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Self-organized game dynamics in complex networks

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Complex networks are ubiquitous and known to profoundly affect the processes that take place on them. From a theoretical perspective, some of the most complex processes studied to date, occurring on complex networks, are related with behavioural dynamics and decision-making, often described by means of social dilemmas of cooperation. Among these, the Prisoner's Dilemma (**PD**) provides the most popular metaphor of such dilemmas, given that its only Nash equilibrium is mutual defection, despite mutual cooperation providing higher returns – thus the dilemma. We may also assume a population dynamics (evolutionary) approach to game theory where agents revise their behaviour based on the perceived success of others, creating a gradient of selection which dictates how cooperation self-organizes through time. Evolutionary Games provide one of the most sophisticated examples of complex dynamics in which the role of the underlying network topology proves ubiquitous. For instance, when cooperation is modeled as a prisoner's dilemma game, cooperation may emerge (or not) depending on how the population is networked (Santos et al., 2012a).

Up to now, it has been hard to characterize in detail the global dynamics by which local self-regarding actions lead to a collective cooperative scenario while relating it to the network topology. Indeed, most network studies have been focused on the analysis of the evolutionary outcome of cooperation – either by means of the numerical analysis of steady states or by the analytical determination of the conditions that lead to fixation – without characterizing the self-organization process by which one of the strategies outcompetes the other. Here we report on new results (Pinheiro et al., 2012a,b; Santos et al., 2012b,a), where we show how to establish the link between individual and collective behavior in the context of evolutionary games in networked populations.

Overall, we show how behavioral dynamics of individuals facing a cooperation dilemma in social networks can be understood as though individuals face a different dilemma in absence of structure. As illustrated in Fig. 1, homogeneous networks promote a coexistence dynamics between cooperators and defectors – akin to the Chicken or Snowdrift game – whereas heterogeneous networks, from single to scale-free networks, favor the coordination between them, similar to the Stag-hunt game. In other words, while agents locally perceive and play a **PD**, globally the dynamics of the population resembles the one obtained from a completely different game, as if, individuals would be locally facing a different dilemma.

To this end we define a time-dependent variable – that we call the average gradient of selection (**AGoS**) – and use it to track the self-organization of cooperators when coevolving with defectors. In finite well-mixed populations, that is, populations in which every individual can interact with every other individual in the population, this gradient of selection ($G(x_C) = T^+(x_C) - T^-(x_C)$) can be computed analytically, as the difference of the probabilities of increasing ($T^+(x_C)$) and decreasing ($T^-(x_C)$) the number of cooperators, for each fraction x_C of cooperators. While this quantity is impossible to be attained analytically for arbitrary network structures, the **AGoS** provides a numerical account of the same variables, offering the change in time of the frequency of cooperative traits under selection. The **AGoS** can be computed for arbitrary intensity of selection, arbitrary population structure and arbitrary game parameterization. We further prove that the global games are not fixed: they change in time, co-evolving with the motifs of cooperators in the population. The evolutionary outcome of such a self-organization process will depend sensitively on this coevolution, which can be followed using a time-dependent **AGoS**.

The scenarios illustrated in Fig. 1 become even richer whenever one takes into account the role of selection pressure (also known as intensity of selection)in the overall evolutionary dynamics of a networked population. Selection pressure provides the relative significance of agent fitness in the evolutionary process, as opposed to an arbitrary or random adoption of strategies. This is important, as selection pressure can be very different depending on the processes at stake. Indeed, in many social interactions, errors in decision making, perhaps induced by stress or exogenous confounding factors, which often translate into a bounded rational be-

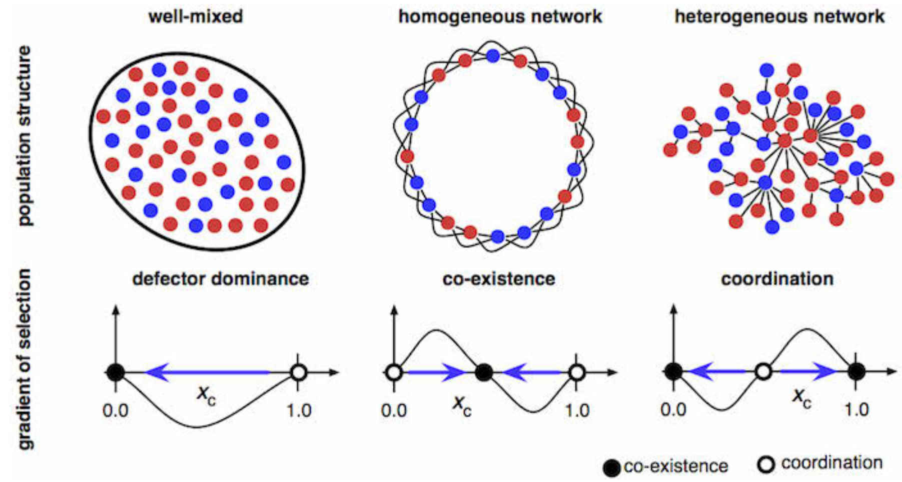

Figure 1: The Average Gradient of Selection (**AGoS**) provides a characterization of the change in time of the fraction of cooperators under natural selection, being positive (negative) when the fraction of cooperators tends to increase (decrease). While in well-mixed populations (left panel), the tragedy of the commons ($x_C = 0$) emerges as the only stable fixed point, homogeneous networks favor the co-existence of cooperators and defectors (middle panel), whereas degree heterogeneous networks (right panel) creates two basins of attraction, as if agents would be locally facing a coordination dilemma.

haviour of the players, may lead to an overall weak selection environment. This contrasts with many other situations where selection is mainly strong such as the dynamics of cultural evolution. Moreover, the fate of cooperation in social networks may depend on how the success of the others is locally perceived, which is related with the number of partners of each player and their social context, turning selection pressure into a central variable in behavioural evolution.

As we show in (Pinheiro et al., 2012b), as opposed to what happens in heterogeneous populations that generate a coordination dynamics for a broad range of selection pressure values, on homogeneous networks the population-wide dynamics depends on the intensity of selection: under strong selection they favour a co-existence like dynamics while under weak selection we recover the *well-mixed* scenario of a **PD**-like dynamics which leads to the demise of cooperation (Fig. 1, left panel). Additional, we were able to identify the existence (on several types of networks) of an optimum level of selection pressure for which cooperation is maximised. The underlying process that leads to this result differs from homogeneous to heterogenous networks. In the first class of networks the optimal selection pressure is associated with the ability of cooperators to form and sustain clusters, while on the second class it is the result of a decoupling in the distribution of intensities of selection between pairs of agents that is present from the natural diversity of fitnesses (Santos et al., 2012a) in the population.

Additionally, the application of the **AGoS** is not limited 2-person games. In fact, as discussed in (Santos et al., 2012b), heterogeneous network structures create multiple internal equilibria when individuals face public goods dilemmas, departing significantly from the reference scenario of a structureless population, approach which can also be extended to N-person games in adaptive networks (Moreira et al., 2013). Finally, we would like to stress that the scope and importance of this methodology goes far beyond the present application to evolutionary games on graphs. The principles can be used to extract any dynamical quantity that describes a process (as long as it is a Markov process) taking place on a network such as the outbreak of epidemics or the spread of information.

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