

Stackelberg-based Coverage Approach in Nonconvex Environments

Bijan Ranjbar-Sahraei, Kateřina Staňková, Karl Tuyls, Gerhard Weiss

Department of Knowledge Engineering

Maastricht University

Email: {b.ranjbarsahraei,k.stankova,k.tuyls,gerhard.weiss}@maastrichtuniversity.nl

Abstract

This paper introduces StaCo: Stackelberg-based Coverage approach for nonconvex environments. This approach structurally differs from existing methods to cover a nonconvex environment, as it is based on a game-theoretic concept of Stackelberg games. Our key assumption is that one robot can predict (short-term) behavior of other robots. No direct communication takes place among the robots, the approach is decentralized. However, the leading robot can direct the system into the optimal setting much more efficiently just by changing its own position. This paper extends our previous work in which we have introduced the StaCo approach for coverage of a convex environment, with a simpler type of robots. We provide theoretical foundations of the approach. We demonstrate its benefits by means of case studies (using the Sim.Lam software). We show situations in which the StaCo approach outperforms the standard approach, which is based on combination of the Lloyd algorithm and path planning.

Keywords: Multi-robot coverage of nonconvex environments, Stackelberg games

Introduction & Literature Overview

Multi-robot control in an unknown environment is an emerging topic of various research fields (e.g., flocking control (Olfati-Saber, 2006), aggregation (Martinoli et al., 1999), multi-robot coverage (Cortes et al., 2004), formation (Ren and Sorensen, 2008)). This paper focuses on multi-robot coverage.

Most of the proposed solution methods for multi-robot coverage are not applicable in practice, as they encounter difficulties such as failing to find the globally optimal solutions and the inability to account for nonconvex environments (Cortes et al., 2004; Martinoli et al., 1999). As a consequence, despite the wide range of existing works in the domain of multi-robot coverage (Breitenmoser et al., 2010; Butler and Rus, 2004; Cortes et al., 2004; Pimenta et al., 2009; Ranjbar-Sahraei et al., 2012; Schwager et al., 2009), there are still only very few in-field deployments.

Some works dealing with control of the system of multiple robots (not necessarily with multi-robot coverage) have tried to tackle the critical issues of nonconvexity and failing to find the global optimum. Ganguli et al. (2007) solve the distributed Art Gallery Problem in a nonconvex environment. In (Ganguli et al., 2009) the problem of the coordination of a group of robots to achieve rendezvous in a non-

convex environment is treated. An optimal control method to drive a team of multiple robots to target sets under collision avoidance and with proximity constraints in a known environment with obstacles is introduced in (Ayanian and Cumar, 2008). An elegant way of tackling the problem of nonconvex environments is introduced in (Caicedo and Žefran, 2008a,b). The nonconvex region is first transformed by a diffeomorphism to a convex region. Subsequently, the standard Voronoi coverage approach is applied on this region. As the authors themselves state, there is one major drawback of this method: The first phase of this method (transforming the region into a convex region) is computationally very expensive. Moreover, in some cases the solution of the transformed problems does not correspond to the solution of the original problem. Pimenta et al. (2008) apply the geodesic distance measure to Voronoi coverage. While this method is very efficient for some types of environments, it is not guaranteed that the optimal solution will be found for all types of nonconvex regions even if this solution is reachable.

One of the most practically applicable approaches for coverage of a nonconvex environment is introduced in (Breitenmoser et al., 2010). This algorithm combines the standard Lloyd algorithm with a local path planning. However, while this algorithm converges to the locally optimal configuration, it might be extremely slow and does not resolve the issues regarding failure to find the globally optimal configuration.

While the performance of the above mentioned algorithms might be improved via more effective algorithmic implementation, fundamental improvements of the settling time and convergence could be made if the structure of the robotic swarm played a role. Motivated by this idea, this paper introduces a game-theoretic approach which can deal with nonconvexity and local optimality issues more efficiently than the existing algorithms. The Stackelberg Coverage (StaCo) approach is based on the game-theoretic concept of Stackelberg games (Staňková, 2009; Staňková et al., 2013). It assumes that one robot is more advanced than the others. This more advanced robot, called a leader, perceives the environment globally. By its own movement, the leader changes the boundaries of the Voronoi regions of the other robots. Subsequently, and without any direct communica-

tion with the other robots, the leader steers the other robots into a more optimal configuration. The main advantage of the StaCo approach is keeping the benefits of decentralized methods while performing almost as well as the centralized methods with respect to the system optimality; it preserves the simplicity of the major population of the robotic swarm, while one robot can predict behavior of the others and act so that the desired behavior is achieved faster and with a higher precision.

This paper extends the results of (Staňková et al., 2013), where we introduced the StaCo approach for multi-robot coverage of convex environments, toward multi-robot coverage in nonconvex environments. Moreover, more realistic robots are considered in the case studies.

Game theory has been successfully applied in various fields; its known applications in the robotic field relate to pursuit-evasion and search problems (Meng, 2008; Raboin et al., 2010). However, application of the Stackelberg games in the multi-robot coverage of nonconvex environments is new.

In the next sections, we will briefly summarize our previous work and introduce the problem of the Voronoi coverage of a nonconvex environment and its properties. Subsequently, we will explain the StaCo approach in nonconvex coverage and analyze its properties. We will also present case studies in which we demonstrate the advantages of the StaCo approach. We will conclude by discussing the achieved results, limitations of the proposed approach, and the future research directions.

StaCo in Convex Environments

In (Staňková et al., 2013) we have introduced the StaCo approach for convex environments, as a specific case of a Stackelberg game with one leader (more advanced robot) and multiple followers (very simple robots). We have shown theoretically and by means of case studies that the proposed approach can never perform worse than the standard coverage algorithms, such as the Lloyd algorithm (Cortes et al., 2004), while most of the time the StaCo approach significantly outperforms the standard approaches (by the means of settling time or by finding the globally optimal configuration when standard approaches fail). Figure 1 from (Staňková et al., 2013), illustrates the performance of the StaCo approach in comparison to the classical coverage proposed by Cortes et al. (2004). All experiments are carried out in convex environments and with robotic swarms of different sizes.

Voronoi Coverage of a Nonconvex Environment

In this section we will informally discuss the problem of nonconvex environment coverage, including a discussion on the existence and the uniqueness of the optimal Voronoi configuration, and uniqueness of this optimal solution.

Problem Formulation

The goal is to deploy a group of networked robots in a nonconvex environment, i.e., an environment including

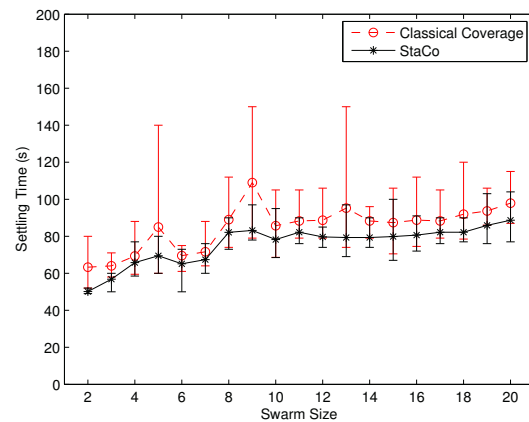


Figure 1: Comparison of the coverage settling time between the proposed StaCo approach and the classical coverage approach for robotic swarms of different sizes in convex environments (Staňková et al., 2013).

free standing obstacles, holes, and/or areas with nonconvex boundaries.

Problem Properties

Existence of the optimal Voronoi configuration: Unlike in a convex environment, the optimal solution does not necessarily exist in a nonconvex environment. This is caused by the fact that the centroids of Voronoi regions are computed in the convex environment, not taking any obstacles into account. However, the centroid of the region might lie on an obstacle or be part of an unreachable region, as shown in Figure 2a. Considering only the situations in which the centroids of the optimally chosen Voronoi regions are reachable, the globally optimal solution exists (but may be impossible to find with standard algorithms).

Uniqueness of the optimal Voronoi configuration: The solution configuration does not need to be unique, as there might be multiple solution configurations that are permutations of each other. See Figure 2b for an example of a circular region with a circular obstacle in the middle. Independent of how many multiple robots would be placed in this region, there exist infinitely many optimal Voronoi tessellations in this region.

StaCo Voronoi Coverage of a Nonconvex Environment

Theoretical Foundation & Properties

In this section we formulate multi-robot coverage problem in a nonconvex environment as a dynamic Stackelberg game with one leader and multiple followers, with additional assumptions on the robot's obstacle avoidance behavior. The approach proposed in this section will be referred to as StaCo: Stackelberg-based Coverage Approach. For more details on Stackelberg-based Coverage of convex environments, see (Staňková et al., 2013).

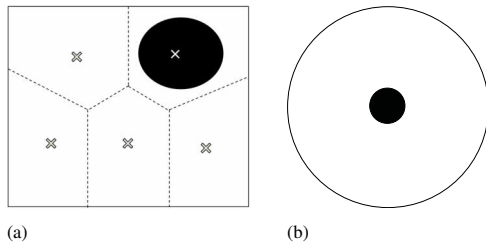


Figure 2: Properties of the Voronoi coverage of a nonconvex environment: (a) Example of the region an obstacle in which the optimal configuration is unreachable. The crosses denote the optimal positions of the robots. (b) Example of a region with (infinitely) many optimal Voronoi tessellations.

Let $\Omega \subset \mathbb{R}^2$ be a convex region. Let n convex obstacles o_1, \dots, o_n be placed in Ω . Let us consider M robots (players) placed in the region at time $t = 0$ in $\Omega \setminus \{\bigcup_{j=1}^n o_j\}$.

One of the players, denoted for the sake of simplicity as player 1, is the *leader*, all other players, denoted by $2, \dots, M$, are the *followers*. The roles of the players are assumed to be fixed during the entire duration of the game.

Let $\mathbf{x}(t) \stackrel{\text{def}}{=} \{x_1(t), x_2(t), \dots, x_M(t)\}$ be the configuration of the robots at time t , with $t \in [0, T]$, $\mathbf{x}(0) = \{x_1(0), x_2(0), \dots, x_M(0)\}$ being the initial configuration of the robots and $\mathbf{x}(T) = \{x_1(T), x_2(T), \dots, x_M(T)\}$ being their final configuration at final time T , with $x_i(t) \neq x_j(t)$ if $i \neq j$. Note that $x_i(t) \in \mathbb{R}^2$ for each $i \in \{1, \dots, M\}$. Let $V_i(t)$ indicate the Voronoi region (cell) in which i -th robot is located at time t . For each $\mathbf{x}(t)$ the Voronoi regions are defined by the *Voronoi partition* of Ω at time t , $\mathcal{V}(t) = \{V_1(t), \dots, V_M(t)\}$, generated by the points $\mathbf{x}(t) = (x_1(t), \dots, x_M(t)) : V_i(t) = \{\omega \in \Omega : \|\omega - x_i(t)\| \leq \|\omega - x_j(t)\|, \forall j \neq i\}$. System dynamics are given by the following system of ordinary differential equations:

$$\dot{x}_i(t) = u_i(t), \quad i = 1, \dots, M \quad (1)$$

where $u_i(t) \in \mathbb{R}^2$ is the control (decision) of the i -th robot at time t . The cost functions for the leader (robot 1) at time t is given by

$$C_1(t) = \sum_{i \in \{1, \dots, M\}} \int_{V_i(t)} \|\omega - x_i(t)\|^2 d\omega. \quad (2)$$

Let us assume from now on that T is defined as the so-called stopping time, i.e., the minimal time such that for each $\tau > T$ the cost $C_1(\tau)$ does not change: $T = \min\{\nu : C_1(\tau) = C_1(\nu) \forall \tau > \nu\}$. Then the leader minimizes $C_1(T)$. Alternatively, the leader might minimize T . The cost function for the follower $j \in \{2, \dots, M\}$ at time t is

$$C_j(t) = \int_{V_j(t)} \|\omega - x_j(t)\|^2 d\omega. \quad (3)$$

The problem of the leader (robot 1) can be then defined as

$$(P_{\text{StaCo}}) \begin{cases} \text{Find } u_1^{(S)}(\cdot) = \arg \min_{u_1(\cdot)} C_1(T), \text{ w.r.t.} \\ u_j(\cdot) = \arg \min_{u_j(\cdot)} \int_{V_j(t)} \|\omega - x_j(t)\|^2 d\omega. \\ \dot{x}_i(t) = u_i(t), \end{cases}$$

with $j = 2, \dots, N, i = 1, \dots, N$. Note that in a nonconvex case, $u_1^{(S)}$ involves both obstacle avoidance and reaching the goal behavior. Therefore, the underlying assumption here is that obstacle avoidance is one of the possible controls in (1). Moreover, we want to see how quickly the optimal Voronoi tessellation is found, i.e., the secondary goal is to minimize T .

Proposition 1. *Let at time t each player i know only state $x_i(t)$ and corresponding $V_i(t)$ and let Hessian of (2) be positive definite at each t . Then the so-called continuous-time Lloyd descent (Cortes et al., 2004)*

$$u_i^*(t) = \kappa \left(\frac{\int_{V_i(t)} \mathbf{x} dx_i}{\int_{V_i(t)} dx_i} - x_i(t) \right), \quad (4)$$

$\kappa > 0$, extended by the standard path planning algorithm for obstacle avoidance (Breitenmoser et al., 2010), asymptotically converges to minimal $C_1(T)$ and to minimal $C_j(T)$ for $j = 2, \dots, M$, provided that the final configuration in which the minimal $C_1(T)$ is reachable (i.e., no optimal x_i lies on an obstacle or in an unreachable region).

Proof. As shown in (Cortes et al., 2004), $u_i^*(t)$ defined by (4) with respect to $\dot{z}_i(t) = u_i(t)$ converges asymptotically to the set of critical points of (2). The critical points of (2) coincide with critical points of (3). If corresponding V_i is finite, this solution is global due to positive definiteness of (2), as follows from (Du et al., 1999). Assuming that the obstacle avoidance is one of the possible moves in (1) for each robot and that the optimal configuration is reachable from the initial configuration, this concludes the proof. \square

Validation of the positive definiteness of (2) is an open problem (Cortes et al., 2004) and even if the convergence to the global optimum is guaranteed, in general no guarantees on the speed of this convergence exist. This leads us to the question whether there exist algorithms that perform better than the standard Lloyd algorithm (combined with the obstacle avoidance (Breitenmoser et al., 2010) as the covered environment is nonconvex) if we allow the leader (robot 1) to have more information about the state and decisions of the followers.

Note that while a certain position might be unreachable using the classical Lloyd algorithm (a robot might, for example, get stuck on an obstacle, while the Lloyd algorithm would lead the robot to continue through the obstacle), it might be reachable using combination of the Lloyd algorithm and path planning (Breitenmoser et al., 2010). In the reminder of the article, we will refer to the combination of the the Lloyd algorithm and a path planner for obstacle avoidance as the *standard approach*, assuming tacitly that

the StaCo approach uses the same obstacle avoidance mechanisms as the standard approach.

The solution of (P_{StaCo}) strongly depends on the so-called information pattern, i.e., the amount of information that each player knows and recalls over her own state, the state of the others, and action made by herself and the others during the game (Başar and Olsder, 1999; Staňková and De Schutter, 2011; Staňková et al., 2013). If at each time $t \in [0, T]$ robot P_1 knows only $\mathbf{x}(t)$, the standard approach and the Stackelberg approach might coincide (unless more locally optimal solutions exist). However, if P_1 has more information available, the StaCo approach will perform better than the standard approach (Staňková et al., 2013). The following proposition extends Proposition IV.1. in (Staňková et al., 2013).

Proposition 2. *Let player 1 know $x_j(\tau)$ and $u_j(\tau)$ (for all $j \neq 1$) for $\tau \in [t, t + \Delta]$, with $\Delta > 0$, where $u_j(t)$ is defined by (4). Let $u_1^{(S)}(t)$ denote the optimal control of player 1, possibly dependent on $u_j(\tau)$, $\tau \in [t, t + \Delta]$. Let $T^{(S)}$, and $C_1^{(S)}(T^{(S)})$ denote the corresponding stopping time and the final payoff for player 1 in such a situation, respectively. Then $C_1^{(S)}(T^{(S)}) \leq C_1^{(L)}(T^{(L)})$, where $C_1^{(L)}$ and $T^{(L)}$ denote the cost of the player 1 if the classical approach, combining the Lloyd algorithm and a path planning, is adopted and the corresponding stopping time, respectively. This inequality holds if the optimal final configuration $x(T)$ is reachable from the initial configuration $x(0)$. Moreover, if $C_1^{(S)}(T^{(S)}) = C_1(T^{(L)})$, then $T^{(S)} \leq T^{(L)}$.*

Proof. The leader's decision is not bound by any restrictions. If all past configurations of the StaCo approach and the Lloyd approach coincide, setting the leader's decision to (4) leads to $T^{(S)} = T^{(L)}$, $C_1^{(S)}(T^{(S)}) = C_1(T^{(L)})$. Note that the Hessian of (2) might not be positive definite with the leader's decision defined by (4). Thus, $u_1^{(S)}(t)$ either coincides with (4) when the standard approach is adopted, or, if this choice would lead to only sub-optimal solution, $u_1^{(S)}(t)$ differs from (4) and leads to a better outcome. This outcome readily follows from Proposition IV.3. in (Staňková et al., 2013). \square

Giving more information to the leader almost always leads to a better outcome for the leader also in a very general setting (Başar and Olsder, 1999; Staňková, 2009), while the StaCo approach never leads to an outcome worse than that reached by standard methods (Cortes et al., 2004; Staňková et al., 2013). This follows from the fact that the classical Lloyd algorithm in which there is no hierarchy among the robots is a special case of the StaCo approach in which the leader does not predict possible position of the other robots and optimizes only locally. Should this behavior be optimal, it would also be adapted by the leader in the StaCo approach.

Implementation

Following the theoretical description provided in the previous section, in this section we will explain implementation

of the StaCo approach for coverage of nonconvex environments.

In StaCo, the leading robot (we assume that this robot is only one, while keeping in mind that the StaCo approach allows for multiple leaders) has a higher computational capability and more information than the following robots. Subsequently, the overall performance of the system is improved, and the StaCo approach can reach the optimal configuration faster than classical coverage approaches. Moreover, the StaCo approach can also reach the global configuration even if the standard approach fails.

The proposed StaCo approach for nonconvex environments combines three different components. The first one is the Lloyd algorithm, which is already used in the classical Voronoi-based coverage approach (e.g., by Cortes et al. (2004)), and has been mentioned also in the previous section. The second component is the Stackelberg game (described in the previous work of the authors (Staňková et al., 2013)). The third component is a local path planner, including object avoidance and wall following behaviors. This component helps the robot to pass nonconvexities (e.g., obstacles) and move efficiently toward its goal.

Decision making of leading and following robots: The leader's prediction of the possible future behavior of other robots and enforcing their optimal behavior via the leader's own movements (without a direct communication) are the main ideas behind StaCo.

The followers follow the simple rules of Lloyd algorithms, as shown in Figure 3a. Each follower continuously computes its Voronoi region center, sets this center as its goal, and tries to reach it, where the goal is a particular point in the 2D-space. Computing the Voronoi center can be done both via having access to global coordination of other robots, or via local communication as proposed by Cortes et al. (2004).

The decision making for the leader is more complex. The leader computes its own movement trajectory efficiently directing the entire group to the best possible configuration. Theoretically, this can be achieved by finding the explicit solution of P_{StaCo} , introduced in previous section. However, computation of such a solution analytically is very complicated and therefore, we compute the approximate solution of P_{StaCo} in a numerical way. In this numerical computation, the leader predicts possible behavior of the other robots as a response to its own behavior only for a fixed time interval and fixed number of directions for the leader's next move. The direction which implies the minimal cost function value is chosen as the immediate leader's goal. The immediate goal will be updated by the same procedure after the a priori fixed time. The sequence of such short-term goals defines the leader's movement trajectory. See Figure 3b for the scheme of the leader's behavior.

Remark 3. Note that the leader's prediction quality is directly influenced by the type and amount of information that is available to the leader. In our experiments, it is assumed that the leader knows the position of other robots and knows their dynamics. Additionally, the leader knows the map of

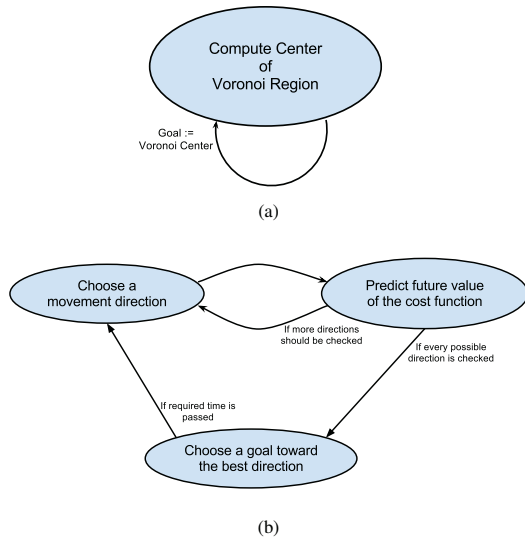


Figure 3: Scheme of robot behaviors/decisions: (a) simple behavior of a follower robot, in which the goal is a specific location in 2D space that the robot tries to reach using a local path planner. (b) the decision making of a leader robot in which the leader figures out which movement direction concludes to the best overall performance of the group.

the environment (more precisely, the position of obstacles). As long as only one robot or a very few number of robots has these abilities among a large number of simple robots, we can consider these assumptions as being practical and feasible, especially due to high capabilities of most modern robots.

Remark 4. Solving the P_{StaCo} problem in a numerical way can be improved in different aspects. First of all, the more movement directions the leader considers, the more accurate the movement, and the higher efficiency. Any metaheuristic which helps the robot in finding the best movement direction can be incorporated into the proposed approach. For example, with use of the A* search algorithm one can penalize choosing a trajectory passing very close to the obstacles, as opposed to an obstacle-free trajectory. Many other search heuristics (e.g., GA and Simulated Annealing) can be used to find the best movement directions in the fast and accurate manner (Resende and de Sousa, 2004). However, study of these techniques is beyond the main scope of this paper, which focuses on overall applicability and efficiency of StaCo.

Local path planner design: Local path planner has an important role in adapting the StaCo approach for convex environment, proposed in previous work of the authors (Štaňková et al., 2013), to coverage of nonconvex environments. Different local path planners are available for autonomous robots (Buniyamin et al., 2011). In (Breitenmoser et al., 2010) the *TangentBug* planner was used to tackle environment nonconvexities. In this paper, we use the hybrid controller proposed by Egerstedt (2000), in which robots follow three basic behaviors of *go-to-goal*, *avoid-obstacle*,

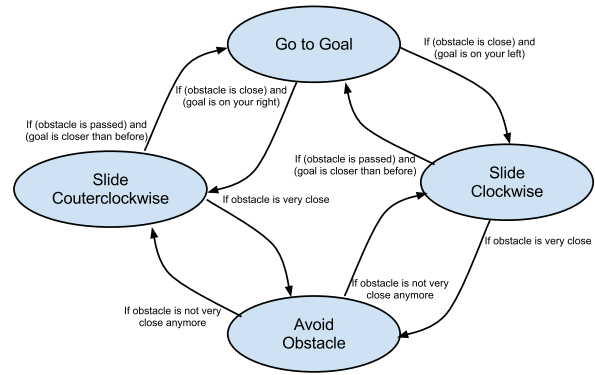


Figure 4: Hybrid automata used as local path planner designed for local path planning in nonconvex environments.

and *sliding along walls*. The hybrid automaton for this behavior-based path planner is shown in Figure 4, where the transitions and resetting values are explained qualitatively. We have adopted this path planner, as it is widely used by other robotic researchers, due to its simplicity of implementation, its robustness to environment changes, and its efficiency in finding the best available trajectory avoiding the obstacles of different shapes. Interested readers are referred to (Egerstedt, 2000) for more details on the design of this local path planner.

While the robot is moving toward its goal (i.e., Voronoi region center), if an obstacle appears in its way, the robot slides on the surface of the obstacle, until it gets to a position from which the goal is closer than it was before detecting the obstacle (i.e., the obstacle is already passed), then it switches back to the standard goal following. If at some point the robot gets very close to the obstacle, a pure repulsive behavior emerges which avoids collision with the obstacle.

Remark 5. The hybrid controller used for local path planning is always able to pass obstacles and move toward the reachable goals (Egerstedt, 2000). However, when the goal is unreachable (i.e., inside of an obstacle or bounded by obstacles), the path planner keeps moving the robot along the borders of the obstacle. In this paper we let the robot move around the obstacle as far as it is required, which will make an average position closer to the goal. However, a useful alternative is to stop the robot after one full cycle around the obstacle, as proposed by Breitenmoser et al. (2010).

Simulations

In this section, we will study the performance of the proposed StaCo approach in comparison to the classical Voronoi-based coverage approach in nonconvex environments introduced by Breitenmoser et al. (2010). Firstly, the simulation environment and the mobile robot platform will be introduced. Secondly, the efficiency of the StaCo approach compared to the classical approach will be illustrated in two case studies.

Simulation Setting

For examining the performance of the proposed coverage approach, Sim.I.am, a MATLAB-based educational software developed by de la Croix and Egerstedt (2013), is used in different coverage scenarios. The mobile robot platform, which is implemented in Sim.I.am, is the Khepera III (K3). The K3 is equipped with 11 infrared (IR) range sensors, nine of which are located in a ring around the robot and two are located on the underside of the robot. The IR sensors are complemented by a set of five ultrasonic sensors.

In the previous work of the authors (Staňková et al., 2013) the simulations of the proposed approach were carried out with a group of mass-less robots (i.e., neither the dynamic nor the kinematic model of the real-world robots were considered). In contrast, in the new simulator the robots (the nonholonomic Khepera robots) are much more realistic. Therefore, compared to (Staňková et al., 2013) and vast majority of papers on the Voronoi coverage, simulations in this paper are much closer to the real-world scenarios.

In the simulation environment, we have access to the array of nine IR sensors that encompass the K3. IR range sensors are effective in the range 0.02 m to 0.2 m only. Since the K3 has a differential wheel drive, it has to be controlled by specifying the angular velocities of the right and left wheels (v_l, v_r). Therefore, the conversion between a unicycle input, the forward and angular speeds, to differentially driven inputs are implemented based on following equation for the i th robot:

$$\begin{aligned} \dot{x}_i^1 &= R(v_l + v_r) \cos(\theta) \\ \dot{x}_i^2 &= R(v_l + v_r) \sin(\theta) \\ \dot{\theta}_i &= R(v_r - v_l)L \end{aligned} \quad (5)$$

where x_i^1 and x_i^2 denote coordination of the robot in horizontal and vertical directions, R is the radius of the wheels, and L is the distance between the wheels, which are known a priori. Wheel encoders are used to provide required information to the odometry of robot. The relevant information needed for odometry is the radius of the wheel, the distance between the wheels, and the number of ticks per full turn of the wheel, which are all implemented internally in the simulator. Note that the equations (5) extend equation (4) in which the robot is considered to have no mass and to be holonomic.

The embedded controller described previously in the form of a hybrid automaton (Figure 4) is used to deal with the local path planning tasks. The transition for moving from “go to goal” behavior to the “sliding mode” happens when a robot is in a distance less than 15 cm , and it will move to the pure repulsive behavior (i.e., obstacle avoidance) when the robot is closer than 6 cm to the obstacle.

The prediction time in which the leading robot finds the approximate best movement direction (Figure 3) is a period of 3 seconds and the robot calculates the final value for moving to 8 different directions (i.e., right, up-right, up, up-left, ..., down right) for this period of time. Note that the way in which the leading robot computes its next step agrees with

the concept of the model predictive control known from the optimal control theory literature (Mayne et al., 2000).

Results

Efficient coverage behavior of StaCo in convex environments is reported in our previous work (Staňková et al., 2013): As shown in Figure 1, the StaCo approach outperforms the classical coverage approaches in most of the environmental settings, and in the worst case StaCo and classical techniques have equal performance.

We use two case studies to show the high performance of StaCo in coverage of nonconvex environments and simultaneously we compare the results with the approach proposed by Breitenmoser et al. (2010).

The two non-trivial case studies for examining the StaCo approach in nonconvex environments are illustrated in Figure 5. In both scenarios five robots are initiated at random positions. The obstacles make the environment nonconvex which consequently makes an efficient coverage difficult.

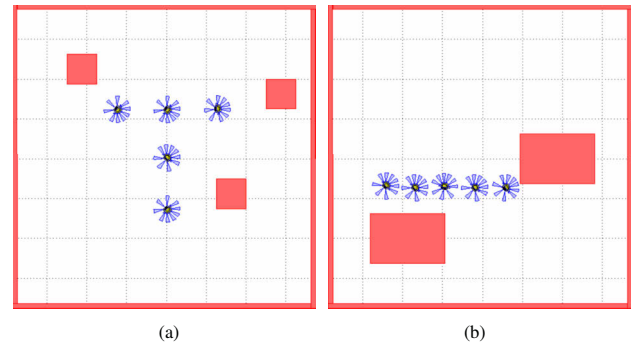


Figure 5: Initial settings for two experiments with five robots: (a) Scenario I. (b) Scenario II.

In both scenarios (Figures 5a and 5b), first the classical coverage approach for nonconvex environments is applied. In this approach, robots move toward their goals while a local path planner is used for obstacle avoidance and obstacle following purposes. Afterwards, we apply the StaCo approach to the same initial configurations, where one (randomly selected) robot acts as the leader. In Figure 5a, the robot in the center and in Figure 5b, the leftmost robot are the leaders. As explained earlier, the leader enforces its decisions on the other robots via its movements in the environment.

The coverage results of initial configurations shown in Figs. 5a and 5b are shown in Figures 6a-6c and Figures 6d-6e, respectively. Firstly, the robot trajectories for both classical coverage and StaCo approaches are shown (Figures 6a and 6d). Subsequently, the final configuration and the final Voronoi tessellation for each approach is illustrated (Figures 6b and 6e). Finally, the cost functions (2) for both approaches are plotted in Figures 6c-6f with respect to time.

As shown in Figures 6c and 6f, the StaCo approach finds the optimal configuration in a short time, while the classical approach is unable to find this optimal configuration

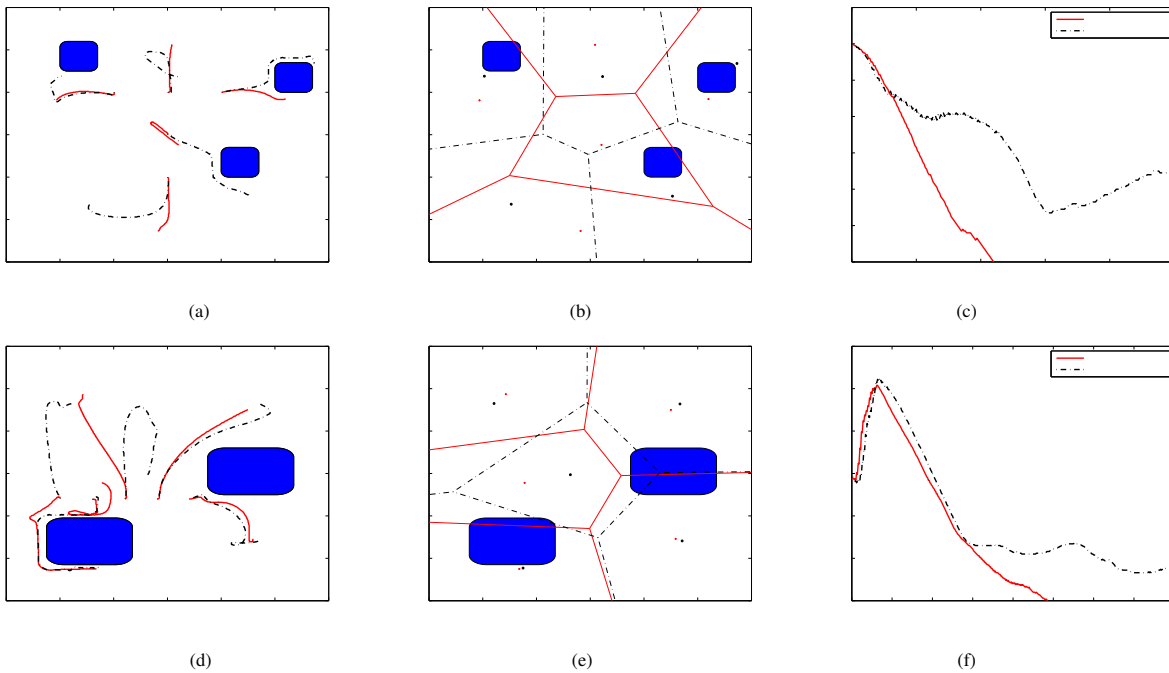


Figure 6: Experimental comparison between classical coverage approach (dash-dotted line) and StaCo approach (continuous line): (a) robot trajectories for Scenario I. (b) final robot configurations and final Voronoi tessellations for Scenario I. (c) cost function comparison for Scenario I. (d) robot trajectories for Scenario II. (e) final robot configurations and final Voronoi tessellations for for Scenario II. (f) cost function comparison for Scenario II.

even in the long term. From the trajectories of the individual robots (Figures 6a and 6d) it can be seen that while the classical approach moves the robots toward the optimal positions blindly (they can not predict whether obstacles will cause problems or not), in the StaCo approach, the leader which has access to more information and higher computation abilities, can enforce the entire group to move to the optimal configuration more efficiently.

Discussion, Conclusions & Future Research

In this paper we have shown the high potential of the StaCo approach in the coverage of a nonconvex environment. In the situations in which the leader can predict the long-term behavior of the other robots, the StaCo approach outperforms the standard approaches for coverage of nonconvex environments. Moreover, we have shown that the StaCo approach outperforms the standard approaches even if the prediction capabilities of the leading robot are very limited. Extending the leader’s prediction horizon will then lead to even better results. The main advantage of StaCo compared to any possible centralised coverage approach, is that StaCo does not rely on any direct communication between robots.

Leader’s predictions might become computationally expensive especially in an environment with many obstacles and/or if the robotic swarm is very large. More advanced optimization methods might then have to be applied to overcome this possible drawback. Our next research step is to address this issue.

Moreover, we plan to implement StaCo in a real-robot setting using a combination of TurtleBots as the leaders and e-pucks as the followers. Although we do not expect implementation problems due to the available advanced robots, we need to explore in detail the level of StaCo precision that can be achieved in in-field scenarios that have different environmental and technical conditions.

Last but not least, our future research will include expressing the optimal leader’s behavior in explicit form and extending the number of leaders (in such a case the hierarchy between the individual leaders might play a role).

References

Ayanian, N. and Cumar, V. (2008). Decentralized feedback controllers for multi-agent teams in environments with obstacles. In *Proceedings of the 2008 IEEE International Conference on Robotics and Automation*, pages 19 – 23.

Başar, T. and Olsder, G. J. (1999). *Dynamic Noncooperative Game Theory*. SIAM, Philadelphia, Pennsylvania.

Breitenmoser, A., Schwager, M., Metzger, J., Siegwart, R., and Rus, D. (2010). Voronoi coverage of non-convex environments with a group of networked robots. In *IEEE International Conference on Robotics and Automation (ICRA)*, pages 4982 – 4989.

Buniyamin, N., Wan Ngah, W., Sariff, N., and Mohamad, Z. (2011). A simple local path planning algorithm for

- autonomous mobile robots. *International Journal of Systems Applications, Engineering & Development*, 5(2):151 – 159.
- Butler, Z. and Rus, D. (2004). Controlling mobile sensors for monitoring events with coverage constraints. In *Proceedings of IEEE International Conference on Robotics and Automation (ICRA)*, pages 1568 – 1573.
- Caicedo, M. and Žefran, M. (2008a). A coverage algorithm for the class of nonconvex regions. In *Proceedings of the 2008 IEEE International Conference on Decision and Control*, pages 4244 – 4249.
- Caicedo, M. and Žefran, M. (2008b). Performing coverage of non-convex domains. In *Proceedings of the 2008 IEEE International Conference on Systems & Control*, pages 1019 – 1024.
- Cortes, J., Martinez, S., Karatas, T., and Bullo, F. (2004). Coverage control for mobile sensing networks. *IEEE Transactions on Robotics and Automation*, 20(2):243 – 255.
- de la Croix, J.-P. and Egerstedt, M. (2013). Sim.Iam. <http://gritslab.gatech.edu/projects/robot-simulator/>. [Online; accessed April-2013].
- Du, Q., Faber, V., and Gunzburger, M. (1999). Centroidal Voronoi tessellations: Applications and Algorithms. *SIAM Review*, 41(4):637 – 676.
- Egerstedt, M. (2000). Behavior based robotics using hybrid automata. In *Hybrid Systems: computation and control*, pages 103 – 116. Springer Berlin Heidelberg.
- Ganguli, A., Cortes, J., and Bullo, F. (2007). Distributed coverage of nonconvex environments. In Saligrama, V., editor, *Networked Sensing Information and Control*, pages 289 – 305. Springer US.
- Ganguli, A., Cortes, J., and Bullo, F. (2009). Multirobot rendezvous with visibility sensors in nonconvex environments. *Robotics, IEEE Transactions on*, 25(2):340 – 352.
- Martinoli, A., Ijspeert, A., and Gambardella, L. (1999). A probabilistic model for understanding and comparing collective aggregation mechanisms. In *Proceedings of the Fifth European Conference on Artificial Life (ECAL99)*, pages 575 – 584.
- Mayne, D., Rawlings, J., Rao, C., and Scokaert, P. (2000). Constrained model predictive control: Stability and optimality. *Automatica*, 36(6):789 – 814.
- Meng, Y. (2008). A game-theory based multi-robot search approach for multiple targets. *IEEE Robotics & Automation Magazine*, 5(4):341 – 350.
- Olfati-Saber, R. (2006). Flocking for multi-agent dynamic systems: Algorithms and Theory. *IEEE Transactions on Automatic Control*, 51(3):401 – 420.
- Pimenta, L. C. A., Kumar, V., Mesquita, R., and Pereira, G. A. S. (2008). Sensing and coverage for a network of heterogeneous robots. In *Decision and Control (CDC), 47th IEEE Conference on*, pages 3947 – 3952.
- Pimenta, L. C. A., Schwager, M., Lindsey, Q., Kumar, V., Rus, D., Mesquita, R. C., and Pereira, G. A. S. (2009). Simultaneous coverage and tracking (SCAT) of moving targets with robot networks. In *Proceedings of the Eighth International Workshop on the Algorithmic Foundations of Robotics (WAFR 08)*, pages 85 – 99.
- Raboin, E., Nau, D., Kuter, U., Gupta, S. K., and Svec, P. (2010). Strategy generation in multi-agent imperfect-information pursuit games. In *Proceedings of the 9th International Conference on Autonomous Agents and Multiagent Systems*, pages 947 – 954.
- Ranjbar-Sahraei, B., Weiss, G., and Nakisae, A. (2012). A multi-robot coverage approach based on stigmergic communication. In *Multiagent System Technologies*, volume 7598 of *Lecture Notes in Computer Science*, pages 126 – 138. Springer.
- Ren, W. and Sorensen, N. (2008). Distributed coordination architecture for multi-robot formation control. *Robotics and Autonomous Systems*, 56(4):324 – 333.
- Resende, M. G. C. and de Sousa, J. P., editors (2004). *Metaheuristics: Computer Decision-Making (Applied Optimization)*. Springer.
- Schwager, M., Rus, D., and Slotine, J. J. (2009). Decentralized, adaptive coverage control for networked robots. *International Journal of Robotics Research*, 28(3):357 – 375.
- Staňková, K. and De Schutter, B. (2011). Stackelberg equilibria for discrete-time dynamic games – Part I: Deterministic games. In *Proceedings of the 2011 IEEE International Conference on Networking, Sensing and Control*, pages 249 – 254, Delft, The Netherlands.
- Staňková, K. (2009). *On Stackelberg and Inverse Stackelberg Games & Their Applications in the Optimal Toll Design Problem, the Energy Market Liberalization Problem, and in the Theory of Incentives*. PhD thesis, Delft University of Technology, Delft, The Netherlands.
- Staňková, K., Ranjbar-Sahraei, B., Weiss, G., and Tuyls, K. (2013). StaCo: Stackelberg-based coverage approach in robotic swarms. In *Proceedings of ADAPTIVE 2013*, pages 71–76.