

# Effects of Local Communication and Topology on Collective Movement Initiation

Brent E. Eskridge

Department of Computer Science and Network Engineering, Southern Nazarene University, Bethany, OK 73008  
beskridge@snu.edu

## Abstract

Collective movement in autonomous systems, such as a team of robots, are frequently implemented using complex interaction rules and have significant communication requirements. These restrictions frequently relegate such systems to static, simplified environments. In contrast, collective movements in natural systems consistently occur in dynamic, complex environments in which significant communication is either impractical or impossible, and have been successfully modeled using simple, local interaction rules. In the work presented here, one such model is extended to include local communication and the spatial distribution of the group so that it can eventually be used as a guide for developing artificial systems capable of cohesive, collective movements. The extended model predicts that a reliance on local communication does not necessarily mean there will be a significant loss in the expected success of collective movement attempts if appropriate interaction rules are chosen. Furthermore, the model predicts that the addition of local communication, in conjunction with the topology of the group, results in higher expected success in attempting collective movements for individuals with central locations in the group as compared to individuals occupying edge locations.

## Introduction

Collective movement is a necessary consequence of living and working in groups. As a result, considerable attention has been paid to the performance benefits of effective collective movements in both artificial and natural systems. However, there has historically been a dichotomy in how artificial and natural systems arrive at collective movement. Artificial systems, such as **multi-robot systems (MRSs)**, generally interact using complex rules that require precise sensor information and significant communication between robots, and are limited to operations in controlled environments. On the other hand, collective movement in natural systems can be successfully modeled using simple, local interaction rules requiring little to no explicit communication between group members and operate in complex, dynamic environments. To achieve the adaptability and simplicity of natural systems, the design and development of artificial systems can take inspiration from natural systems, especially in the area of collective movement.

In MRSs, coordinated actions such as collective movements are generally achieved through explicit, global communication (Balch and Arkin, 1994; Ampatzis et al., 2008). While explicit, global communication can be straightforward to implement, there are significant problems with its application in MRSs. Not only is it sensitive to environmental conditions, but explicit communication has problems scaling to large teams of robots (Anderson and Papanikolopoulos, 2008). In the worst case, the computational complexity of coordinating  $n$  individuals using explicit, global communication is  $O(n^2)$  (Klavins, 2003). Limitations in communication are also a factor in collective movements in natural systems. Although small groups exhibit explicit, global communication, its use in a group cannot reliably scale from the individual to the group (Couzin, 2009). As a result, large groups in nature use local communication and frequently rely on environmental cues, a form of implicit communication (Drapier et al., 2002).

Researchers have recently proposed a number of models for collective movement based on observations of natural systems (Jacobs et al., 2011; Pillot et al., 2011; Sueur et al., 2011; Sueur and Deneubourg, 2011). However, the majority of these cited models were developed based on observations of groups of less than 15 members and assume global communication. As a result, for a model to be useful where global communication is not an option, it must be amenable to the addition of local communication. The ease with which local communication can be added to a model is primarily determined by the complexity of the rules governing the interactions between individuals. If a model's rules are relatively simple, then adding local communication is reduced to the task of limiting the interaction rules currently governing a given individual, based on the individuals with which it is communicating. Group information, such as an individual's actions, is propagated throughout the group using the relationships among group members as determined by the group's spatial distribution, or the group's *topology*.

For this initial investigation, the model proposed by Gautrais (2010) was chosen for extension. While there are many candidate models, some of which already use local commu-

Parameter	Value
$\tau_o$	1290
$\alpha_c$	0.009
$\gamma_c$	2.0
$\varepsilon_c$	2.3
$\alpha_f$	162.3
$\beta_f$	75.4

Table 1: These model parameters were determined through direct observation of collective movement attempts in white-faced capuchin monkeys (Petit et al., 2009; Gautrais, 2010).

nication, this model’s use of simple, yet effective interaction rules facilitated the addition of local communication. Furthermore, it is anticipated that the simplicity of this model will also facilitate its application to artificial systems, such as a team of robots. To evaluate the effects of local communication and topology on the model’s predictions, simulations were performed with two variations of the model for a range of group sizes. The results of these simulations show that the extended model predicts only a small reduction in the mean expected success percentage for collective movement attempts, if the appropriate definition of an individual’s local neighborhood is chosen. Furthermore, the extended model predicts that the addition of local communication and topology results in a comparatively higher mean expected success rate in collective movement initiations for individuals occupying central positions in the group, when compared to individuals occupying edge positions.

## Collective Movement Model

The model chosen for extension was developed through observations of collective movement attempts in a group of ten white-faced capuchin monkeys (Petit et al., 2009; Gautrais, 2010), and was later confirmed in observations of sheep groups ranging in size from 2–8 members (Pillot et al., 2011). It uses three interaction rules to govern the decision-making process involved in starting collective movements. The first rule assumes that all individuals within the group can initiate a collective movement attempt with a rate of  $1/\tau_o$  (see Table 1). While this assumption may not hold for groups with dominant leaders, studies have shown that it is a viable assumption for egalitarian animal groups, such as the capuchin monkeys used in the model’s development.

Since the model assumes global communication, once an individual initiates a collective movement, the remaining individuals are assumed to have observed the initiation attempt and have the opportunity to follow the initiator. The second rule describes the rate at which followers join the collective movement attempt and is calculated by  $1/\tau_r$ . The time con-

stant  $\tau_r$  for the following rate is calculated by the following:

$$\tau_r = \alpha_f + \beta_f \frac{N - r}{r} \quad (1)$$

where  $\alpha_f$  and  $\beta_f$  are constants determined through direct observation (see Table 1),  $N$  is the number of individuals in the group, and  $r$  is the number of individuals following the initiator (Petit et al., 2009; Gautrais, 2010). Note that as the number of individuals following the initiator increases, the rate at which individuals join the movement also increases.

Not all initiation attempts are successful as initiators often cancel and return to the group. The third rule calculates this cancellation rate by the following:

$$C_r = \frac{\alpha_c}{1 + (r/\gamma_c)^{\varepsilon_c}} \quad (2)$$

where  $\alpha_c$ ,  $\gamma_c$ , and  $\varepsilon_c$  are constants determined through direct observation (see Table 1), and  $r$  is the number of individuals following the initiator. Note that as the number of individuals following the initiator increases, the rate at which the initiator cancels an initiation decreases. Also, simulations of the model include the implicit assumption that a successful collective movement requires all of the members of the group to participate, since there is a non-zero probability of canceling even if all but one member participates. While this is not necessarily the case in nature, cohesive, collective movements are the primary objective of this work and, as such, incomplete movements are considered failures.

Fundamental to these rules is the concept of mimetism, in which an individual’s probability of choosing an action is related to the number of individuals already performing the action (Pyritz et al., 2011). A variety of types of mimetism have been observed in natural systems and are usually differentiated by an individual’s choice of *whom* to mimic. In this model, *anonymous mimetism*, or *allelomimetism*, is used since individuals do not use the identity of group members when choosing whom to mimic. Anonymous mimetism is particularly useful for groups in which membership frequently changes and information such as an individual’s reputation may not be available. While information regarding a specific robot’s capabilities could be used in determining whom to mimic in a MRS, an anonymous model is useful as it represents a worst-case scenario.

## Extending the Model

Scaling up the number of individuals within the group presents a choice regarding how a larger group size affects the model. Since it was developed for a small group with global communication, the model assumes each individual directly observes and interacts with every other individual within the group. However, when the number of individuals is scaled up, this assumption may no longer hold since spatial and cognitive constraints can limit the number of neighbors with which an observer interacts. This primarily affects

the calculation of the following rate constant,  $\tau_r$  (see Equation 1), which uses the size of the entire group,  $N$ .

One option to consider is that the model does not need modification and that the assumptions regarding global communication are correct, regardless of group size. Furthermore, since white-faced capuchin monkeys are commonly found in groups of less than 20 members, with few reaching 30 members, their behaviors may not *need* to work with large groups (Fragaszy et al., 2004). However, while global communication is the easiest to implement and is reasonable given past work, evidence from nature does not support its use in large groups. For example, Pillot et al. (2011) noted that crowding in a group prevents an individual from observing all but its closest neighbors.

If global communication is not an option, then the flow of information between individuals becomes an important factor in the success of a collective movement attempt. Since the number of individuals with which an observer interacts is limited, only the individuals observing an initiation attempt are capable of following. This presents a choice regarding the value of  $N$  in the calculation of the following rate constant,  $\tau_r$ , from Equation 1. The first option is to use the size of the entire group, denoted  $G$ , so that  $N = G$ . While this results in the same following constant as with global communication, the number of individuals capable of following the initiator is now limited to the number of individuals observing the initiator. As a consequence, the odds of the initiator canceling increase since the number of potential followers is limited. This choice also includes an implicit assumption that, although the number of group members with which an individual interacts is limited, group members know the size of the group and know the state of all the other members of the group. As such, this choice represents a logical contradiction, but it is still a useful choice against which the predictions of other models can be compared.

An alternative to using the size of the entire group is to use the number of individuals with an individual directly interacts. Ballerini et al. (2008) have shown that starlings appear to interact with, on average, a fixed number of neighbors. If this is true for other groups, then the number of nearest neighbors, denoted  $N_c$ , could be used so that  $N = N_c$ . The following rate constant  $\tau_r$  would then be independent of the group size, unlike the previous option of using  $N = G$ . Intuitively, this appears to be a better choice since individuals are unlikely to be capable of observing all the individuals within a large group and are frequently found to mimic their closest neighbors.

Restricting the group to local communication introduces other side effects into the model beyond simply limiting which individuals can observe an initiator. First, since group members can be unaware that a movement has been initiated, unaware individuals are free to initiate a movement of their own. As a result, multiple initiators can be present at any given time within the group and competing for follow-

ers. Furthermore, since a movement attempt is considered successful if all the individuals choose to depart, either as an initiator or a follower, then it is entirely possible for a successful collective movement to be comprised of multiple groups, each with its own initiator. While this may not result in the desired *cohesive*, collective movement, an investigation into multiple group movements is reserved for future work. Lastly, the potential presence of multiple initiators means that a movement attempt is only considered a failure if *all* the initiators cancel. As long as one initiator remains, there is potential for success.

## Numerical Implementation

To evaluate the effects of scaling up the group size, numerical simulations were performed using three different models. The first was the original model that assumed global communication within the group and the group size for the following rate calculations (i.e.,  $N = G$ ). Since global communication was assumed, the topology of the group did not have any effect on the simulation. While, as previously mentioned, this option seemed unlikely in many cases, it did provide a baseline against which the other models could be compared. The second model assumed only local communication within the group, but still used the group size for following rate calculations (i.e.,  $N = G$ ). The last model also assumed only local communication, but used the number of directly interacting neighbors for following rate calculations (i.e.,  $N = N_c$ ). While empirical observations of natural systems consistently yield  $N_c$  values in the range 6–7 (Ballerini et al., 2008), a value of  $N_c = 10$  was used for these simulations to remain consistent with the original model and minimize the number of confounding variables.

For each model, group sizes from 10 to 100 individuals were evaluated. For the local communication models, thirty different evaluations were performed for each group size, each with different initial conditions, namely, a different random seed and topology, since the topology of the group influenced the results. In each evaluation, individuals were assigned random locations in a two-dimensional plane within a distance of 10 of the origin. These locations were then used to determine the  $N_c = 10$  nearest neighbors for each individual and, therefore, the topology of the evaluation. While there are other methods for building random networks, this approach was used since it is the one that will be used when higher fidelity simulations are performed involving movement of individuals in a two-dimensional environment. For all three models, a single evaluation consisted of 20,000 simulations, each constituting a single attempt at a collective movement. All individuals had approximately the same number of initiation attempts as the initiation rates for all individuals were the same. Furthermore, the following and cancellation rates were the same for every individual in the group with the only differences between individuals being their nearest neighbors, as determined by their locations

within the group. The model parameters used were the same as those used in the original model (see Table 1), which were determined through direct observation of collective movement attempts in white-faced capuchin monkeys (Petit et al., 2009; Gautrais, 2010).

To quantify the effects of scaling the group size and using the group's topology, a variety of metrics were used. The primary measures of success were the overall percentage of initiation attempts that proved to be successful, referred to as the *success frequency*, and the relative success of individual initiations. The relative success of an individual in initiating collective movements was calculated as follows:

$$\text{Leadership} = \frac{\text{Number of moves led by } L}{\text{Total number of moves}} \quad (3)$$

which is the same as previous work (Gautrais, 2010). To assess an individual's significance within the group due to its position in the topology, two different measures were used. The first measure was the eigenvector centrality of the individual, which is a common measure of significance used in social network analysis (Wey et al., 2008). It quantifies how closely the individual is connected to the other individuals within the group. It is especially useful for collective movements and in highly connected networks (Sueur and Petit, 2008; Kasper and Voelkl, 2009). The second measure used was based on the topology of the group and an individual's interacting, nearest neighbors. This measure, referred to as an individual's *mimicking neighbors*, was the total number of individuals for whom a given individual was one of  $N_c = 10$  nearest neighbors. Using this measure, individuals that had a larger number of mimicking neighbors had more influence within the group than those with fewer mimicking neighbors, since they had the potential to transmit information throughout the group faster.

## Results and Analysis

For a group size of 10, there was no statistically significant difference between any of the three models. This indicates that the modifications made to the model to accommodate local communication did not alter the model to the extent that it was unable to make the same predictions. However, as the group size was increased, the differences between the model predictions were readily apparent. Figure 1 shows the predicted success frequency versus group size for each model. As the group size was increased, the predicted success frequency for both the global communication model and the local communication model using  $N = N_c$  increased until they both reached an asymptotic limit of slightly less than 0.5. On the other hand, the predicted success frequency for the local communication model using  $N = G$  dropped as the group size was increased. While the model using global communication predicted higher success frequencies than both local communication models at a statistically significant level for group sizes larger than 25

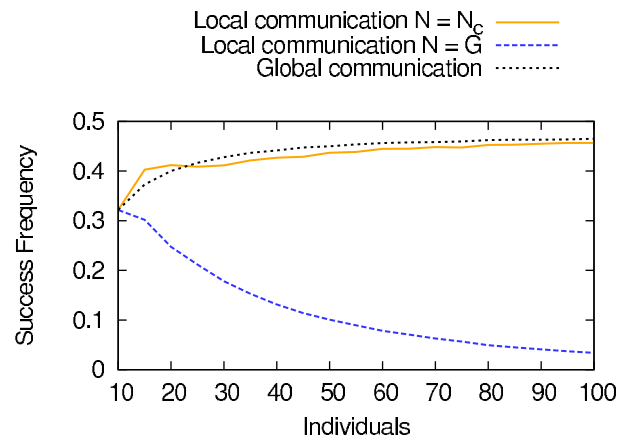


Figure 1: The frequency of successful collective movements as a function of group size for each treatment are shown. Confidence levels are omitted for clarity.

(Student's t-Test,  $p \approx 0$ ), the practical difference between the global communication model and the local communication model using  $N = N_c$  is less significant. Using Cohen's  $d$  statistic to determine the effect size between the two models, the largest predicted effect size was  $d = 2.75$  for a group size of 45. Although this is traditionally considered a large effect size, the combination of 30 evaluations and a group size of 45 produced a large number of samples for each model and resulted in a small standard deviation. In practical terms, the use of the local communications model using  $N = N_c$  resulted in at most a predicted loss in success frequency of 4.17% over all group sizes. On the other hand, the local communication model using  $N = G$  resulted in a predicted loss in success frequency of 93.7% for a group size of 100. Given these predictions, and the fact that collective movements of thousands of individuals are frequently observed in nature, the remainder of the reported results are for the local communication model for which  $N = N_c$ .

Figure 2 shows two measures of an individual's significance within the group, namely the eigenvector centrality and the number of mimicking neighbors, vs. the expected leadership success (see Equation 3) of the individual for group sizes of  $G = 20, 60,$  and  $100$ . A blue line denotes the line of best-fit for each set of data. For both measures, there was a clear correlation between the significance of the individual within the group and its expected leadership success in each group size. Although the mean expected leadership success of individuals decreased as the number of individuals increased, this was to be anticipated since there were more individuals capable of initiating movements.

The Pearson product-moment correlations between the expected leadership success of an individual and each of the two significance measures are shown in Figure 3. Each correlation was statistically significant with  $p \approx 0$ . Correlations between the number of mimicking neighbors and the ex-

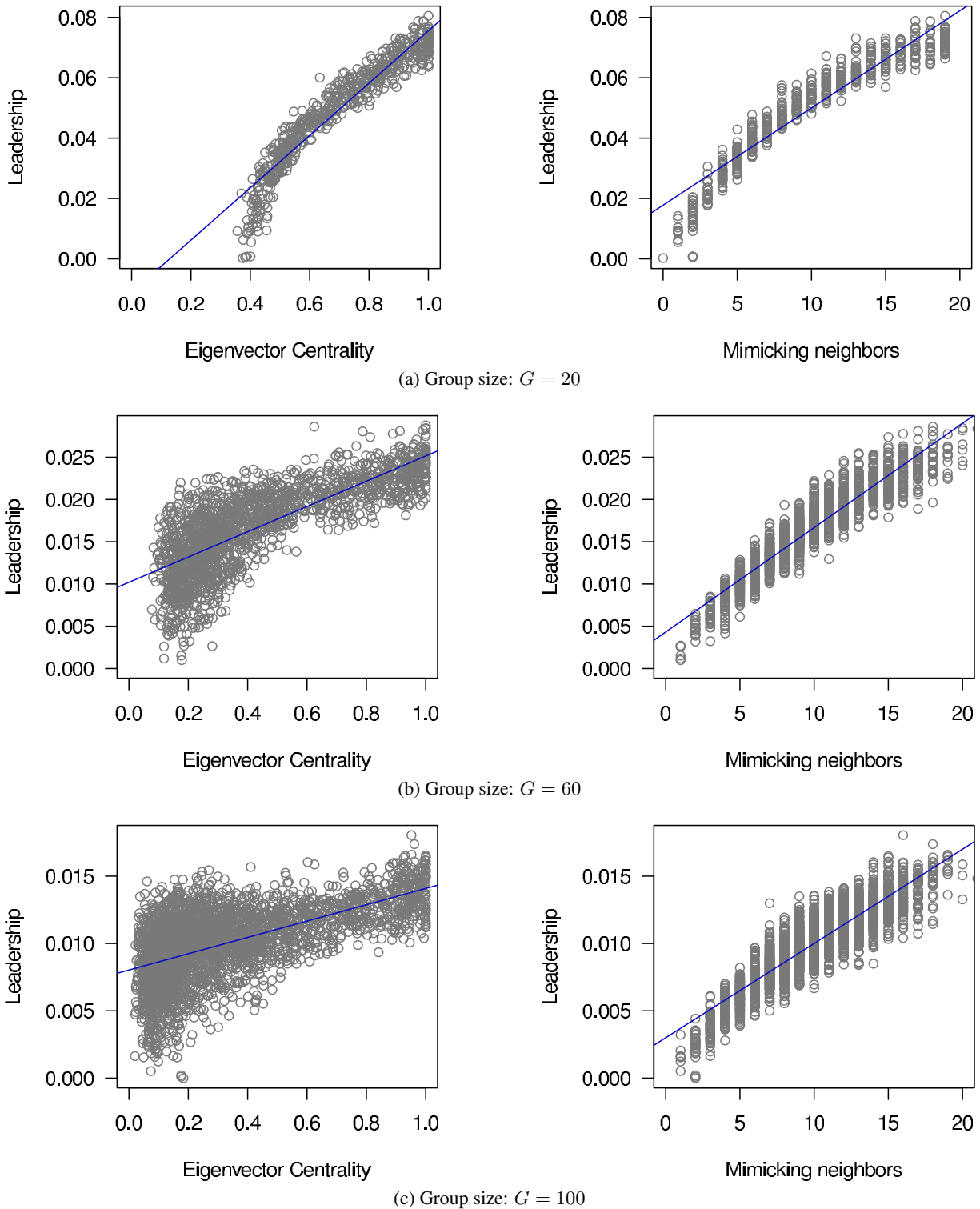


Figure 2: The two measures of an individual's significance within the group, namely the eigenvector centrality and the number of mimicking neighbors, vs. the expected leadership success of the individual for the local communication model using  $N = N_c$  are shown. Each circle represents an individual from a single evaluation. Blue lines indicate the line of best-fit.

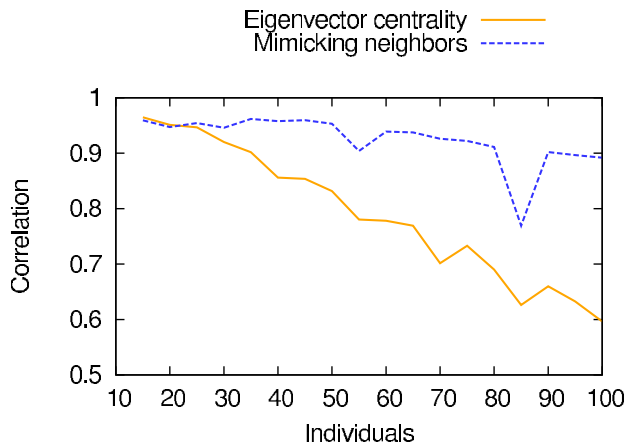


Figure 3: The correlation between two measures of an individual's significance within the group, namely the eigenvector centrality and the number of mimicking neighbors, and the expected leadership success of the individual are shown. These results only represent simulations for the local communication model in which  $N = N_c$ . Confidence levels are omitted for clarity.

pected leadership success was stronger than the correlations for the eigenvector centrality for large group sizes. While these linear correlations are statistically significant, non-linearities are visible for low measures of significance. For example, the leadership versus eigenvector centralities of approximately 0.5 and larger, while a non-linear correlation is visible for values less than 0.5. These non-linearities are present in the results for each of the group sizes evaluated. The loss of correlation as the group size increased could be attributed to the fact that, with a larger group size, outliers are more likely to be present within the group. One study saw a similar increase in noise as the group size was scaled up, even though the groups sizes were small compared to these simulations (Pillot et al., 2011).

Figure 4 illustrates this correlation between the number of mimicking neighbors and the expected leadership success using the topology for a run using a group size of 45 and the local communication model in which  $N = N_c$ . In this figure, the size of the individual represents the number of mimicking neighbors with larger sizes denoting more mimicking neighbors. The color denotes the individual's expected leadership success (see Equation 3), with orange denoting low success and blue denoting high success. While the individuals centrally located within the group have higher expected leadership success, it is not their location *per se* that correlates with their success. Rather, their expected leadership success is correlated with the number of mimicking individuals, which is a byproduct of their location within and the distribution of the group.

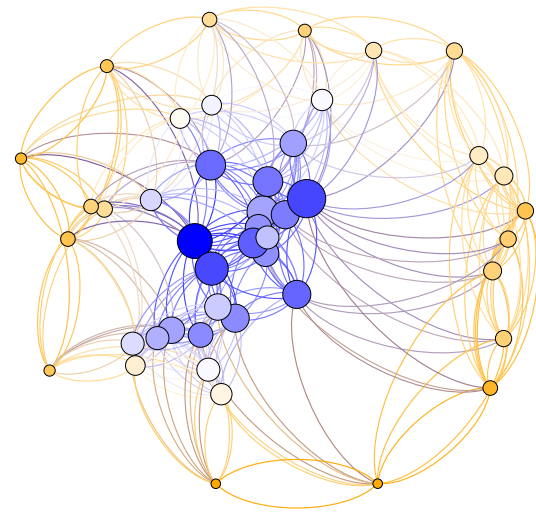


Figure 4: The group topology for a simulation using a group size of 45 is shown. An individual's color denotes its expected leadership success and ranges from orange, denoting low success, to blue, denoting high success. The size of the individual represents the number of mimicking neighbors with larger sizes denoting more mimicking neighbors.

While there was a noticeable drop in the correlation for the number of mimicking neighbors for a group size of 85, further analysis revealed that the topology from a single evaluation with a tightly-knit group of individuals produced outlier results (see Figure 5). Performing the correlation analysis with Spearman's rank correlation, which is less sensitive to outliers, resulted in a correlation of 0.876, which is more consistent with the trend observed in Figure 3.

## Discussion

There are a number of conclusions that can be drawn from these predictions. First, the restriction of a group to local communication results in a minimal drop in the mean expected probability of success for collective movement attempts, depending on the interaction rules used. This is significant as it means that the requirement for global communication that is present in many models can be removed with only a small drop in performance, given the appropriate environment and inter-individual interactions. The model in which  $N = N_c = 10$  predicts that the mean expected probability of a successful collective movement would be only slightly less than the model using global communication. On the other hand, the local communication model in which  $N = G$  predicts that the probability of success would drop to less than 5% as the number of group members approaches 100. As was previously discussed, the difference in these predictions is due to the rate at which individuals follow the initiator. In the model using  $N = G$ , individuals base their decision-making on the actions of the entire

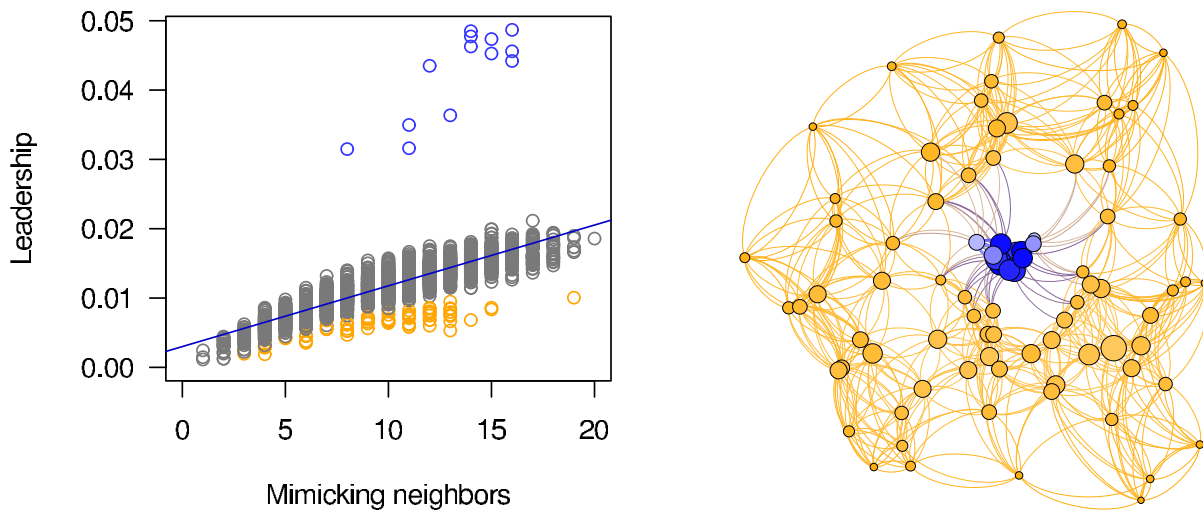


Figure 5: The number of mimicking neighbors vs. the leadership success of individuals in all simulations using a group size of 85 and  $N = N_c$  is shown on the left. Individuals from a single run representing outlier results are emphasized in color. Individuals in blue had high leadership success and individuals in orange had lower success. The topology of the individuals in the simulation is shown on the right with the same coloring scheme. The size of an individual correlates to the number of mimicking neighbors with larger individuals having more.

group, even though the number of group members which they observe and with which they interact is limited. As was noted earlier, this presents a logical contradiction in the model, the result of which is a loss in predicted frequency of successful collective movement initiations.

Second, the addition of local communication and topology to the collective movement model resulted in statistically significant correlations between the centrality of an individual within the group and the individual's leadership success. While there are other studies that have shown that individuals in a central location within a group of capuchins are more successful leaders than those occupying edge positions (Leca et al., 2003), this is the first known work to demonstrate that a model using local communication and topology is sufficient to predict those results. Although social considerations may determine the spatial location and distribution of a group (Bode et al., 2012), both local communication models use anonymous mimetism. Therefore, one can conclude that this model predicts that the use of local communication in a spatial distribution of individuals is sufficient to produce consistent leadership by centrally located individuals, even in the presence of anonymous mimetism. However, these models assume identical cancellation rates for all members of the group, which may not always be the case.

Lastly, it should be noted that these results rely on the constraint that the topology, and, therefore, the communication network, was fixed throughout an entire simulation. Since there was no actual movement involved in the simulation, the neighbors with which an individual interacts re-

mains constant, regardless of their participation in a collective movement. Future work will determine whether the effects of removing this constraint.

## Conclusions and Future Work

Effective, cohesive collective movements provide a variety of benefits for group members. Although there has historically been a difference between how these movements are modeled in natural and artificial systems, there are compelling motivations to use models derived from collective movements in natural systems to inform the design of artificial systems. To that end, the work presented here has extended a simple model of collective movement to operate effectively with large groups and serve as a guide for developing artificial systems capable of cohesive, collective movements. To accommodate larger groups using realistic physical constraints, the model was modified to use local communication and the topology of the group. Based on simulations with varying group sizes, the extended model predicts that restricting the group to only local communication can result in only a slight drop in the predicted success frequency of collective movements, if the appropriate interaction rules are used. In particular, the local communication model in which individuals base their decisions on the actions of their nearest neighbors predicts only a minimal loss in success frequency, while the model in which individuals base their decisions on the actions of the entire group predicts that collective movement initiations would rarely, if ever, succeed. In addition to these success frequency predictions, the extended model predicts that the combination

of local communication and topology results in significantly higher expected leadership success for individuals that are centrally located within the group as compared to individuals located at the edge of the group.

This work represents the initial stages of research into promoting emergent leadership and cohesive, collective movements in robot teams and there are a variety of opportunities for future work. First, the motivations of individuals to initiate a movement and follow an initiator should be explored in combination with how these individual differences influence the success of collective movements. While there has been significant work in this area already (Sumpter, 2009; Sueur and Deneubourg, 2011), of primary interest are motivations that either have analogues, or that can give inspiration for analogues, in multi-robot systems. Second, although the extended model accounted for local communication in following, it does not do so for canceling. Although the canceling rate is minimal for movements comprising 10 or more individuals, it is still non-zero and will require modification for it to be entirely consistent with the use of local communication. Lastly, the simulations should be extended to include actual movement. In some animal species, edge individuals exhibit greater leadership success because of the freedom of movement afforded by being on the edge of the group (Ramseyer et al., 2009). It would be interesting to learn how this freedom of movement alters the effects of local communication and topology that are predicted by the stationary simulations used here.

### Acknowledgments

The author wishes to thank Ingo Schlupp, Jacques Gautrais, Blake Jordan, Elizabeth Valle, and Mo Niazi for their contributions and insights. This work was supported by NSF grant No. BCS-1124837.

### References

- Ampatzis, C., Tuci, E., Trianni, V., and Dorigo, M. (2008). Evolution of signaling in a multi-robot system: Categorization and communication. *Adaptive Behavior*, 16(1):5–26.
- Anderson, M. and Papanikolopoulos, N. (2008). Implicit cooperation strategies for multi-robot search of unknown areas. *Journal of Intelligent and Robotic Systems*, 53(4):381–397.
- Balch, T. and Arkin, R. C. (1994). Communication in reactive multiagent robotic systems. *Autonomous Robots*, 1(1):27–52.
- Ballerini, M., Cabibbo, N., Candelier, R., Cavagna, A., Cisbani, E., Giardina, I., Lecomte, V., Orlandi, A., Parisi, G., Procaccini, A., Viale, M., and Zdravkovic, V. (2008). Interaction ruling animal collective behavior depends on topological rather than metric distance: Evidence from a field study. *Proceedings of the National Academy of Sciences*, 105(4):1232–1237.
- Bode, N., Franks, D., and Wood, A. (2012). Leading from the front? social networks in navigating groups. *Behavioral Ecology and Sociobiology*, pages 1–9.
- Couzin, I. D. (2009). Collective cognition in animal groups. *Trends in Cognitive Sciences*, 13(1):36–43.
- Drapier, M., Chauvin, C., and Thierry, B. (2002). Tonkean macaques (*Macaca tonkeana*) find food sources from cues conveyed by group-mates. *Animal Cognition*, 5(3):159–165.
- Fragaszy, D., Visalberghi, E., and Fedigan, L. (2004). *The complete capuchin: The biology of the genus Cebus*. Cambridge University Press.
- Gautrais, J. (2010). The hidden variables of leadership. *Behavioural Processes*, 84(3):664–667.
- Jacobs, A., Sueur, C., Deneubourg, J., and Petit, O. (2011). Social network influences decision making during collective movements in brown lemurs (*Eulemur fulvus fulvus*). *International Journal of Primatology*, 32:721–736.
- Kasper, C. and Voelkl, B. (2009). A social network analysis of primate groups. *Primates*, 50(4):343–356.
- Klavins, E. (2003). Communication complexity of multi-robot systems. *Algorithmic Foundations of Robotics V*, pages 275–292.
- Leca, J., Gunst, N., Thierry, B., and Petit, O. (2003). Distributed leadership in semifree-ranging white-faced capuchin monkeys. *Animal Behaviour*, 66(6):1045–1052.
- Petit, O., Gautrais, J., Leca, J.-B., Theraulaz, G., and Deneubourg, J.-L. (2009). Collective decision-making in white-faced capuchin monkeys. *Proceedings of the Royal Society B: Biological Sciences*, 276(1672):3495–3503.
- Pillot, M.-H., Gautrais, J., Arrufat, P., Couzin, I. D., Bon, R., and Deneubourg, J.-L. (2011). Scalable rules for coherent group motion in a gregarious vertebrate. *PLoS ONE*, 6(1):e14487.
- Pyritz, L., King, A., Sueur, C., and Fichtel, C. (2011). Reaching a consensus: Terminology and concepts used in coordination and decision-making research. *International Journal of Primatology*, 32(6):1268–1278.
- Ramseyer, A., Boissy, A., Dumont, B., and Thierry, B. (2009). Decision making in group departures of sheep is a continuous process. *Animal Behaviour*, 78(1):71–78.
- Sueur, C. and Deneubourg, J. (2011). Self-organization in primates: Understanding the rules underlying collective movements. *International Journal of Primatology*, 32(6):1413–1432.
- Sueur, C., Deneubourg, J., and Petit, O. (2011). From the first intention movement to the last joiner: Macaques combine mimetic rules to optimize their collective decisions. *Proceedings of the Royal Society B: Biological Sciences*, 278(1712):1697–1704.
- Sueur, C. and Petit, O. (2008). Organization of group members at departure is driven by social structure in macaca. *International Journal of Primatology*, 29(4):1085–1098.
- Sumpter, D. J. (2009). Group behaviour: Leadership by those in need. *Current Biology*, 19(8):R325–R327.
- Wey, T., Blumstein, D. T., Shen, W., and Jordán, F. (2008). Social network analysis of animal behaviour: A promising tool for the study of sociality. *Animal Behaviour*, 75(2):333–344.