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# Adapting the Bak-Sneppen Model to a Dynamic and Partially Connected Grid of Hierarchical Species

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## Abstract

This paper describes and investigates a swarm intelligence system with similarity-oriented behavioral rules, hierarchical clustering and evolution by random mutation. The evolutionary scheme is based on the Bak-Sneppen model of co-evolution between interacting species. The swarm of species, in this case, is randomly distributed on a 2-dimensional grid of nodes. The number of nodes is larger than the swarm size and the species are allowed to move on the grid. The rule that defines the movement of the species through the gird is based on the similarity between the species' fitness values and the ranking of those same values within the entire population. Meanwhile, the fitness values are modified using the rules of a 2-dimensional Bak-Sneppen model. The system is intended to be a framework for metaheuristics with spatially structured populations and we show that it displays the desired characteristics for that purpose. Furthermore, these characteristics emerge as global patterns from the local interaction of the species. Without requiring the tuning of control parameters to precise values, the system seems to self-organize into a critical state between randomness and order.

## Introduction

*Self-organization* is a concept that includes a wide range of systems and dynamics. It is used in the realm of physics, chemistry, mathematics, biology and even in social sciences. In general, the term refers to a process through which a system increases its complexity without any external action. Although the complexity sciences have not yet devised a mathematical language that explains the origins and dynamics of self-organization, it may be stated that self-organization describes the property of systems whereby unexpected global patterns emerge from local rules. This paper presents a self-organized model of a population of simple entities that displays coherent global behavior emerging from local rules. The model was designed with the main objective of being applied as a dynamic and self-regulated base-structure for non-panmictic (or structured structured) population-based metaheuristics. The resulting system is a type of swarm intelligence − see Kennedy and Eberhart (2001).

Swarm intelligence algorithms are self-organized systems in which unsophisticated distributed entities interact locally, causing global patterns to emerge. The interaction may be restricted to the communication between the entities, or it may use an environment as a medium for that communication. When the entities interact with (and via) the environment, the system is said to be *stigmergic*, a term introduced by Grassé (1959) to describe the ability of social insects in using the environment as a communication medium.

Fernandes et al. (2012) have recently described a new swarm intelligence discrete system with stigmergic local rules. The system consists of a population of $n$ simple individuals (or particles) moving and interacting on a 2-dimensional grid of nodes. Stigmergy is modeled by providing the particles with the capacity of depositing and following *marks* that carry information about the particle. The structure is defined by local spatial neighborhood and results in a partially connected and dynamic grid of individuals. Each individual is assigned with a random value in thr range [0,1]. This value is called *fitness*.

The motivation behind the work by Fernandes et al. (2012) is to create a dynamic framework for non-panmictic Evolutionary Algorithms (EAs), as defined by Tomassini (2005). EAs belong to a class of metaheuristics based on the Darwinian theories of evolution by natural evolution that use a population of possible solutions (individuals) to a problem. The population evolves by selection, recombination and mutation towards optimal regions of the search landscape. In panmictic EAs, every individual is allowed to interact with every other individual in the population. However, large-scale problems or deceptive functions with multiple local optima may require other type of structures. Therefore, in recent years, non-panmictic EAs, also known as *spatially structured EAs* (see Tomassini (2005)), are gaining increasing attention by the community. This class of EAs restricts the interaction according to a pre-defined or evolving structure that connects the population of solutions. They permit to control the genetic diversity of the population and avoid premature convergence, but they also require extra designing and tuning efforts. In addition, the chosen structure affects the connectivity and the performance of the algorithm.

One possible approach to overcome the rigid connectivity of the traditional structures without being trapped in complicated network design is to use the self-organizing and emergent properties of complex adaptive systems. The work by Fernandes et al. (2012) is an attempt to model the desired characteristics of a dynamic and self-regulated population structure for non-panmictic EAs. In fact, complex properties, such as dynamic clusters of particles displaying pink noise patterns, have been observed while testing the model. However, the experiments in Fernandes et al. (2012) are restricted to a stationary version of the model, i.e., the fitness values of the individuals do not change during the run.

This paper extends the study by Fernandes et al. (2012) and investigates the behavior of the system when populations of time-varying fitness values interact on the grid and generate the structure. The rules for varying the fitness values were taken from the Bak-Sneppen model of co-evolution between interacting species, a complex system proposed by Bak and Sneppen (1993): in each time-step, the fitness value of the worst individual and the fitness values of its neighbors (if any) are replaced by random values in the range $[0,1]$. In other words, the worst individual and its adjacent neighbors in the habitat are mutated.

The Bak-Sneppen model is an example of Self-Organized Criticality (SOC), a theory that has been proposed by Bak et al. (1987) for explaining a class of systems that self-organize into a critical state without requiring the tuning of control parameters. When in the critical behavioral region, these systems display typical signatures, such as scale-invariance, power-law relationships between events and their intensity (or duration) and output variables with pink noise power spectrum.

The Bak-Sneppen model has all the above referred signatures. Like other SOC systems, it doesn't require parameters that need to be tuned. Furthermore, its global behavior can be described as a population of fitness values that evolve during the run. The average fitness of the population tends to grow and the gap $G(t)$ of the system, which is the maximum of the minimum fitness before time-step $t$, is increased during the run until it reaches a specific range (that depends on the topology of the population). These characteristics make the Bak-Sneppen a good candidate for being implemented on the framework proposed by Fernandes et al. (2012) in order to investigate if the behavior observed in the stationary version is maintained in a population of time-varying fitness values. Moreover, the resulting model provides the opportunity to study a version of the Bak-Sneppen model that, to the extent of our knowledge, has not yet been proposed. This new version is characterized by a dynamic topology and by the self-regulated and hierarchical clustering of species.

In this paper, the experiments were designed for describing the properties of the new system, for analyzing the system's behavior in search for complexity and self-organization signatures, and for testing the robustness of the system to changes in the fitness distribution of the population.

The remainder of the paper is structured as follows. The following section addresses SOC and describes the original Bak-Sneppen model. Then, the proposed system is described and contextualized within the current research on spatially structured populations. The subsequent section describes the experiments and the system's dynamic behavior. The final section concludes the paper and outlines future lines of research.

## SOC and The Bak-Sneppen Model

SOC is a critical state formed by self-organization in a long transient period at the border of order and chaos. While *order* means that the system is working in a predictable regime where small disturbances have only local impact, *chaos* is an unpredictable state very sensitive to initial conditions or small disturbances. In complex adaptive systems, complexity and self-organization usually arise at that transition region between order and chaos, or *on the edge of chaos*, as it is sometimes stated. SOC systems are dynamical with a critical point at the region between order and chaos as an attractor. However, and unlike many physical systems, which have a parameter that needs to be tuned in order to obtain the critical state, SOC systems are able to self-tune to the critical point.

In a SOC system, small disturbances can lead to the so-called *avalanches*, that is, events that are spatially or temporally spread through the system. This happens independently of the initial state. Moreover, the same perturbation may lead to small or large avalanches, which in the end will display a power-law proportion between the size of the events and its abundance.

This means that large (catastrophic) events may hit the system from time to time and reconfigure it. These power-law relationships between the size of the events and their frequency are widespread in Nature. Earthquake distribution, for instance, follows the Gutenberg-Richter law, which is a power-law proportion between the magnitude of the earthquakes that occurred in a specific area during a specific period of time, and the frequency of those earthquakes. *Pink noise*, or $1/f$ noise, also displays power-law behaviour (as opposed to *white noise*, which is chaotic).

One may distinguish three types of power-laws arising from physical systems. For instance, the power spectral density distribution (like the pink noise) is described by:

$$P(f) \propto \frac{1}{f^{\alpha}} \tag{1}$$

where $f$ is the frequency, $P(f)$ is the power of that frequency and $\alpha$ is a real number between 0 and 2.0, but usually close to 1.0. If $\alpha = 0$ then $P(f)$ is named white noise; if $\alpha = 2.0$ then it is named red noise or Brownian noise; when $\alpha = 1.0$ then the function $P(f)$ describes pink noise. In general, this function describes which frequency is the most dominant in the temporal behaviour of the system under consideration: the power spectral density is just the square of the Fourier transform of the signal under consideration.

Another power-law arises in size distributions (like the Guttenberg-Richter law, for instance):

$$N(s) \propto \frac{1}{s^{\tau}} \tag{2}$$

where $s$ is the size of an event (or magnitude) and $N(s)$ reflects a distribution of frequency of such events.

A third kind of power-law is identified in the temporal distribution of events, where $\tau$ is either the duration of the event, or the time between events, as described by equation (3):

$$N(\tau) \propto \frac{1}{\tau^{\gamma}} \tag{3}$$

SOC may be the common link between a wide range of natural phenomena operating at the region between order and chaos that exhibit these power-law relationships, a scaleinvariant behavior that does not need to be tuned. The first system were SOC was identified is a cellular automaton called *sand pile* and it is described by Bak et al. (1987). Later, Bak and Sneppen (1993) introduced the model of co-evolution between interacting species: the Bak-Sneppen model.

In nature, different species in the same eco-system are related trough several features (food chains, for instance). They co-evolve, and the extinction of one species affects the species that are related to them, in a chain reaction that can reach huge proportions. Fossil records suggest that the size of extinctions events is in power-law proportion to its frequency. It is also known that the biological history of life on Earth is punctuated by catastrophic extinction events. The Bak-Sneppen model aims at understanding and explaining the mechanisms underlying mass extinction. It consists of a number of species, each one with a fitness value assigned and each one connected to other species (neighbors). Every time step, the species with the worst fitness and its neighbors are eliminated from the system and replaced by individuals with random fitness.

This description may be translated to a mathematical model. The system is defined by $n^d$ fitness numbers $f_i$ arranged on a $d$-dimensional lattice (ecosystem) with $n$ cells. At each time step, the smallest $f$ value and its $2 \times d$ neighbours are replaced by uncorrelated random values drawn from a uniform distribution (in other words, the worst species is removed from the population and its neighbors are mutated). The system is thus driven to a critical state were most species have reached a fitness above a certain threshold and the avalanches produce non-equilibrium fluctuations in the configuration of the fitness values. The complex behavior is observed even in the one-dimensional case, were species are arranged in a chain, an each one has two neighbors.

Since its proposal, the model has been thoroughly investigated by the community and several extensions and modifications have been described. In the seminal paper by Bak and Sneppen, the research is focused on the 1-dimensional version of the system. Higher dimensional models have been since then investigated. De los Rios et al. (1998), for instance, study the high dimensional Bak-Sneppen model ($d \geq 2$) and conclude that the system shows a rich behavior with four qualitatively different regimes as a function of dimensionality: $d \leq 2$, $2 < d < 4$, $4 \leq d < 8$ and $d \geq 8$.

In this paper, we have used the rules of a Bak-Sneppen model with $d = 2$. However, the resulting system is not a standard 2-dimensional Bak-Sneppen model. In our model, the position of species is dynamic and the grid is partially connected, i.e., each species may have four or less species in its von Neumann neighborhood. This leads necessarily to a different behavior and the dynamics observed in the 2-dimensional model may not occur. However, we are mainly interested in the behavior of the proposed system as a potential framework for spatially structured EAs and therefore we search for signatures of dynamic clustering and robustness to changes. A theoretical analysis and empirical validation of the Bak-Sneppen model for determining critical exponents and the gap function is left for future work.

## The System

The proposed framework is a discrete system with a swarm of heterogeneous individuals controlled by a set of local rules. The rules define the actions of a population of $n$ particles that move on a 2-dimensional toroidal grid of nodes with size $X \times Y$. In each time-step, every particle tries to move to a neighboring node. The rules that model the system are the following.

At $t = 0$, the particles are assigned a random *fitness* value in the range $[0,1]$ and then randomly distributed in a $X \times Y$ grid of nodes. Then, at each time-step, each particle moves to an adjacent free node (if any), leaving a mark with information on its status in the previous node. In this paper, the status is the fitness of the particle. The particles decide where to go by inspecting their Moore neighborhood. If there are no free nodes in the neighborhood (i.e., all the cells are occupied by particles), the particle stays in that same node until the next iteration. If there are free cells, the particle checks for marks. If it finds no marks, it just randomly chooses a destination node between the free neighboring nodes. If marks are found with better fitness than the particle's fitness, the particle moves to the node with the mark that minimizes the difference between its fitness and the fitness on the mark. Whenever a particle changes its position, it leaves a mark in its previous location. The marks only remain in the *habitat* for a time-step. In summary, communication by dropping and following information is the base-rule of the proposed system. The system is modeled with a stigmergic behavior.

The particles are ranked according to their fitness. This strategy is imposed with the objective of establishing a hierarchy in the self-organization of the clusters: worst particles tend to follow better particles (the better individuals are *leading the way*).

In each time-step (which comprises the update of every particle's position), the particle with lowest fitness is mutated (i.e., its fitness is replaced by a random value with uniform distribution within the range $[0,1]$), as well as the fitness of its neighbors. The position of the neighbors is defined by the von Neumann neighborhood of the particle (with range 1). This is the standard Bak-Sneppen model on 2-dimensional habitats. The only difference is that in this case the number of neighbors of the worst particle that are also mutated is not necessarily $2 \times d = 4$. This is the maximum number of individuals that are mutated. If the worst particle is isolated (if there are no particles in its von Neumann neighborhood) there are no more mutations in that time-step except for the particle itself. Therefore, in each time-step, $p$ particles are mutated, with $p \leq 1 + 2 \times d$.

This basic set of rules drives the system towards a dynamic global pattern that displays signs of self-organization. A structure of particles, formed by clusters and paths, emerges on the habitat. However, these clusters are far from being static and, in a few generations, the distribution of the whole swarm may change dramatically (while maintaining a typical configuration of clusters and paths). The swarm's behavior is not ordered (nor chaotic). Please remember that convergence to a behavioral region between order and chaos is a signature of self-organization.

Algorithm

1. Randomly place $n$ particles in a grid of node with size $X \times Y$
2. Randomly attribute a fitness value to each particle
3. Find the particle with the lower fitness value. Mutate its fitness and the fitness of its neighbors (von Neumann neighborhood).
4. Rank the particles by increasing fitness
5. For each particle do
   6. check Moore neighborhood for marks and other particles
   7. if no marks in the neighborhood
      8. move to a free cell in the neighborhood (if any)
   9. if there are marks in the neighborhood
      10. move to the site of the nearest fitness mark which is better than its own fitness
      11. leave a mark in the previous site
      12. erase the mark in the new site
13. if stop criteria not met return to 3

Besides dynamic clusters, there are other signatures that suggest that the model comprises a *hidden order* that emerges from local rules. The following section tries to detect and describe those signatures under static and dynamic populations.

Please note that the only parameters that need to be set are the population size $n$ and the grid size. If the ratio between $n$ and the grid size is set within a specific range (large enough to allow communication between the particles, while not so large so that the particles hardly move on the grid), the system self-organizes without requiring the tuning of control parameters. The following section shows that dynamic global patterns emerge within a wide range of population and grid size values. But first, let us discuss the motivation behind the proposed model.

## Motivation

Genotypic representation, operators, selection schemes and population size are typical panmictic EAs moduli that require design choices. However, a population structure may be also introduced in the design scheme of this class of algorithms. This structure specifies a network of acquaintances for individuals to interact, that is, mating or selection is restricted to neighborhoods within the network structure. Spatially structured EAs include fine-grained approaches such as cellular EAs and course-grained approaches such as island models.

The initial objective of spatially structured EAs was to develop a framework for studying massive parallelization – see Gordon and Whitley (1993). Afterwards, the need to provide traditional EAs with a proper balance between exploration and exploitation motivated several lines of research that explore the potentiality of different population structures in maintaining genetic diversity. Population structures were primarily devised as static regular lattices: every individual has a fixed number of potential interaction partners. Later on, complex population structures have been also studied − by Giacobini et al. (2005) and Payne and Eppstein (2006), for instance. However, these standard *cellular EAs* have some drawbacks: synchronicity (in most cases) and a strong dependence on the problem since the genetic diversity promoted by a prefixed topology is uncorrelated to the problem structure.

Dynamic population structures have only recently raised the interest of researchers. To the extent of our knowledge, only few works address explicitly the issue of dynamic population structures in cellular EAs. Alba and Dorronsoro (2005) dynamically change the ratio that defines the neighborhood of interaction. Since the ratio may affect selection pressure, the authors analyze its influence on the balance between exploration and exploitation. However, the base-structure of the cellular EA (i.e. a grid lattice) is maintained throughout the run.

Whitacre et al. (2008) focus on two important conditions missing in EA populations: a self-organized definition of locality and interaction epistasis. With that purpose in mind, they propose a dynamic structure and conclude that these two features, when combined, provide behaviors not observed in the canonical EAs or traditional spatially structured EAs. The most noticeable change in the behavior is an unprecedented capacity for sustainable coexistence of genetically distinct individuals within a single population. The authors state that the capacity for sustained genetic diversity is not imposed on the population; instead, it emerges as a natural consequence of the dynamics of the system.

Laredo *et al.* (2010) propose a framework for EAs based on peer-to-peer networks (see Steinmetz and Wehrle (2005) for a survey on peer-to-peer networks). Within a simulated environment, they model the dynamics of real networks and conclude that their system is able to achieve better performance than traditional EAs on a wide range of problems, while being scalable and resilient to the volatility of nodes in the network.

The work by Fernandes et al. (2012), extended in this paper with a Bak-Sneppen model, has some minor similarities to that by Whitacre et al. (2008), since the structural characteristics of complex systems within an EA population are also recreated. However, while in Whitacre et al. (2008) the structure co-evolves with the EA until it reaches a stable self-organized state, the system described here does not converge to rigid or nearly-rigid state. Instead, it aims at a system working in a critical state where links are frequently created and destroyed and where new emergent patterns appear at high rate.

We demonstrate that the proposed system has indeed emergent properties that could prove useful for spatially structured EAs, or other spatially structured population-based metaheuristics. In this paper, the dynamics of the system and its self-organizing behavior are studied under dynamic populations: the fitness values vary through the run according to the rules of the Bak-Sneppen model. Such dynamics are intended to model the behavior of EAs on the proposed framework. Therefore, it is expected that the outcome of the experiments can provide information on the self-organizing properties of the system and on the limits of those properties.

## Experiments and Discussion

This section investigates the dynamic behavior of the system. Visual descriptions of the patterns that emerge from the interaction of the particles are given. Output patterns are analyzed in search for self-organization signatures. The degree of clustering throughout the entire run is inspected, as well as the distance of the particle to their neighbors (measured in variation between fitness values).

The system was tested with stationary and time-varying populations. The experiments with static populations have been described by Fernandes et al. (2012); therefore, in this paper we only give an overview of the results and conclusions in that study in order to contextualize the discussion. The stationary model is described by the pseudo-code given in the previous section after removing step 3.

The main goals of this section are: 1) check if the self-organizing properties are maintained with time-varying fitness values; 2) investigate the properties of the dynamic and partially connected Bak-Sneppen model and compare it to the standard models.

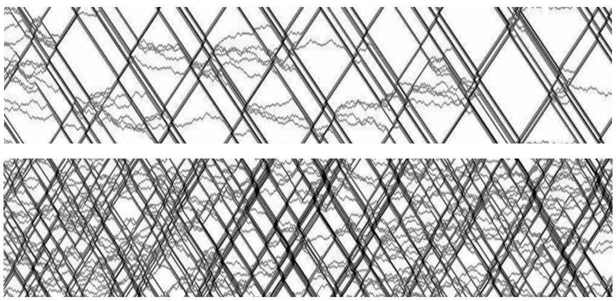


**Figure 1.** Space-time diagrams of a 1-dimensional habitat. $X \times Y$: $150 \times 1$. Swarm: 25, 50 (top to bottom).

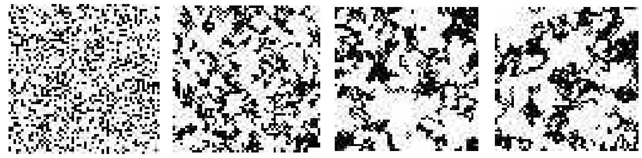
### Stationary Fitness Values

Although the model has been designed has a 2-dimensional framework for EAs, the 1-dimensional version may be constructed by setting $X$ or $Y$ to 1 (see the pseudo-code in the previous section). The 1-dimensional version displays interesting and complex behavior, has shown in Figure 1. The graphics represent the space-time diagrams of the system. These diagrams are usually used to track the spatial configuration of a cellular automaton over a number of time-steps. In this case, the diagrams may illustrate the chaotic and order factors of the system.

Results with grid size $150 \times 1$ and $n = 25$ and $n = 50$ are shown in Figure 1. The leftmost row of the cells is the 1-dimensional lattice set up with a random initial distribution of particles. Each successive row going right is the updated lattice at the next time step. The diagrams show a mixture of order and randomness which is typical, for instance, of class 4 cellular automata. Some clusters of particles move up or down, while free particles randomly move through the grid until they are "captured" by a cluster. Meanwhile, clusters disaggregate, freeing more "wandering" particles. These are typical signatures of complexity and activity between order and randomness. If these traits emerge in a 1-dimensional environment, it is expected that, at least, a similar degree of complexity is present in the 2-dimensional system.

In order to investigate the 2-dimensional model, the grid was then set to $60 \times 60$ and the swarm size to $1200$ (meaning that the ratio between particles and nodes is 1:3.

Figure 2 depicts the distribution of the particles on the grid at different time-steps between $t = 0$ and $t = 1000$. The



| $t = 0$ | $t = 25$ | $t = 750$ | $t = 1000$ |
|---|---|---|---|
| ($k = 1.18$) | ($k = 3.47$) | ($k = 4.11$) | ($k = 4.02$) |

**Figure 2**. Position of particles and average degree of clustering $k$. $X \times Y$: $60 \times 60$; $n = 1200$.

average degree $k$ of clustering is given. This variable measures the number of particles in each particle's Moore neighborhood. The average $k$ is the degree value averaged over the entire population.

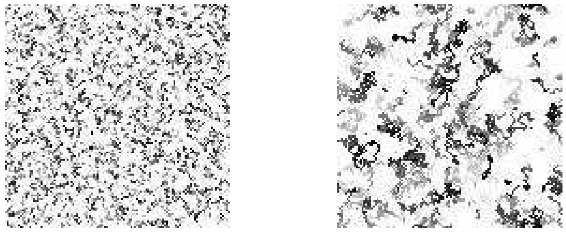
The images in Figure 2 show that the particles are able to self-organize into a dynamic structure of clusters and paths. This assumption is confirmed by the $k$ values, which, starting from $k = 1.18$, tend to grow, reaching $4.02$ after $1000$ iterations. The graphics also confirm that the particles do not only aggregate in small clusters, they also form trails between the clusters. In fact, in most of the time-steps, large parts of the population are connected. This is a key result for the project of designing a dynamic self-organized framework for spatially structured EAs, since information may flow quickly through the population.

Another important outcome is observed in the snapshots of later iterations. Averaged $k$ is similar at iterations $750$ and $1000$. In fact, at this later stage, $k$ does not tend to increase. However, the distribution of the particles is clearly different in the two *snapshots* of the system. That is, even after converging to the maximum range of $k$ values, the swarm continues to reorganize and reshape the clusters. The system is in a state of dynamic equilibrium. Clusters form, but they may disaggregate at any moment, and the particles move to another region of the habitat where they will cluster again with other particles.

Figure 3 shows the distribution of fitness values on the grid by plotting the particles with a grey-level proportional to their fitness. Comparing the distributions at an early and later stage we see that the particles do not only self-organize into clusters; they also tend to cluster according to the fitness, creating structures of particles with similar fitness.

A quantitative analysis of the system was conducted by investigating its output variables, namely the average degree of clustering $k$ and the average distance $d$ to the neighbors. The Fourier Transform of $k$ and $d$ was calculated for a representation of the signal in the frequency domain. For the Fourier Transform, $4096$ samples of the signals were used, from $t = 1000$ to $t = 5095$. This way, the spectral density leaves out the transient phase, from the random configuration at $t = 0$ to the self-organized state. The observation and analysis of the spectral density showed that large regions of the spectra are reasonably approximated by power-laws.

The power spectra were plotted in log-log coordinates, as is customary, since the logarithmic transform renders the power spectrum a straight line whose slope can be easily estimated. The slope $\alpha$ of the power-law in both cases was found to be close to 1, which is the slope of pink noise. The more general case, which displays a spectral density $S(f) = constant/f^{\alpha}$, where $0 < \alpha \leq 2$, is sometimes referred simply as $1/f$ noise.



$t = 10$ $t = 10000$

**Figure 3.** Distribution of fitness values on the grid. Lighter grey areas correspond to particles with lower fitness. $X \times Y$: $60 \times 60$; $n = 1200$.

If we investigate the spectrum of $k$ and $d$ that emerges from a random structure, we find an almost flat density, a signature of white noise. The stigmergic rule supply the system with a typical trait of complex adaptive system and self- organization in near-equilibrium state between order and chaos.

Table 1 show the slopes of the power-laws used for fitting the data obtained by different ratios between the grid size and the number of particles. The relationship between intensity and frequency of $k$ and $d$ is similar when the ratio is in the range [1:24, 1:2]. Outside this range, $\alpha$ tends to decrease.
This is an expected result, due to the physical constraints of the system. On one hand, the swarm requires critical mass to interact. On the other hand, the particles require space to move. However, the model seems to be robust. In order to study its robustness, the swarm was tested with a fixed ratio between the population size and the number of nodes. Several combinations of $n$ and grid size were used. The slopes of the power-laws used for fitting $k$ and $d$ spectrum are in Table 2. With $n = 33$ the slope of the power-law decreases, but for $n > 33$ the power-laws are very similar. The properties of the signals are stable for three orders of magnitude. The system is robust as long as the ratio is within a specific range. The complete description of these experiments, as well as other details on the results with the stationary version of the model, are given by Fernandes et al. (2012).

Table 1. Slope $\alpha$ and $r$-squared of the power-law that fits the $\boldsymbol{k}$ and $d$ spectral density for different ratios between $n$ and the number of nodes on the grid ($X \times Y$).

| $n$: $nodes$→ | 1:24 | 1:12 | 1:6 | 1:3 | 1:2 | 1:1.5 | 1:1.2 |
|---|---|---|---|---|---|---|---|
| $k$ | **1.18 (0.76)** | **1.23 (0.76)** | **1.23 (0.76)** | **1.20 (0.76)** | 1.07 (0.70) | 0.88 (0.60) | 0.56 (0.60) |
| $d$ | 0.82 (0.60) | **1.00 (0.72)** | **0.97 (0.68)** | **1.01 (0.69)** | **1.00 (0.69)** | 0.93 (0.64) | 0.42 (0.60) |

Table 2. Slope $\alpha$ and $r$-squared. $n$: $nodes$ is fixed and equal to 1: 3.

| $n$ → | 33 | 75 | 147 | 300 | 616 | 1200 | 2408 | 4800 |
|---|---|---|---|---|---|---|---|---|
| $k$ | 1.15 (0.72) | 1.29 (0.77) | 1.18 (0.75) | 1.22 (0.77) | 1.18 (0.74) | 1.20 (0.76) | 1.17 (0.74) | 1.18 (0.76) |
| $d$ | 0.87 (0.62) | 1.04 (0.70) | 1.04 (0.71) | 1.10 (0.75) | 1.03 (0.70) | 1.01 (0.69) | 1.02 (0.69) | 0.97 (0.69) |

### Time-Varying Fitness Values

In this paper, the model was tested with the Bak-Sneppen mutation rules (i.e., including the step 3 of the pseudo-code given in the previous section). The size of the grid was set to $60 \times 60$ and the swarm is comprised of 1200 individuals.

The first analyses aim at comparing the behavior of the system with stationary and non-stationary fitness values. For that purpose, the spectra of the output variables ($k$ and $d$) were computed and compared with the spectral densities of the stationary version.

Figure 4 compares the spectral density of the average distance between neighboring particles in each time-step. The introduction of the mutation rules based on the Bak-Sneppen model does not affect significantly the distribution of frequencies.

Figure 5 shows the spectral density of the connectivity degree $k$. Again, introducing a mutation mechanism in the original model does not affect the general behavior of the swarm and the clustering dynamics. These results demonstrate that it is possible to obtain an emergent behavior consisting of dynamic clustering based on similarity and hierarchy using not only a population of stationary fitness values, but also an evolving population. This is an important result since an EA, by definition, is a population of solutions that, in average, improves over time. If an EA is implemented on a population of the model, and if the intensity of changes is maintained within a certain boundary (here, the number of fitness values that change in each time-step is in the range [1,5]), it is expected that global patterns that emerge from the proposed model also appear in the model-based EA.

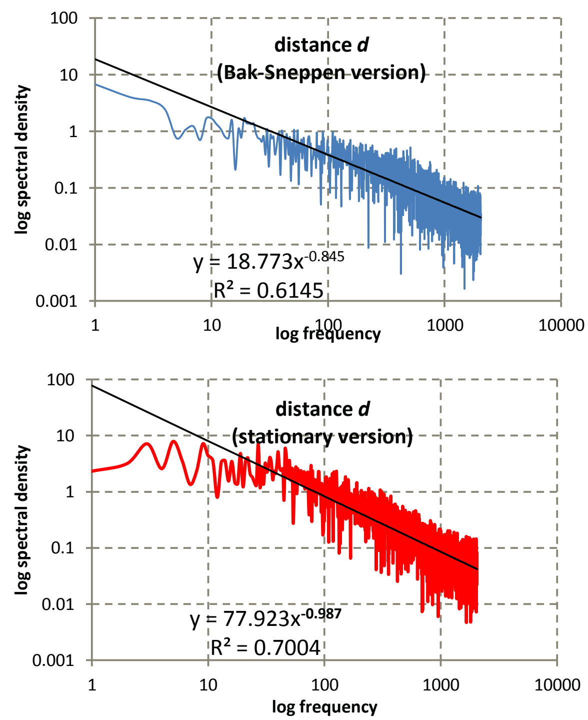



**Figure 4.** Comparing the spectral density of the average distance $d$ that emerges from the stationary and non-stationary fitness versions of the model.

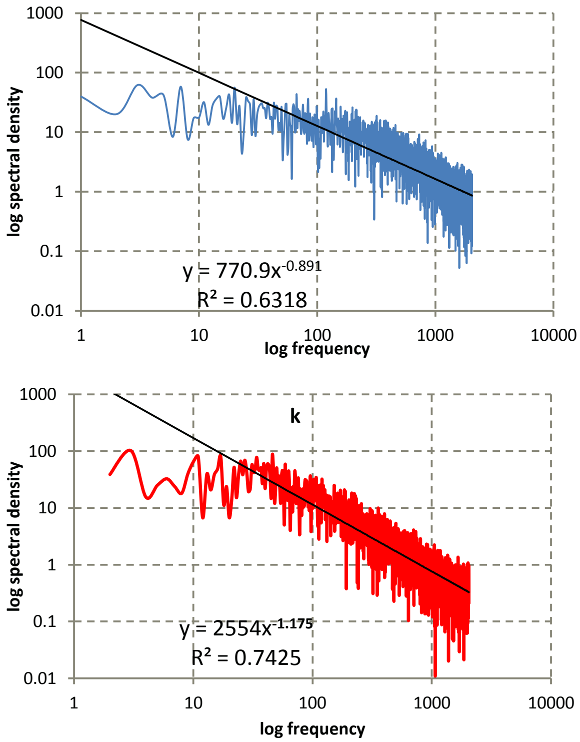



**Figure 5.** Comparing the spectral density of the average connectivity degree k that emerges from the stationary and non-stationary fitness versions of the model.

The evolution of the population can be visualized by plotting the average and minimum fitness of the population, as well as the gap function $G(t)$. Figure 6 shows the evolution of 1200 particles on a $60 \times 60$ grid, while Figure 7 shows the evolution of a population of 1200 on $30 \times 40$, i.e., Figure 7 displays the behavior of a standard 2-dimensional Bak-Sneppen model. The average fitness of the partially connected model evolves to higher values. The proposed model reaches an average fitness values of approximately 0.8, while the standard 2D model stays below 0.7 (a result observed in several runs with different random seeds).

The gap function also grows faster and reaches higher values. In the several runs conducted for this study, the critical value of the gap function was found to be $f_c$~0.6. The dynamics of the proposed model is clearly different from the standard 2D model. The sparser connection between the particles is a reasonable explanation for the differences in the evolutionary rates (please remember that in our model there are $p \leq 1 + 2 \times d$ particles that are mutated in each time-step, while in the standard 2D Bak-Sneppen model there are $p = 1 + 2 \times d$ mutations). The effects of the local movement rules are harder to measure, but since the particles cluster according to the fitness values, better particles tend to gather in the same regions, and therefore the mutation of the worst individuals will tend affect also *weak* neighbors, thus leading to a faster evolution of the population's fitness values.

One of the SOC signatures of the Bak-Sneppen model is the power-law relationship between the duration of the species' periods of stasis (time-steps between successive mutations) and their frequency. The proposed model displays

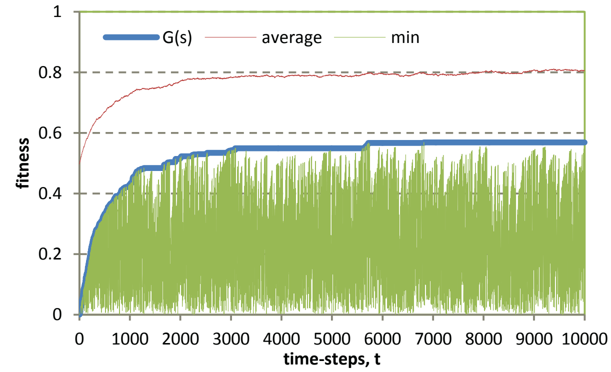



**Figure 6.** Evolution of 1200 particles on a $60 \times 60$ grid. Average fitness, minimum fitness and gap function.

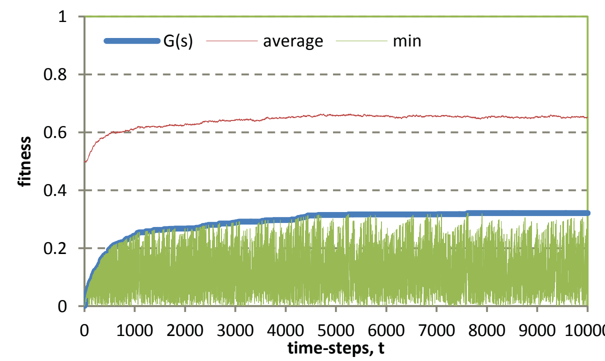



**Figure 7.** Evolution of 1200 particles on a $30 \times 40$ grid (standard 2D Bak-Sneppen model).

the same signature. The exponent of the power-law is approximately $3/2$, as seen in Figure 8. This is the same exponent obtained with the standard 2-dimensional model, while the 1-dimensional Bak-Sneppen system, in our experiments, displays a power-law with exponent approximately $7/4$..

The model maintains the characteristics of the stationary version proposed by Fernandes et al. (2012). Global patterns of clusters connected by paths tend to emerge. These clusters are highly dynamic, and in a few generations the distribution of the particles in the habitat may dramatically change (we believe there is an avalanche-based self-organized phenomenon behind the massive reconfigurations of the system but we haven't yet identified i). The output variables of the system display pink noise spectral densities. Furthermore, the proposed model maintains the characteristics of standard 2-dimensional Bak-Sneppen models. The average fitness of the population tends to grow with time, and the gap function converges to a specific critical value. The power-law observed in the distribution of distances between successive mutations also appears in the proposed model, with the same exponent as the 2-dimensional Bak-Sneppen model.

## Conclusions and Future Work

This paper describes an evolutionary extension of the self-organized swarm intelligence system proposed by Fernandes

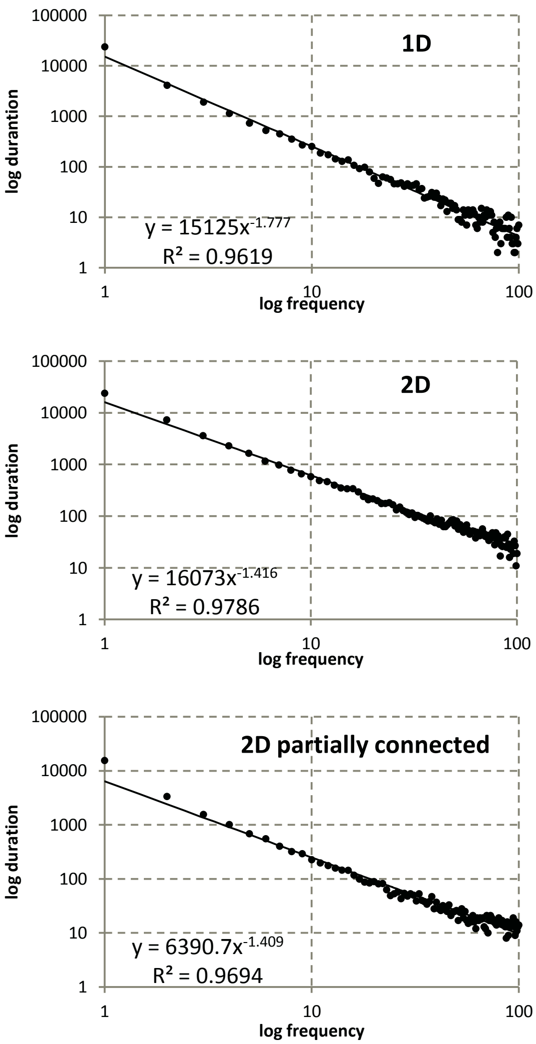



**Figure 8.** Duration of the periods of stasis (periods in-between mutations).

et al. (2012). The system is a swarm of simple particles that interact on a heterogeneous grid of nodes. The particles communicate via the grid, and move according to simple rules. A fitness value is assigned to each particle. In each time-step, the fitness values of the worst particle and its neighbors are mutated. This is the basic rule of a Self-Organized Critically (SOC) model known as the Bak-Sneppen model of co-evolution between interacting species.

The system has been designed as a base-framework for spatially structured Evolutionary Algorithms (EAs). The original model (without the Bak-Sneppen mutation rules) displays a complex behavior illustrated by dynamic clustering of the particles, catastrophic reconfigurations of the distribution of the particles on the grid, and output variables with pink noise spectral densities. The model proposed in this paper maintains the main characteristic of the stationary fitness values version. This conclusion is very important for the project of designing a spatially structured framework for EAs based on the proposed system. Furthermore, the system displays the same SOC signatures as the standard 2-dimensional Bak-Sneppen model.

In the future, the research will be focused on two main lines of work. Firstly, an EA will be implemented on the model and compared to standard spatially structured EAs. Secondly, the behavior of the system as an (hypothetical) SOC system will be studied. Traits such as the critical fitness threshold and the critical exponents of the model will be investigated. Furthermore, we believe that there is an avalanche-based phenomenon triggering the massive reconfigurations of the system (particles' positions on the grid). In a future research, we will try to identify that phenomenon, its origin, and study its distribution in search for self-organization signatures.

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