

Spatial Organisation of Cooperation with Contingent Agent Migration

Pierre Buesser and Marco Tomassini

HEC Faculty, Information Systems Department, University of Lausanne, Switzerland

Abstract

In the framework of game theory and cooperation, we study standard two-person population games when agents in the population are allowed to move to better positions in a two-dimensional diluted grid. We show that cooperation may thrive for small interaction radius and when mobility is low. Furthermore, we show that, even when the agents cannot change their game strategy, interesting spatial patterns do emerge as players explore their neighborhood in order to find a better place to migrate to. In the Prisoner's Dilemma and Stag-Hunt games, when the losses experienced by cooperators against defectors as well as the game and migration radius are large enough, players move in a coherent way because clusters of cooperators followed by defectors form. On the other hand, in the Hawk-Dove game or when the migration radius is small, players end up blocked into stationary clusters.

Introduction

Systems whose parts are contained in physical space are very important in biological and social sciences since most interactions among living beings or artificial actors take place in such a space. Thus game-theoretical interactions among spatially embedded agents distributed according to a fixed structure in the plane have been studied in detail since the pioneering works of Axelrod (Axelrod, 1984) and Nowak and May (Nowak and May, 1992). The related literature is very large; see, for instance, the review article by Nowak and Sigmund (Nowak and Sigmund, 2000) and references therein for a synthesis. Most of this work was based on populations of agents arranged according to planar regular grids for mathematical simplicity and ease of numerical simulation. Recently, some extensions to more general spatial networks have been discussed in (Buesser and Tomassini, 2012). Strategic behavior on fixed spatial structures is important but, in the majority of real situations both in biology and in human societies, actors have the possibility to move around in space. Many examples can be found in biological and ecological sciences, in human populations, and in engineered systems such as ad hoc networks of mobile communicating devices or robot teams. Mobility may have positive or negative effects on cooperation, depending on several

factors. For instance, early on Enquist and Leimar (Enquist and Leimar, 1993) studied a model in which space is not explicitly represented but assortment of strategies is made non-uniform by introducing the possibility of abandoning a non-profitable relationship and searching for another partner, thus modifying the homogeneous well-mixed original population structure. Their main conclusion was that mobility may seriously restrict the evolution of cooperation. In the last decade there have been several new studies of the influence of mobility on the behavior of various games in spatial environments representing essentially two strands of research: one in which the movement of agents is seen as a random walk, and a second one in which movement may contain random elements but it is purposeful, or strategy-driven. In the present study we focus on situations where, instead of randomly diffusing, agents possess some basic cognitive abilities and they actively seek to improve their situation by moving in space represented as a discrete grid in which part of the available sites are empty and can thus be the target of the displacement. This approach has been followed, for example, in (Helbing and Yu, 2009, 2008; Jiang et al., 2010; Chen et al., 2011; Aktipis, 2004). The mechanisms invoked range from success-driven migration (Helbing and Yu, 2009), adaptive migration (Jiang et al., 2010), flocking behavior (Chen et al., 2011), and cooperators walking away from defectors (Aktipis, 2004). The general qualitative message of this work is that purposeful contingent movement may lead to highly cooperating stable or quasi-stable population states if some conditions are satisfied. Despite all the above work, the quantitative results strongly depend on the assumptions made and on the details of the models.

Our approach here is inspired by the work of (Helbing and Yu, 2008, 2009) which they call "success-driven migration" and which has been shown to be able to produce highly cooperative states. In this model, locally interacting agents playing either defection or cooperation in a two-person Prisoner's Dilemma are initially randomly distributed on a grid in equal proportions with a certain density such that there are empty grid points. Agents are updated one at a time.

When chosen for updating, the agent evaluates the current payoff she would accumulate by playing two-person games with all her current neighbors but she can also “explore” an extended square neighborhood by testing all the empty positions up to a given distance. If the player finds that it would be more profitable to move to one of these positions then she does it, choosing the best one among those tested, otherwise she stays at her current place. Helbing and Yu find that robust cooperation states may be reached by this single mechanism, even in the presence of random noise in the form of random strategy mutations and random agent relocation. Our study builds upon this work in several ways. In the first place, whilst Helbing and Yu had a single game neighborhood, we systematically investigate game neighborhood and migration neighborhood, showing that only some values of this pair of parameters allow the evolution of cooperation using success-driven migration. Second, we present systematical results for a whole game phase space including the Hawk-Dove class of games, the Stag Hunt coordination class, and the Prisoner’s Dilemma class, while only the Hawk-Dove and the Prisoner’s Dilemma are studied in Helbing and Yu (2008). We find that fully cooperative states can be reached for the standard neighborhoods and for several migration distances in the Stag Hunt case, while cooperation can also be achieved in the Prisoner’s Dilemma for a non-negligible part of its game space. Mobility is less beneficial in the hawk-dove game where cooperation levels are on the average only slightly better than in the static, motionless case. Finally, we also study the extreme case of system evolution when agents cannot change their initially attributed strategy and are only allowed to test free cells within their migration radius in order to possibly move to more profitable regions. Here cooperation cannot evolve by definition but we are interested in the dynamical patterns that may form, i.e. whether or not the agent distribution remains uniform during the dynamics. In this case, in the Prisoner’s dilemma and Stag Hunt games we find that players move in a coherent way because clusters of cooperators followed by defectors are formed. On the other hand, in the Hawk-Dove game or when the radius within which players move is small, players end up blocked into stationary clusters.

Evolutionary Games and Migration in Two-Dimensional Space

The Games Studied

We investigate three classical two-person, two-strategy, symmetric games classes, namely the Prisoner’s Dilemma (PD), the Hawk-Dove Game (HD), and the Stag Hunt (SH). These three games are simple metaphors for different kinds of dilemmas that arise when individual and social interests collide. The Harmony game (H) is included for completeness but it is not a dilemma since cooperation is trivially the NE. The main features of these games are summarized here

for completeness; more detailed accounts can be found elsewhere e.g. (Weibull, 1995; Hofbauer and Sigmund, 1998; Vega-Redondo, 2003). The games have the generic payoff matrix M (equation 1) which refers to the payoffs of the row player. The payoff matrix for the column player is simply the transpose M^T since the game is symmetric.

$$\begin{matrix} & C & D \\ C & \begin{pmatrix} R & S \\ T & P \end{pmatrix} \end{matrix} \quad (1)$$

The set of strategies is $\Lambda = \{C, D\}$, where C stands for “cooperation” and D means “defection”. In the payoff matrix R stands for the *reward* the two players receive if they both cooperate, P is the *punishment* if they both defect, and T is the *temptation*, i.e. the payoff that a player receives if he defects while the other cooperates getting the *sucker’s payoff* S .

In order to study the usual standard parameter space (Santos et al., 2006; Roca et al., 2009), we restrict the payoff values in the following way: $R = 1, P = 0, -1 \leq S \leq 1$, and $0 \leq T \leq 2$.

For the PD, the payoff values are ordered such that $T > R > P > S$. Defection is always the best rational individual choice, so that (D, D) is the unique Nash Equilibrium (NE) and also the only fixed point of the replicator dynamics (Weibull, 1995; Hofbauer and Sigmund, 1998). Mutual cooperation would be socially preferable but C is strongly dominated by D .

In the HD game, the order of P and S is reversed, yielding $T > R > S > P$. Thus, when both players defect they each get the lowest payoff. Players have a strong incentive to play D , which is harmful for both parties if the outcome produced happens to be (D, D) . (C, D) and (D, C) are NE of the game in pure strategies. There is a third equilibrium in mixed strategies which is the only dynamically stable equilibrium (Weibull, 1995; Hofbauer and Sigmund, 1998).

In the SH game, the ordering is $R > T > P > S$, which means that mutual cooperation (C, C) is the best outcome and a NE. The second NE, where both players defect is less efficient but also less risky. The difficulty is represented by the fact that the socially preferable coordinated equilibrium (C, C) might be missed for “fear” that the other player will play D instead. The third mixed-strategy NE in the game is evolutionarily unstable (Weibull, 1995; Hofbauer and Sigmund, 1998).

Finally, in the H game $R > S > T > P$ or $R > T > S > P$. In this case C strongly dominates D and the trivial unique NE is (C, C) . The game is non-conflictual by definition and does not cause any dilemma, it is mentioned to complete the quadrants of the parameter space.

There is an infinite number of games of each type since any positive affine transformation of the payoff matrix leaves the NE set invariant (Weibull, 1995). Here we study the customary standard parameter space (Santos et al., 2006; Roca

et al., 2009), by fixing the payoff values in the following way: $R = 1$, $P = 0$, $-1 \leq S \leq 1$, and $0 \leq T \leq 2$. In the TS -plane each game class corresponds to a different quadrant depending on the above ordering of the payoffs as depicted in Fig 1, left image, and the figures that follow. The right part of Fig 1 shows the standard replicator dynamics results for a well mixed population (Weibull, 1995).

Population Structure

The euclidean two-dimensional space is modeled by a discrete square lattice of side L with toroidal borders. Each vertex of the lattice can be occupied by one player or be empty. The density is $\rho = N/L^2$, where $N \leq L^2$ is the number of players. Players can interact with k neighbors which lie at a distance smaller or equal than a given constant R_g . Players can also migrate to empty grid points at a distance smaller than R_m . The relationships between the neighborhoods defined as above and the customary square Moore neighborhoods of increasing order are illustrated in Fig. 2.

Payoff Calculation and Strategy Update Rules

Here it is specified how individual's payoffs are computed and how agents decide to revise their current strategy. We take into account that each agent i interacts locally with a set of neighbors V_i lying closer than R_g . Let $\sigma_i(t)$ be a vector giving the strategy profile at time t with $C = (1, 0)$ and $D = (0, 1)$ and let M be the payoff matrix of the game (equation 1). The quantity

$$\Pi_i(t) = \sum_{j \in V_i} \sigma_i(t) M \sigma_j^\top(t) \quad (2)$$

is the cumulated payoff collected by player i at time step t .

We use an asynchronous scheme for strategy update and migration, i.e. players are updated one by one by choosing a random player in each step with uniform probability and with replacement. Then the player migrates with probability p or updates its strategy with probability $1 - p$. Several update rules are customary in evolutionary game theory (Roca et al., 2009). Here we shall use imitative strategy update protocol which consists in switching to the strategy of the neighbor that has scored best in the last time step. This *imitation of the best* (IB) policy can be described in the following way: the strategy $\sigma_i(t)$ of individual i at time step t will be

$$\sigma_i(t) = \sigma_j(t - 1), \quad (3)$$

where

$$j \in \{V_i \cup i\} \text{ s.t. } \Pi_j = \max_{k \in \{V_i \cup i\}} \{\Pi_k(t - 1)\}. \quad (4)$$

That is, individual i will adopt the strategy of the player with the highest payoff among its neighbors including itself. If there is a tie, the winner individual is chosen uniformly at random.

A final remark is in order here. The above model rules are common in numerical simulation work, which has the advantage that the mathematics is simpler and results can be compared with previous work. However, they are homogeneous among the agents and there is no learning. It is far from clear whether they are able to model real situations in biological systems and especially human societies. However, we feel that these considerations are outside the scope of the present numerical investigation.

Strategy Imitation and Migration rules

When player i is chosen for update, she changes her strategy with probability $1 - p$ or migrates with probability p . If the pseudo-random number drawn dictates that i should migrate, then she considers N_{test} randomly chosen positions in the disc of radius R_m around itself in order to take into account her bounded rationality. $N_{test} = 20$ has been used in all the simulations. For each trial position the player computes the payoff that she would obtain in that place with her current strategy. The positions already occupied are just discarded from the possible choices. Then player i stays at her current position if she obtains there the highest payoff, or migrates to the most profitable position among those explored during the test phase. If several positions, including her current one, share the highest payoff she chooses one at random. The protocol described in Helbing and Yu (Helbing and Yu, 2009) is slightly different: the chosen player chooses the strategy of the best neighbor including himself with probability $1 - r$, and with probability r his strategy is randomly reset. Before this imitation step i deterministically chooses the highest payoff free position in a square neighborhood surrounding the current player and including himself. If several positions provide the same expected payoff, the one that is closer to the old position of i is selected.

Algorithm 1: migration of player i

```

for  $j \in [1, N_{test}]$  do
    choose random position  $x_j$  in  $V_i$ 
    if  $x_j$  is free then
        compute the expected payoff  $\Pi(x_j)$  of player  $i$ 
        at  $x_j$ 
    choose the best  $\Pi(x_j)$ ; if several  $x_j$  share the same
     $\Pi(x_j)$  choose one at random and migrate to this
    position
    
```

Mobility Measure

In order to assess if a player has a definite direction of motion with respect to time we will use the following mobility measure. Mobility is defined as $M = \max_{t \in [0, \tau]} (D_t) / L$ where τ is the time interval for a player to travel a total distance L if she moves the maximal distance R_m at each time step in the same direction. D_t is the Euclidean distance from

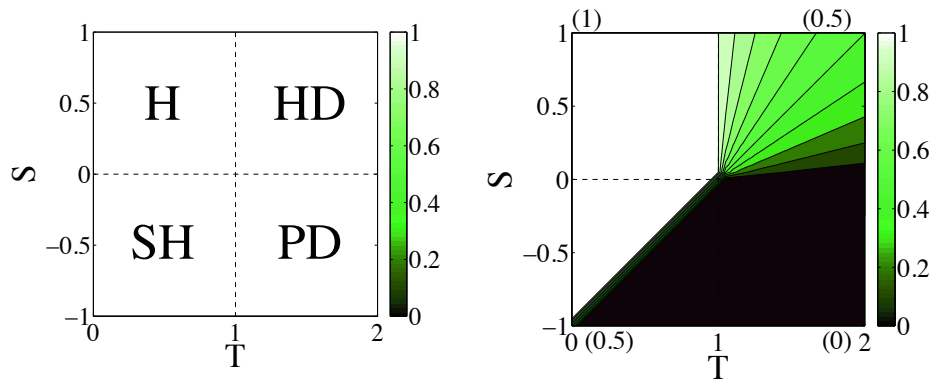


Figure 1: (Color online) Left image: The games phase space (H= Harmony, HD = Hawk-Dove, PD = Prisoner’s Dilemma, and SH = Stag Hunt) as a function of S, T ($R = 1, P = 0$). Right image: cooperation at steady state in a well mixed population for comparison purposes. Lighter tones stand for more cooperation. Figures in parentheses next to each quadrant indicate average cooperation in the corresponding game space.

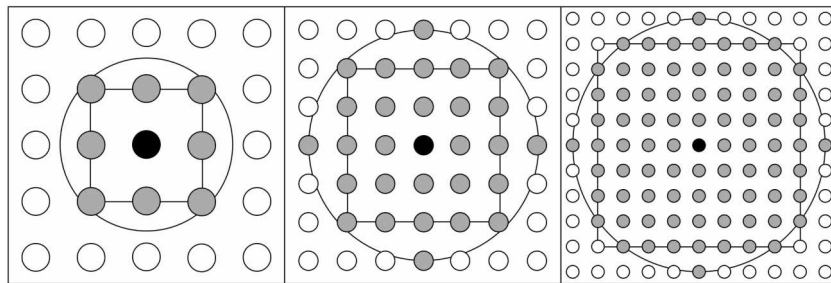


Figure 2: Relationships between the neighborhoods defined by the radii R_g and R_m and the Moore square neighborhoods. Left: with the 1.5 radius the neighborhood is identical with the standard Moore neighborhood at distance one. Middle: radius 3 is almost equivalent to a Moore neighborhood at distance two marked as a square. Right: with radius 5 the closer Moore neighborhood has distance four.

the initial position to the position at time t . The interval τ is not taken from the beginning of the simulation but rather after a time sufficient for the mobile patterns to form. Thus M measures the ratio between the maximal distance over time reached by a player from her initial position and the maximal distance that it is possible to reach in the best case. We multiplied this measure by four in order to increase the contrast in the images. However, this measure is not a strict indicator of coherent motion as moving clusters can collide and change direction.

Simulation Parameters

The TS -plane has been sampled with a grid step of 0.1 and each value in the phase space reported in the figures is the average of 50 independent runs. The evolution proceeds by first initializing the population by creating a player in each cell of the underlying lattice with probability ρ . Then the players’ strategies are initialized uniformly at random such that each strategy has a fraction of approximately 1/2 unless otherwise stated. For each grid point, agents in the popula-

tion are chosen sequentially at random with replacement to revise their strategies or positions. Payoffs are constantly updated. To avoid transient states, we let the system evolve for a period of $\tau = 1000$ time steps, for each time step $N = 1000$ players are chosen for update. At this point almost always the system reaches a steady state in which the frequency of cooperators is stable except for small statistical fluctuations. We then let the system evolve for 50 further steps and take the average cooperation value, or the mobility, in this interval. We repeat the whole process 50 times for each grid point and, finally, we report the average cooperation values over those 50 repetitions.

Results

Strategy Evolution and Mobility

In this section we discuss cooperation results with the IB rule and adaptive migration and explore the influence of different radii R_m and R_g and the density ρ . Fig. 3 left image displays the cooperation level in the ST -planes with the

Downloaded from http://direct.mit.edu/isal/proceedings-pdf/ecal2013/25/192/1901531/978-0-262-31709-2-ch029.pdf by guest on 29 September 2023

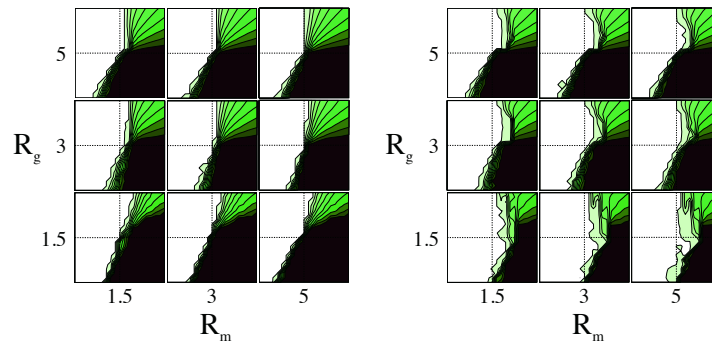


Figure 3: (Color online) Average cooperation levels with IB strategy revision rule as a function of R_g and R_m with $p = 0.5$ and $\rho = 0.5$. Left image: Random migration. Right image: best fitness migration rule. The size of the population is 1000 players. In all cases the initial fraction of cooperators is 0.5 randomly distributed among the occupied grid points.

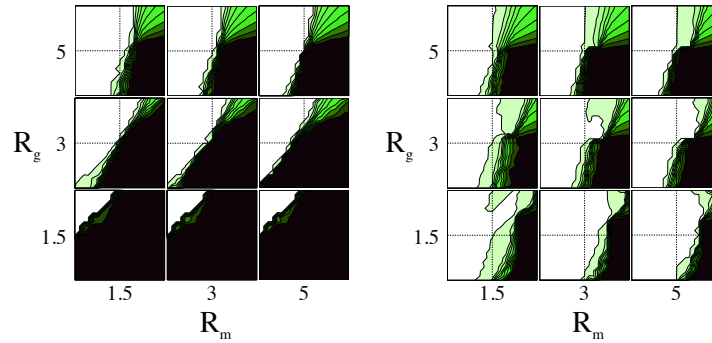


Figure 4: (Color online) Average cooperation levels with IB strategy revision rule as a function of R_g and R_m with $p = 0.5$ and $\rho = 0.1$. Left image: Random migration. Second image: best fitness migration rule. The size of the population is 1000 players. In all cases the initial fraction of cooperators is 0.5 randomly distributed among the occupied grid points.

IB rule and a density $\rho = 0.5$ for several combinations of R_g and R_m . For the sake of comparison, and in order to have a baseline case, Fig. 3 left image shows the case in which migration is not dictated by success but, rather, it is simply random, i.e. the target of migration will be a free cell randomly drawn among those contained in the R_m disk. The right image depicts cooperation levels when migration is success-driven. We see that, for $R_g = 1.5$, full cooperation is achieved in the SH quadrant for all R_m in the case of contingent migration while cooperation is notably lower in the random migration case for all R_m . For the PD cooperation remains nearly constant through R_m for $R_g = 1.5$, or slightly improves with smaller R_m in the contingent migration case with average values in the quadrant of about 0.3. In contrast, it is almost zero in the random diffusion case. Increasing the game radius R_g doesn't help and all average values tend to fall independent of R_m . This is because enlarging the neighborhood of a player is a step towards the mixed population in which cooperation results are worse, as can be seen in Fig. 1. We have observed that the increase in cooperation for $R_g = 1.5$ with “intelligent” migra-

tion is essentially due to the formation of cooperator clusters that remain relatively stable throughout evolution thanks to the possibility for cooperators to join one of those clusters. With larger R_g values, small cooperator clusters are easier to break and large C clusters, which would help cooperation to establish itself in the cluster, cannot form and defection prevails at least in the PD case. The Hawk-Dove game, due to its mixed-strategy equilibrium benefits less from success-driven migration as the two other games.

Density is a parameter that heavily influences the evolution of cooperation Vainstein et al. (2007); Sicardi et al. (2009), also in the presence of intelligent migration Helbing and Yu (2009); Jiang et al. (2010). Too high densities are detrimental because they tend to limit the mobility of agents to a point that only cooperator clusters that appear owing to statistical fluctuations in the initial population compositions can eventually remain stable. It appears that low and intermediate densities give more freedom to the population for moving around and to search for better positions. Figure 4 right image shows average cooperation results for the IB strategy revision rule and the same combinations of radii

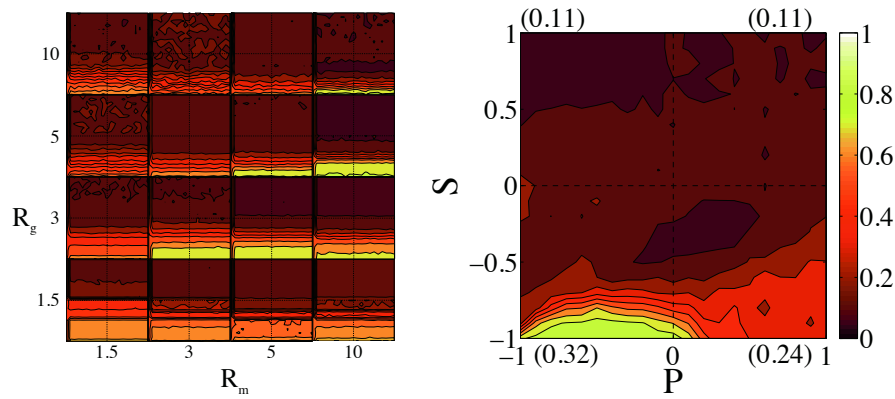


Figure 5: (Color online) Left: average mobility levels in the ST-plane as a function of R_g and R_m . Right: mobility in the SP-plane for $T = 1$, $R_m = 10$ and $R_g = 10$. The best fitness migration rule is used. The size of the population is 1000 players and $\rho = 0.1$. Cooperators fraction is 0.5 randomly distributed among the population. Lighter tones stand for more mobility.

but for $\rho = 0.1$ instead of 0.5. With $\rho = 0.1$ cooperation generally increases. In this case defectors attack clusters with a smaller rate since they are more diffused in space and move randomly until they find a cluster of cooperators. The advantage of intelligent migration with respect to random motion is even more marked here by comparing with the left image. In the latter defection appears to be even stronger than in the mixed population, but this is rather special and is due to the fact that the system is very diluted which causes most encounters to be between just two players.

Mobility Only: Emergence of Dynamic Clusters

In this section we study the emergence of dynamic clusters. These clusters are formed by a cohesive group of cooperators followed by defectors. The left image of Fig. 5 displays the mobility of nodes (see Sect. 5) for several ST-planes as a function of the game and migration radius. Lighter tones stand for higher mobility and indicate that such dynamic clusters may form. It can be observed that the dynamic clusters tend to appear with low S . The horizontal stripes of constant M can be explained by the fact that, as long as $P = 0$, all positive values of T are identical since the best target position for migration remains the same. On the other hand, when P is comparable to T or larger, defectors form clusters among themselves and stop following cooperators, which causes M to decrease. In contrast, when P is negative enough, defectors repel each other and they can not gather behind cooperators. These effects are reflected in the averages shown in the right image of Fig. 5.

We display dynamic clusters for some particular runs in Figs. 6 and 7. Figure 6 shows clusters that have formed after a number of time steps and that are already stable as a function of S with $R_g = 5$ and $R_m = 10$. The corresponding game can be inferred from Fig. 5 left image. From left to right the images show situations with increasing cluster mobility. There is a sort of mobility transition such that,

while the first two images show clusters that do not move, the rightmost one corresponds to a situation in which the clusters are much more dynamical.

Figure 7 shows the clusters appearance when mobility is high (compare with Fig. 5 left image) as a function of the radius of play R_g for the same game as above, which is in the PD region. One can see that there is a direct relationship between increasing R_g and the cluster size. With a given R_m , which is here 10, when R_g is comparatively small, clusters do form but they are continuously destroyed and reformed in an other places without a definite motion.

The Effect of Strategy Update In the limiting case $p \rightarrow 1$, i.e. very little strategy update with respect to migration, dynamical patterns form before any significant strategy update. Fig. 8 displays the ST-plane in that case. It can be observed that cooperation is lost for the lower values of S . This loss of cooperation can be related to an increase in mobility by comparing Fig. 4 (right) with Fig. 8 and by remarking that the loss of cooperation between these two cases correspond to the relatively high levels of mobility seen in Fig. 5 for this area of the game space. In the case $p = 0.5$ the dynamical patterns cannot fully form since the strategy evolution is too fast. In fact as clusters of cooperators form defectors are attracted towards them. Considering only the case in which cooperation thrives, if p is high enough the incoming defectors are transformed into cooperators directly while approaching the cluster. Thus the cluster remains static and grows. On the other hand, when p is small the migrating defectors cumulated around the cluster will eventually cause it to move.

Conclusion

In the framework of game theory we have studied the evolution of cooperation in spatially structured populations when a given focal player can only interact with players contained

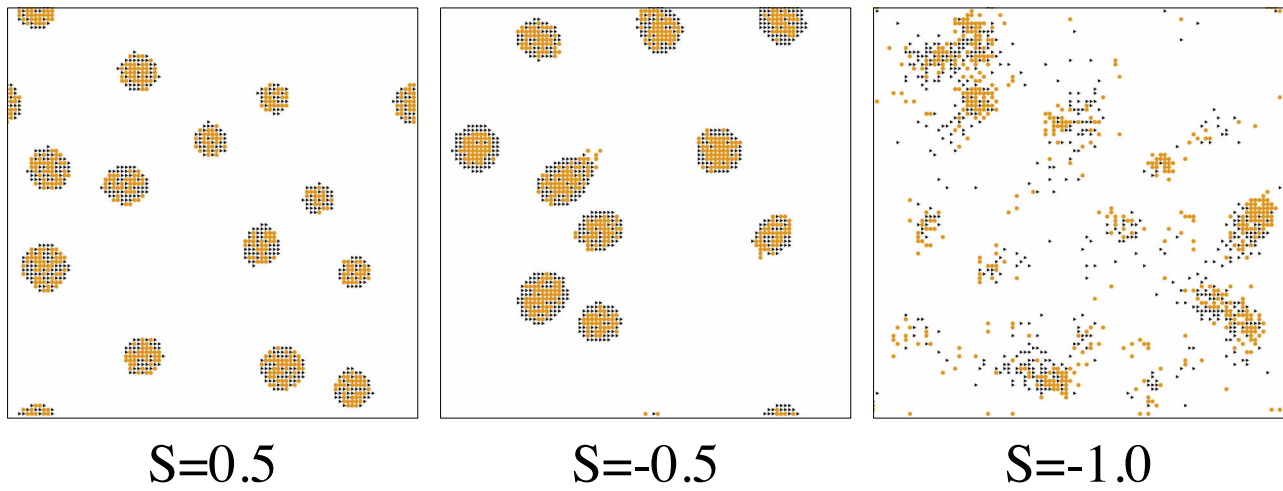


Figure 6: (Color online) Cluster for $R_g = 5$ and $R_m = 10$, $T = 1.5$. Cooperators are represented as orange circles and defectors as black triangles.

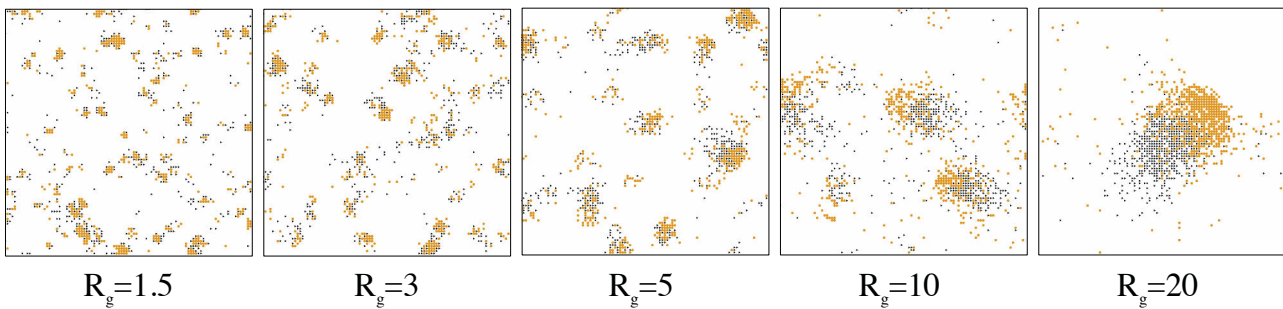


Figure 7: (Color online) Cluster for $R_m = 10$, $T = 1.5$, $S = -1.0$ for different R_g values. Cooperators are represented in orange and defectors in black.

in a radius R_g centered on the focal player that is small with respect to the space available. This locality of interactions is a realistic feature of actual populations and markedly differs from the customary well mixed population. Besides being able to adapt their strategy with probability p , in our model players can also move around to unoccupied places in the underlying two-dimensional grid also with probability $1 - p$. The amount of displacement is determined by the migration radius R_m . Migration depends on the payoff, i.e. a player that has decided to migrate can examine a number of free positions around it within the radius R_m , earn a potential payoff by fictitious play with the neighbors at that position, and finally choose to migrate to the position that provides the best payoff among those tested. We show that an equal amount of this strategy and of strategy mutation in the original position gives rise to full cooperation in the SH game

space and, to comparatively high values in the more difficult PD game space. This is particularly striking when compared with the baseline case in which strategy revision is identical but migration is to a randomly chosen free cell in the disk of radius R_m .

We have also investigated pattern formation in the population under the effects of intelligent migration only. In this case too we start from a 50 – 50 random distribution of cooperators and defectors. However, now cooperation cannot evolve since strategy changes are not allowed. What we do observe is a very interesting and intricate phenomenon of dynamical or almost static pattern formation that is related to the underlying game played and that also depends on the R_g and R_m radii. We have analyzed the nature and dynamics of these clusters and we have shown that mobility of agents can be high when the sucker payoffs S reaches negative enough

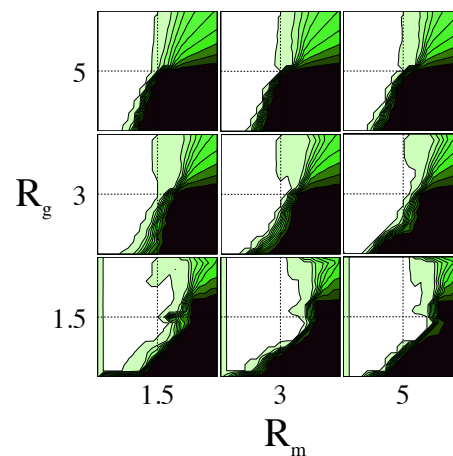


Figure 8: (Color online) Average cooperation levels with best fitness migration rule and IB strategy update rule as a function of R_g and R_m , $\rho = 0.1$, and $p = 0.99$. The size of the population is 1000 players. The initial fraction of cooperators is 0.5 randomly distributed among the occupied grid points.

values compared to the reward payoff R . The temptation T has only to be positive as the punishment P is null in our settings. For high interaction radius R_g and migration radius R_m the motion is coherent and the cooperators tend to gather and move in the same direction with swarms of defectors following them. When R_m is small players can be blocked into clusters. On the other hand, when R_g is low and R_m high the clusters are constantly destroyed and reformed in different places. For both R_g and R_m low small clusters are formed and the motion is not definite. Future work should include the study of the effect of strategy update and mobility noise in the dynamics, as well as the use of different strategy update and, possibly, migration rules.

Acknowledgments

The authors wish to thank the Swiss National Foundation for their financial support under contract n. 200021-14661611.

References

- Aktipis, C. A. (2004). Know when to walk away: contingent movement and the evolution of cooperation. *J. Theor. Biol.*, 231:249–2160.
- Axelrod, R. (1984). *The Evolution of Cooperation*. Basic Books, Inc., New-York.
- Buesser, P. and Tomassini, M. (2012). Evolution of cooperation on spatially embedded networks. *Phys. Rev. E*, 86:066107.
- Chen, Z., Gao, J., Kai, Y., and Xu, X. (2011). Evolution of cooperation among mobile agents. *Physica A*, 390:1615–1622.
- Enquist, M. and Leimar, O. (1993). The evolution of cooperation in mobile organisms. *Anim. Behav.*, 45:747–757.
- Helbing, D. and Yu, W. (2008). Migration as a mechanism to promote cooperation. *Advances in Complex Systems*, 11:641–652.
- Helbing, D. and Yu, W. (2009). The outbreak of cooperation among success-driven individuals under noisy conditions. *Proc. Natl. Acad. Sci. USA*, 106:3680–3685.
- Hofbauer, J. and Sigmund, K. (1998). *Evolutionary Games and Population Dynamics*. Cambridge, N. Y.
- Jiang, L.-L., Wang, W.-X., Lai, Y.-C., and Wang, B.-H. (2010). Role of adaptive migration in promoting cooperation in spatial games. *Physical Review E*, 81:036108.
- Nowak, M. A. and May, R. M. (1992). Evolutionary games and spatial chaos. *Nature*, 359:826–829.
- Nowak, M. A. and Sigmund, K. (2000). Games on grids. In Dieckmann, U., Law, R., and Metz, J. A. J., editors, *The Geometry of Ecological Interactions: Simplifying Spatial Complexity*, pages 135–150. Cambridge University Press, Cambridge, UK.
- Roca, C. P., Cuesta, J. A., and Sánchez, A. (2009). Evolutionary game theory: temporal and spatial effects beyond replicator dynamics. *Physics of Life Reviews*, 6:208–249.
- Santos, F. C., Pacheco, J. M., and Lenaerts, T. (2006). Evolutionary dynamics of social dilemmas in structured heterogeneous populations. *Proc. Natl. Acad. Sci. USA*, 103:3490–3494.
- Sicardi, E. A., Fort, H., Vainstein, M. H., and Arenzon, J. J. (2009). Random mobility and spatial structure often enhance cooperation. *J. Theor. Biol.*, 256:240–246.
- Vainstein, M. H., Silva, A. T. C., and Arenzon, J. J. (2007). Does mobility decrease cooperation? *J. Theor. Biol.*, 244:722–728.
- Vega-Redondo, F. (2003). *Economics and the Theory of Games*. Cambridge University Press, Cambridge, UK.
- Weibull, J. W. (1995). *Evolutionary Game Theory*. MIT Press, Boston, MA.