

# Lévy-like Distribution Shown by Intermittent Search Model with Misunderstanding Switch Pattern

Hisashi Murakami<sup>1</sup> and Yukio-Pegio Gunji<sup>1</sup>

<sup>1</sup>Kobe University  
hssh415@gmail.com

## Abstract

In an intermittent random search, in which slow motion to detect the target is discretely separated from the motion to migrate to another feeder, the high efficiency of the Lévy strategy is generally found, meaning that the time interval of phase switching is chosen from the Lévy distribution. Though the Lévy strategy is consistent with the searching behavior of real animals, some researchers claim that the Lévy-like distributions exhibited by animals are not necessarily produced by a Lévy process. Here, we propose an intermittent two-phase search model that does not include a Lévy process. In this model, the agent is basically a correlated random walker (CRW), but it memorizes its trajectory and counts the number of crossovers in a trajectory. If the number exceeds a threshold, the agent resets the memory of trajectories and makes ballistic movement in the direction uncorrelated to the past. We also show that this model can optimize the trade-off between macro search (exploration) and micro search (exploitation), which is shown by the CRW. Finally, we demonstrate that another intermittent search model that uses an ambiguous rule to switch the two phases can show a Lévy-like distribution of time intervals.

## Introduction

It is interesting to try to understand how living organisms navigate to targets in a natural environment, where resources are usually unpredictably distributed such that there is limited information about their locations (Viswanathan et al., 2011). The Lévy walk (LW) is considered the most important model of this type of random search, which is a special random walk in which each step length is chosen from a power-law distribution with a heavy tail (a so-called Lévy distribution) (Viswanathan et al., 2008, Reynolds and Rhodes, 2009). The LW shows a scaled step length  $l$  such that  $P(l) \sim l^{-\mu}$  with  $1 < \mu < 3$ , where  $\mu$  represents the power-law exponent. In the foraging simulations, if prey is abundant and, thus, predictable, it is known that classical random walks such as Brownian motion can yield higher encounter rates than LW. In contrast, when preys are sparsely and unpredictably located, LW is more efficient than classical random walks.

In this sense, LW has a reliable theoretical advantage, but is it consistent with empirical data? Indeed, it has been reported that, among diverse organisms, experimental evidence of LW can be found (Humphries et al., 2010, Cole, 1995, Viswanathan et al., 2008). It is most evident in the wandering

albatross. In the first of a series of pioneering works by Viswanathan et al., albatross behavior was tracked by using a humidity sensor attached to one leg of each bird (Viswanathan et al., 1996). Flight-time intervals were measured by wet periods, and dry points were considered to be landings on the water to catch fish. A reinvestigation using GPS, however, showed that most long flights in fact consisted of rest time, when the bird was in its nest, and concluded that there is no power-law distributed step length in wandering albatross (Edwards et al., 2007). Nevertheless, the latest study, which used the same method as above but examined the birds one by one, found that individuals foraging for sparse food exhibited certain Lévy movement patterns eventually (Humphries et al., 2012).

A trajectory of LW describes a search pattern composed of many small step clusters, interspersed by longer relocations known as “saltations” (O’Brien et al., 1990). This pattern can be intuitively described as an intermittent random search strategy in which slow motion is used to detect the target and a discretely separated motion is used to migrate to another feeding location (Bénichou et al., 2011). For example, if we lose a tiny object (e.g., a key) in a huge field, we can consider two simple ways to detect the key: a slow, careful search and a rough, fast one. In the former case, we can search accurately, but we would spend a very long time in the field. In the latter case, we may detect the key quickly, but in many cases, the lack of accuracy would result in just as long of a search time as the slow search. This illustrates a trade-off between the exploitation of old certainties and the exploration of new possibilities, which is frequently found in biology (March, 1991). To balance this dilemma, we would eventually choose a combined strategy, i.e., an intermittent strategy.

In studies of the intermittent strategy, it is difficult to determine the optimal way to switch between the different motions. The Lévy strategy also plays an important role. Bartumeus and his colleague compared a correlated random walk (CRW) (Kareiva and Shigesada, 1983), which is known as a natural way to model the emergence of angular correlations in animal trajectories coming from local scanning, with an intermittent model based on a CRW but incorporating uncorrelated reorientations with a time interval whose length is chosen from the Lévy distribution (Bartumeus and Levin, 2008). Then, they showed that this Lévy intermittent model is more efficient than the non-intermittent version.

Through these theoretical and empirical studies, the Lévy strategy has been established as a key to understanding animal search behavior. Consequently, it has been proposed that the Lévy strategy must be a strong target for natural selection. This is the so-called Lévy foraging hypothesis (Viswanathan et al., 2011). Still, some researchers claim that the Lévy-like distribution shown by animals is not necessarily produced by a Lévy process. Indeed, few models in which an agent walks deterministically and interacts with complex distributed targets can show an LW pattern (Santos et al., 2007). Moreover, Benhamou used combined exponential distributions to suggest that there is no guarantee that a Lévy-like distribution is based on a Lévy process (Benhamou, 2007).

Here, we will show a simple intermittent model that is not based on a Lévy distribution but does possess the principal features of an intermittent strategy, i.e., it shows two different phases. In this model, the number of crossovers in a trajectory is regarded as the extent to which the agent implements local search, and it also represents the threshold used to switch between the two phases. We demonstrate how this model can strike a balance in a trade-off between macro search (exploration) and micro search (exploitation), and we compare the model with a CRW. Finally, we describe another intermittent search model that uses an ambiguous switching rule. The ambiguity results from a stochastically generated long trail, and it generates a search in which the agent wastes too much time. Moreover, we demonstrate that the model can show a Lévy-like distribution of the time intervals.

## Results

### Basic models

First, we present a simple model that includes the main factor of intermittent search, in which an agent iterates to form local scanning behavior (here, we call this the exploitation phase). The local scanning is interspersed by longer relocations or saltations (exploration phase) in continuous space and discrete time. We refer to the entire model as EERW for short.

In the exploitation phase, the agent basically moves as a CRW. Here, angular correlations are introduced on the basis of a circular Gaussian distribution ( $-1.0 \leq g \leq 1.0$ ) centered at the value  $g = 0$  (maximum probability), although other distributions (e.g., a wrapped Cauchy distribution) might be as good (Bartumeus and Levin, 2008). The turning angle is represented by  $\theta = g\pi$ . At each step, the angle of the agent is determined by combining the turning angle with the previous angle. The standard deviation ( $SD$ ) of the Gaussian distribution controls the directional persistence or correlation length of the random walk (Bartumeus et al., 2005, Viswanathan et al., 2005).

In intermittent models, the agents should migrate to another feeder if the local scanning is finished. Hence, in EERW, the number of crossovers in a trajectory represents the extent to which the agent has searched the surrounding area, although switching between phases has usually been implemented by means of a stochastic process such as the Lévy distribution in previous models (Bartumeus and Levin, 2008). In other

words, each agent memorizes its trajectory and counts the number of crossovers in the trajectory. The threshold number of crossovers needed to switch between the two phases is represented by  $NC$ . Note that  $NC$  can be reasonably estimated as the extent of the local search, as we discuss later. If the number of crossovers exceeds the  $NC$  threshold, the agent makes a ballistic movement in a direction uncorrelated to its past, i.e., the turning angle is chosen from a uniform distribution  $\theta \in [-\pi, \pi]$ . Ballistic movement is continued until the time steps are proportional to the steps of the exploitation phase. The longer the agent stays in the exploitation phase, the larger the area searched by agent is, so the agent should spend as much time on local search as it does on relocation. The proportionality constant is represented by  $P$ . Because of its finite memory, the agent resets its memory of the trajectories, returns to the exploitation phase and starts walking again as a CRW at the new location.

The parameters in our model are listed below:

$SD$ : standard deviation of the Gaussian distribution with respect to directional persistence

$NC$ : threshold number of crossovers in a trajectory

$P$ : proportionality constant with respect to distance in the exploration phase

$l$ : step length

In this paper, we fixed the step length at  $l=0.5$ .

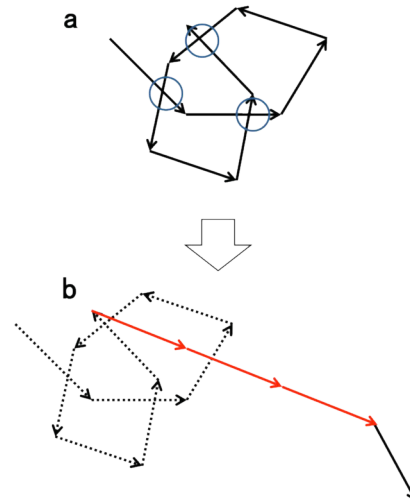


Figure 1 Schematic diagram of phase transition in the EERW model: (a) Steps of the Exploitation phase are represented by black arrows, and crossovers are surrounded by blue circles. (b) Steps of the Exploration phase are represented by red arrows, and steps of the previous and following Exploitation phases are represented by dashed and solid black arrows, respectively.

Fig. 1a shows the procedure from exploitation phase to exploration phase. In this case, the agent implements the exploitation phase as a CRW, and its trajectory has three crossovers, i.e.,  $NC = 3$ . When the agent switches to the exploration phase, it makes a ballistic movement comprising three steps, so  $P$  has a value of approximately 0.33 because there are nine steps spent in the exploitation phase.

Fig. 2 shows a series of snapshots of whole trajectories of the CRW and EERW models at  $T = 1000-100000$  with  $SD = 0.3$ ,  $l = 0.5$ ,  $NC = 10$  and  $P = 0.3$ . These simulations are implemented in continuous 2D space with no boundary conditions, but in Fig. 2, they are displayed as if they were in a space of  $200 \times 200$  with a wrapped boundary. It is easy to see that the search area of the EERW is broader than that of the CRW, even though they have the same search steps. In many cases, the agent with the search tasks has a perceptual range with a certain radius, in which the agent can detect a target (Viswanathan et al., 2011). Then, the search efficiency is estimated by the number of targets captured in this range. We may partially regard the search areas as representative of search efficiency. However, if the search area were the most important factor in a random search, it is strange that ballistic movement would be the most optimal strategy. In the next section, we show that EERW can balance a trade-off between macro search (exploration) and micro search (exploitation), which is shown by the CRW.

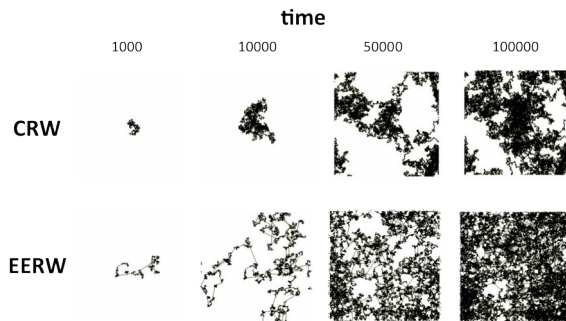


Figure 2 Snapshot of time development of the trajectories of the CRW (upper side) and EERW (lower side) in a continuous two-dimensional space of  $200 \times 200$  with wrapped boundary condition. Time proceeds from left to right. For both simulations,  $SD = 0.3$  and  $l = 0.5$ , and for EERW,  $NC = 10$  and  $P = 0.3$ . The trajectory of the CRW concentrates and overlaps at the center of the field, whereas that of the EERW is sparsely distributed but covers the entire field.

**A trade-off between macro search and micro search**

Biological systems, from individual organisms to groups of animals, are subject to a trade-off between exploitation and exploration at various levels. Especially in open environments, the decision of whether to stay in a known environment or explore a new environment is a difficult one (March, 1991, Gunji et al., 2011, Bénichou et al., 2011). In a random (intermittent) search, the agent would move to another field if the local search were finished. Therefore, the exploitation-exploration dilemma would correspond to the relationship between micro and macro search. In fact, as mentioned above, if the agent moves by means of a strategy leaning toward micro search, it would spend a long time searching a huge field. Moreover, even if the agent detects some targets, those targets cannot be the most abundant resource. On the other hand, if the agent moves by means of a strategy leaning toward macro search, it also takes a long time

to detect targets because of search inaccuracy. Hence, there is a trade-off between micro search and macro search.

In this sense, the classical random walk is the strategy most biased against micro search, whereas ballistic movement is the most biased against macro search. Now, we show that CRW displays a micro-macro search trade-off, because as the parameter  $SD$  approaches zero, the behavior of CRW gets closer to ballistic movement. Conversely, as  $SD$  becomes larger, the behavior approaches a classical random walk. Here, we estimate the extent to which an agent implements micro search by means of the number of total crossovers in a trajectory of 5,000 simulation time steps, and we estimate the extent to which the agent implements macro search by means of closure areas. The closure areas are measured by total neighborhood areas with radii  $r = 1.0$ , where each arrival point of the agent is centered. Overall, the trade-off between micro search, represented by the total number of crossovers, and macro search, represented by the closure areas, is easy to see (Fig. 3). Patterns of CRW are generated with no boundary condition by varying the parameter  $SD$  from 0.01 to 1.0 in increments of 0.01. By comparing the patterns of CRW with those of EERW, we see that EERW can balance exploitation with exploration, and the patterns of EERW are generated under the same conditions as those of CRW, except that  $NC = 15$  and  $P = 0.3$ .

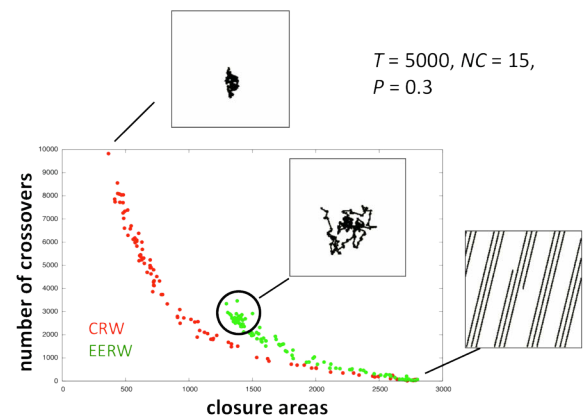


Figure 3 Performance of the CRW (red points) and EERW (green points) with respect to the closure areas and the number of crossovers. Snapshots of trajectories of the CRW with  $SD$  values of 0.0 and 1.0 are represented by top box and bottom box, respectively. Additionally, trajectories of the EERW with  $SD$  0.5 are represented by a box connected to a black circle, which indicates that the EERW can balance the trade-off.

Bartumeus and colleague showed that the Lévy intermittent model could be more efficient than the CRW (Bartumeus and Levin, 2008). In their study, the Lévy intermittent model was compared to the CRW model with two different values of a parameter that controls the directional persistence of the model. However, the CRW can display various behaviors, from persistent, ballistic movement to an uncorrelated classical random walk. We examined the CRW with most of the possible  $SD$  values, revealing that the EERW intermittent model without a Lévy process can balance the trade-off of the

CRW between exploitation and exploration, as calculated by two quantities: number of closure areas and number of crossovers.

**Another intermittent search model**

In the study of random search, some models have assumed that agents have memory or learning skills (Ferreira et al., 2012, MacNamara and Houston, 1987). Their memories and learning abilities are also assumed to be finite. However, there has been no attempt to assume ambiguity of memory or misunderstanding of learning. In this section, we first introduce ambiguity and/or misunderstanding in the form of a rule to switch between search phases in the EERW. Second, we show that such a modified EERW (MEERW) results in a Lévy-like distribution of time intervals for phase switching. Finally, we discuss the difference between LW and MEERW.

In the EERW model, the number of crossovers in a trajectory represents the extent to which the agent implements local search. Hence, there was a threshold number of crossovers,  $NS$ , at which the agent switched phases. In contrast, in the MEERW model, which in most ways is the same as the EERW model (Fig. 4a), the value of the threshold  $NS$  is dynamically varied by two types of misunderstanding of the rule.

One such misunderstanding occurs when a long trail is stochastically generated without enough crossovers. We regard it as a ballistic movement (Fig. 4b). If such the long trail is generated, it is assumed that the exploitation phase was already implemented, even though the number of crossovers did not exceed the threshold  $NS$ , and exploration phase is regarded as a trail entailing the reset of the memory of the trajectory. Then,  $NS$  is decremented by one because the agent misunderstands a shorter local search (and the generated trail) as a rule. We here define such a long trail as a series of tracks comprising  $N_t$  tracks, in which the inner product of each track and the next one is greater than  $IP$ .

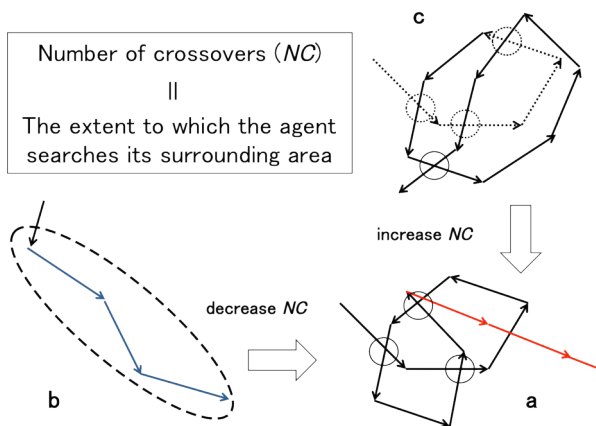


Figure 4 Schematic diagram of MEERW. (a) The MEERW model is basically same as the EERW, in which  $NC$  indicates how long an agent searches the local area. (b) A long trail stochastically generated decreases  $NC$ . (c) Excessive search time increases  $NC$ .

For the other misunderstanding, we introduce an additional memory restriction: an agent can memorize only  $N_m$  tracks as a trajectory, so it can make crossovers only with the memorized tracks (Fig. 4c). Moreover, if the agent spends  $N_s$  steps without switching phases, then  $NS$  is incremented by one because the agent misunderstands the longer local search as a rule.

In this paper, we fixed the parameters at  $N_t = 15$ ,  $IP = 0.85$ ,  $N_m = 10$  and  $N_s = 100$ .

Now, we demonstrate the Lévy-like distribution of time intervals for phase switching by comparing the EERW and MEERW models. For EERW, we measured the time steps spent in the exploitation phase as the time interval because the ratio of time steps spent in the exploitation phase to those spent in the exploration phase is constant. For MEERW, we basically measure time steps in the same way as for EERW, but if the stochastic long trail is generated, we also measure time steps from the start of the exploitation phase to the time that the trail is generated. Fig. 5 shows the frequency distribution of time intervals in a one million time-step simulation with  $SD = 0.2$  and  $P = 0.3$ . The exponent  $\mu$  is computed as the slope of a regression line for the range of values where power-law behavior (straight line in a log-log plot) is observable. The exponent  $\mu$  for EERW is 5.81 with  $R^2 = 0.957$ . For MEERW, it is 2.38 with  $R^2 = 0.943$ .

In the Lévy strategy with the Lévy intermittent model, the exponent  $\mu$  of the tail of the power-law distribution should be in the interval  $1 < \mu < 3$ . For  $1 \geq \mu$ , the distribution is not defined. For  $\mu \geq 3$ , provided the conditions of the Generalized Central Limit Theorem, the tail converges to a Gaussian distribution. In the latter case, the time interval will show an intrinsic characteristic scale. In this sense, the distribution of MEERW is considered to be Lévy-type but that of EERW is not.

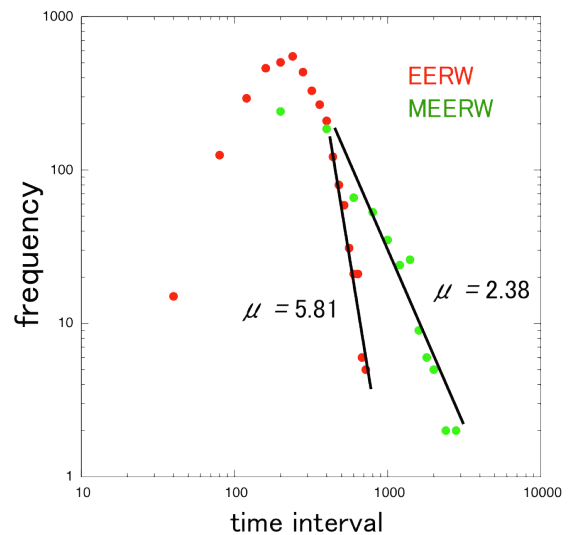


Figure 5 Power-law distributions of EERW and MEERW.

Indeed, the distribution of EERW cannot be Lévy-type. In Table 1, we estimated the exponent  $\mu$  for EERW, varying the

parameter  $NC$  from 1 to 20 and  $SD$  from 0.1 to 0.9. The results show that, for all combinations of  $NC$  and  $SD$ , the exponent  $\mu$  is greater than 3 and has a strong correlation ( $R^2 > 0.9$ ).

		$NC=1$	$NC=10$	$NC=20$
$SD=0.1$	$\mu$	5.472	5.066	4.602
	$R^2$	0.95724	0.92954	0.90619
$SD=0.5$	$\mu$	6.465	6.303	8.426
	$R^2$	0.9613	0.95194	0.93657
$SD=0.9$	$\mu$	7.771	9.071	7.691
	$R^2$	0.94909	0.95885	0.93114

Table 1 Estimation of the exponent  $\mu$  of EERW with the parameters  $NC$  and  $SD$ .

What is the difference between LW and MEERW? It is hard to distinguish the LW model from the MEERW model when we set the step length  $l$  to be very small relative to the parameter  $P$ . Fig. 6 shows a snapshot of a MEERW trajectory with  $l=0.005$  and  $P=3.0$ . It is easy to see that it has the features of an LW trajectory, such as varying step size with some small-step clusters interspersed with longer steps. The main difference between LW and MEERW lies in the behavior exhibited at the arrival point.



Figure 6 Snapshot of a MEERW trajectory with  $l = 0.005$  and  $P = 3.0$ .

### Discussion

The fact that local clusters are connected by saltations in animal searches and/or LW suggests that there are rules for the detection of targets (Bénichou et al., 2011). The intermittent search strategy assumes that agents move to another field if the local search is finished, which is consistent with the clustering phenomenon. This strategy implies that the agent has two different phases. However, the time interval that elapses before phase switching is given by some stochastic process, such as a Lévy process, rather than a rule.

In this paper, we started with the simple intermittent model EERW, which was not based on a Lévy process but instead was equipped with the principal features of the intermittent strategy (i.e., there were two different phases). In EERW, the

switch between phases is provided as a rule such that, if the number of crossovers exceeds a threshold  $NC$ , the agent resets the memory of trajectories and makes ballistic long trails in a direction uncorrelated with the past. We demonstrated that EERW could balance a trade-off between macro search (exploration) and micro search (exploitation), and we compared the EERW model with a CRW.

Finally, we constructed a MEERW by incorporating ambiguity or misunderstanding of the rule, in which a threshold  $NC$  is dynamically varied by the stochastically generated long trail and excessive search times. As a result, MEERW showed Lévy-like distribution. Moreover, depending on the parameter values  $l$  and  $P$ , MEERW could behave much like an LW model.

An LW model has already been constructed without a Lévy process, yet there must have been deterministic walks and interactions with a complex distribution of targets (Santos et al., 2007). Thus, our model is the first attempt to investigate the hypothesis that an LW can be generated in the absence of a Lévy process and without deterministic walks.

### References

Bartumeus, F. and Levin, S.A. (2008) Fractal reorientation clocks: Linking animal behavior to statistical patterns of search. *PNAS*. 105(49): 19072–19077.

Bartumeus F., da Luz M.G.E., Viswanathan G.M. and Catalan J. (2005) Animal search strategies: a quantitative random walk analysis. *Ecology*. 86(11): 3078-3087.

Benhamou, S. (2007) How many animals really do the Lévy walk? *Ecology*. 88(8): 1962–1969.

Bénichou, O., Loverdo, C., Moreau, M. and Voituriez, R. (2011) Intermittent search strategies. *Rev. Modern Phys.* 83: 81–129.

Cole, B.J. (1995) Fractal time in animal behavior: The movement activity of *Drosophila*. *Anim Behav.* 50:1317–1324.

Edwards, A.M., Phillips, R.A., Watkins, N.W., Freeman, M.P., Murphy, E.J., Afanasyev, V., Buldyrev, S., da Luz, M.G.E., Raposo, E.P., Stanley, H.E. and Viswanathan, G.M. (2007) Revisiting Lévy flight search patterns of wandering albatrosses, bumblebees and deer. *Nature* 449:1044–1048.

Ferreira, A.S., Raposo, E.P., Viswanathan, G.M. and da Luz M.G.E. (2012) The influence of the environment on Lévy random search efficiency: Fractality and memory effects, *Physica A*. 391:3234–3246.

Gunji, Y.P., Shirakawa, T., Niizato, T., Yamachiyo, M. and Tani, I. (2011) An adaptive and robust biological network based on the vacant-particle transportation model. *J Theor Biol.* 272:187-200.

Humphries, N.E., Queiroz, N., Dyer, J.R.M., Pade, N.G., Musyl, M.K., Schaefer, K.M., Fuller, D.W., Brunnschweiler, J.M., Doyle, T.K., Houghton, J.D.R., Hays, G.C., Jones, C.S., Noble, L.R., Wearmouth, V.J. Southall, E.J. and Sims, D.W. (2010) Environmental context explains Lévy and Brownian movement patterns of marine predators. *Nature*. 465:1066–1069.

Humphries, N.E., Weimerskirch, H., Queiroz, N., Southall, E.J. and Sims, D.W. (2012) Foraging success of biological Lévy flights recorded in situ. *PNAS*. 109(19):7169-7174.

- Kareiva, P.M., Shigesada, N. (1983) Analyzing insect movement as a correlated random walk. *Oecologia*. 56:234–238.
- March, J.G., (1991) Exploration and exploitation in organizational learning. *Organization Science*. 2:71–87.
- MacNamara, J.M., Houston, A.I. (1987) Memory and the efficient use of information, *J. Theoret. Biol.* 125:385–395.
- O'Brien, W.J., Browman, H.I. and Evans, B.I. (1990) Search strategies of foraging animals. *Am Sci* 78:152–160.
- Reynolds, A.M. and Rhodes, C.J. (2009) The Lévy flight paradigm: random search patterns and mechanisms. *Ecology*. 90(4):877-887.
- Santos, M.C., Boyer, D., Miramontes, O., Viswanathan, G.M., Raposo, E.P., Mateos, J.L. and da Luz, M.G.E. (2007) The origin of power-law distributions in deterministic walks: the influence of landscape geometry, *Phys. Rev. E*. 75:061114-061120.
- Viswanathan, G.M., da Luz M.G.E., Raposo, E.P. and Stanley, H.E.(2011) *The Physics of Foraging: An Introduction to Random Searches and Biological Encounters*, Cambridge University Press, Cambridge
- Viswanathan, G.M., Raposo, E.P. and da Luz M.G.E. (2008) Lévy flights and superdiffusion in random search: the biological encounters context, *Phys. Life Rev.* 5:133–162.
- Viswanathan, G.M., Afanasyev, V., Buldyrev, S.V., Murphy, E.J., Prince, P.A. and Stanley, H.E. (1996) Lévy flight search patterns of wandering albatrosses, *Nature* 381:413–415.
- Viswanathan, G.M., Raposo, E.P., Bartumeus F., Catalan J, da Luz, M.G.E. (2005) Necessary criterion for distinguishing true superdiffusion from correlated random walk processes. *Phys Rev E*. 72:1– 6.