement of components of spin is a well-known operation, if one does not follow the projection postulate, and if the physical quantities for the measurement are the components of spin, one cannot confirm which are the partners of the measurement outcomes, except for $|\Psi^{(-)}\rangle_{12}$ in (7), since when one obtains states other than $|\Psi^{(-)}\rangle_{12}$, the law of conservation of spin angular momentum does not apply. This fact gives an indication for a correct formulation of teleportation without resort to the projection postulate.

Summarizing the previous arguments, in order to seek a correct teleportation scheme from our standpoint and to establish a condition to realize it, we start with
the system which is in the maximally entangled state of three spin 1/2 particles, where the independent base states are lexicographically ordered.

**Condition 1 (spin state democracy)**
The state of entire system must be in the state

\[ |\Psi\rangle_{123} = |\uparrow\rangle_1 |\uparrow\rangle_2 \left( c_1 |\uparrow\rangle_3 + c_2 |\downarrow\rangle_3 \right) + |\downarrow\rangle_1 |\uparrow\rangle_2 \left( c_3 |\uparrow\rangle_3 + c_4 |\downarrow\rangle_3 \right) + |\uparrow\rangle_1 |\downarrow\rangle_2 \left( c_5 |\uparrow\rangle_3 + c_6 |\downarrow\rangle_3 \right) + |\downarrow\rangle_1 |\downarrow\rangle_2 \left( c_7 |\uparrow\rangle_3 + c_8 |\downarrow\rangle_3 \right) \]

\[ = \left( c_1 |\uparrow\rangle_1 + c_5 |\downarrow\rangle_1 \right) |\uparrow\rangle_2 |\uparrow\rangle_3 + \left( c_2 |\uparrow\rangle_1 + c_6 |\downarrow\rangle_1 \right) |\uparrow\rangle_2 |\downarrow\rangle_3 \]

\[ + \left( c_3 |\uparrow\rangle_1 + c_7 |\downarrow\rangle_1 \right) |\downarrow\rangle_2 |\uparrow\rangle_3 + \left( c_4 |\uparrow\rangle_1 + c_8 |\downarrow\rangle_1 \right) |\downarrow\rangle_2 |\downarrow\rangle_3 \] \hspace{1cm} (9)

for each composition of subsystems 1, 2, 3 and 1, 2, 3.

**Condition 2 (quantum state swapping)**
The states of subsystems must be in equal states, namely, \( |\downarrow\rangle_1 = |\downarrow\rangle_3 \) and \( |\uparrow\rangle_3 = |\uparrow\rangle_2 \). Then we have \( c_2 = c_3 = c_5, c_4 = c_6 = c_7, \) and arbitrary values are allowed for \( c_1 \) and \( c_8 \). These conditions imply that

\[ |\Psi\rangle_{123} = \left( c_1 |\uparrow\rangle_1 + c_2 |\downarrow\rangle_1 \right) |\downarrow\rangle_3 + \sqrt{2}|\phi'\rangle_1 |\Psi^{(+)}\rangle_{23} \]

\[ + \left( c_7 |\uparrow\rangle_1 + c_8 |\downarrow\rangle_1 \right) |\downarrow\rangle_3 - |\downarrow\rangle_1 \left( c_7 |\uparrow\rangle_3 + c_8 |\downarrow\rangle_3 \right) \]

\[ = |\downarrow\rangle_1 \left( c_1 |\uparrow\rangle_3 + c_2 |\downarrow\rangle_3 \right) + \sqrt{2}|\Psi^{(+)}\rangle_{12} |\phi'\rangle_3 \]

\[ + |\downarrow\rangle_1 \left( c_7 |\uparrow\rangle_3 + c_8 |\downarrow\rangle_3 \right) \] \hspace{1cm} (10)

where \( |\phi'\rangle = c_2 |\uparrow\rangle + c_7 |\downarrow\rangle \). We can recognize the fact that condition 1 is necessary to realize condition 2.

**Condition 3 (orthogonality)**
The states of subsystems must be orthogonal. Since the states \( |\downarrow\rangle, |\uparrow\rangle \) and \( |\Psi^{(+)}\rangle \) are already orthonormal, we must only impose this condition between the states \( (c_1 |\uparrow\rangle + c_2 |\downarrow\rangle) \), \( (c_7 |\uparrow\rangle + c_8 |\downarrow\rangle) \) and \( |\phi'\rangle \). This condition is satisfied by choosing

\[ c_1 = -e^{2i\alpha} c_7^*, \quad c_8 = -e^{2i\beta} c_2^* \] \hspace{1cm} (11)

where \( \alpha \) and \( \beta \) stand for \( \arg c_2 \) and \( \arg c_7 \), respectively. With these selections, \( c_1 |\uparrow\rangle + c_2 |\downarrow\rangle \) and \( c_7 |\uparrow\rangle + c_8 |\downarrow\rangle \) represent the same state, \( |\psi'\rangle = -c_7^* |\uparrow\rangle + c_2^* |\downarrow\rangle \), with the exception of the irrelevant phase factors, and we have \( \langle \phi'|\psi'\rangle = 0 \). If this condition is not satisfied, we cannot prepare the ancilla independently of the unknown state \( |\phi'\rangle \). The state \( |\psi'\rangle \) can be obtained from \( |\phi'\rangle \) by a unitary operation. Then, we have the normalized full state,

\[ |\Psi\rangle_{123} = \frac{1}{2} \left\{ |\psi'\rangle_1 \left( e^{2i\alpha} |\uparrow\rangle_{23} - e^{2i\beta} |\downarrow\rangle_{23} \right) + \sqrt{2}|\phi'\rangle_1 |\Psi^{(+)}\rangle_{23} \right\} \]

\[ = \frac{1}{2} \left\{ \left( e^{2i\alpha} |\uparrow\rangle_{12} - e^{2i\beta} |\downarrow\rangle_{12} \right) |\psi'_3\rangle + \sqrt{2}|\Psi^{(+)}\rangle_{12} |\phi'_3\rangle \right\} . \] \hspace{1cm} (12)
However, in order to also prepare the ancilla independently of $\alpha$ and $\beta$, and to use the law of conservation for the identification of the partners of measurement outcomes, we must impose a further condition.

**Condition 4 (construction of mixture)**

Let us restart with the mixture

$$\hat{\rho}_{1,23} = \frac{1}{4} \left\{ |\psi\rangle_1 \langle \psi| \otimes |1\rangle_{23}\langle 1| + |\psi\rangle_1 \langle \psi| \otimes |-1\rangle_{23}\langle -1| + 2|\phi\rangle_1 \langle \phi| \otimes |\Psi^{(+)}\rangle_{23}\langle \Psi^{(+)}| \right\}, \quad (13)$$

which can be constructed from the states appearing in (12), where $|\phi\rangle = a|\uparrow\rangle + b|\downarrow\rangle$, $|\psi\rangle = -b^*|\uparrow\rangle + a^*|\downarrow\rangle$ for normalization, and $\langle \phi|\psi\rangle = 0$.

By partial tracing, we have

$$\left\{ \begin{array}{l}
\hat{\rho}_1 = \text{Tr}^{(23)}(\hat{\rho}_{1,23}) = \frac{1}{2} \left\{ |\psi\rangle_1 \langle \psi| + |\phi\rangle_1 \langle \phi| \right\}, \\
\hat{\rho}_{23} = \text{Tr}^{(1)}(\hat{\rho}_{1,23}) = \frac{1}{4} \left\{ |1\rangle_{23}\langle 1| + |-1\rangle_{23}\langle -1| + 2|\Psi^{(+)}\rangle_{23}\langle \Psi^{(+)}| \right\}. \quad (14)
\end{array} \right.$$  

Recomposing the subsystems 1,23 with 12,3, we obtain $\hat{\rho}_{12,3}$, from which we have

$$\left\{ \begin{array}{l}
\hat{\rho}_3 = \text{Tr}^{(12)}(\hat{\rho}_{12,3}) = \frac{1}{2} \left\{ |\psi\rangle_3 \langle \psi| + |\phi\rangle_3 \langle \phi| \right\}, \\
\hat{\rho}_{12} = \text{Tr}^{(3)}(\hat{\rho}_{12,3}) = \frac{1}{4} \left\{ |1\rangle_{12}\langle 1| + |-1\rangle_{12}\langle -1| + |\Psi^{(+)}\rangle_{12}\langle \Psi^{(+)}| + |\Psi^{(-)}\rangle_{12}\langle \Psi^{(-)}| \right\}. \quad (15)
\end{array} \right.$$  

After the measurement of $\hat{S}^{(12)}_z$, whose operator is given by $(\hat{S}^{(12)}_z \otimes \hat{I}^{(3)})$, $\hat{\rho}_{12,3}$ is converted into

$$\hat{\rho}_{12,3} \rightarrow \hat{\rho}_{12,3,\infty} = \frac{1}{4} \left\{ |1\rangle_{12}\langle 1| + |-1\rangle_{12}\langle -1| \otimes \left\{ |b^2\rangle\uparrow_3 \langle \uparrow| + |a^2\rangle\downarrow_3 \langle \downarrow| \right\} + \frac{1}{2} |\Psi^{(+)}\rangle_{12}\langle \Psi^{(+)}| \otimes \left\{ 2|a^2\rangle\uparrow_3 \langle \uparrow| + 2|b^2\rangle\downarrow_3 \langle \downarrow| \right\} \right.$$  

$$+ ab^*|\uparrow\rangle_3 \langle \downarrow| + a^*b|\downarrow\rangle_3 \langle \uparrow| \right\} \right\} + \frac{1}{2} |\Psi^{(-)}\rangle_{12}\langle \Psi^{(-)}| \otimes \left\{ 2|a^2\rangle\uparrow_3 \langle \uparrow| + 2|b^2\rangle\downarrow_3 \langle \downarrow| \right\} \right.$$  

$$- ab^*|\uparrow\rangle_3 \langle \downarrow| - a^*b|\downarrow\rangle_3 \langle \uparrow| \right\}. \quad (16)$$

Finally, we obtain

$$\left\{ \begin{array}{l}
\hat{\rho}_{3,\infty} = \text{Tr}^{(12)}(\hat{\rho}_{12,3,\infty}) = \hat{\rho}_3, \\
\hat{\rho}_{12,\infty} = \text{Tr}^{(3)}(\hat{\rho}_{12,3,\infty}) = \hat{\rho}_{12}. \quad (17)
\end{array} \right.$$
Therefore, we can safely determine the partners of measurement outcomes of $S_z^{(12)}$ through the law of conservation of spin angular momentum owing to the relations $\rho_1 = \rho_{3,\infty} = \rho_3$, and the fact that both of $\rho_{12,\infty}$ and $\rho_{23}$ are mixtures consisting of eigenstates of $S_z$. These mixtures have the same probabilities with respect to the eigenvalues of $S_z$. In this scheme, we have assumed that the averaged value of the sum of z-components

$$S_z^{(1)} + S_z^{(2)} + S_z^{(3)}$$

is conserved. After the measurement of $S_z^{(12)}$, generally, the allowed state vectors of subsystem 3 are

$$\begin{align*}
|\phi''\rangle &= ae^{-i\theta_1} |\uparrow\rangle + be^{-i\theta_2} |\downarrow\rangle, \\
|\psi''\rangle &= -b*e^{-i\theta_2} |\uparrow\rangle + a*e^{i\theta_2} |\downarrow\rangle,
\end{align*}$$

under this conservation law, where $\theta_1$ and $\theta_2$ are arbitrary. However, $|\phi''\rangle$ and $|\psi''\rangle$ give $\rho_3$ in (15) only for $\theta_1 = \theta_2 \pmod{2\pi}$. Since $\rho_3$ is a mixture, and since there is no phase relation between $|\phi\rangle$ and $|\psi\rangle$, generally $\theta_1 = \theta_2 \neq 0$ is scarcely established. This condition means a rotation about z-axis preserving the orthgonality. Therefore, usually we may assume that $\theta_1 = \theta_2 = 0$. In addition, though $\rho_3$ can be constructed from arbitrary linear combinations of $|\phi\rangle$ and $|\psi\rangle$, these combinations do not satisfy the conservation law. Therefore, under this conservation law, we can confirm that the teleported states are $|\phi\rangle$ or $|\psi\rangle$, corresponding to the measurement outcomes 0 and 1, -1, respectively. Thus we can achieve a teleportation of unknown states $|\phi\rangle$ and $|\psi\rangle$ within the framework of present-day quantum mechanics. In order to accomplish this teleportation, it is sufficient to create entanglements between $|\Psi^{(+)}\rangle_{23}$ and $|\phi\rangle_1 ; |1\rangle_{23}$, $|1\rangle_{23}$ and $|\psi\rangle_1$, respectively, and to join these entanglements with an intensity ratio of 2:1:1. Thus, we have established a set of four conditions for a correct teleportation. The reason for the failure of the original teleportation scheme with respect to the spin, when it departs from the projection postulate, is ascribed to the aristocracy of the EPR singlet. Even if we start from a mixture of type $\rho_{12}$ in (15) as a substitute of $\rho_{23}$ in (14), we can obtain similar results.