DOI: http://dx.doi.org/10.7551/978-0-262-31709-2-ch103

Evolution of Spatial Pattern Formation by Autonomous Bio-Inspired Cellular Controllers

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Abstract

In this paper, a gene regulatory network called FGRN (Fractal Gene Regulatory Network) and a reaction-diffusion system called AHHS (Artificial Homeostatic Hormone System) are investigated for spatial pattern formation. The two bio-inspired controllers possess similar and different features in terms of their underlying processes, structures, and communication abilities. By comparing their behaviours and capabilities in pattern formation, we provide a deeper understanding of the effects of their features. The controllers are evolved and investigated for producing various patterns in presence of different implicit positional information as well as developing a memory in order to keep the desired pattern when the positional information is eliminated. The behaviours of the controllers in each case are discussed and a preliminary test of robustness is performed. The experiments represent a positive impact of diffusion process in AHHS that is compensated by the complex structure of FGRN in producing patterns in presence of positional information and a negative effect of diffusion process in memory capability.

Introduction

Formation of spatial patterns is a challenging subject both in biological and artificial organisms. Different forms with various levels of complexity are found everywhere in nature. One of the challenges in developmental biology is to understand the underlying processes that control the pattern formation (Jaeger and Martinez-Arias (2009)). On the other hand, from the point of view of multi-modular robotics, a proper behaviour emerges from a proper pattern of roles assigned to the modules across the body of a robot.

A problem encountering pattern formation is symmetry-breaking. In biological organisms it happens at early developmental phases. As it was suggested by Wolpert (1968) and is found in embryos, e.g. fruit fly Drosophila melanogaster (Driever and Nusslein-Volhard (1988); Ephrussi and Johnston (2004)), the polarization of an organism is induced by some maternal cue in the form of morphogen gradients. By using these gradients in the environment of the organism, some information is provided that is used for localization of the organism's units (cells) and participates in the process of development. The same concept is useable in artificial organisms (e.g. localization of modules in a modular robot).

For subdivisioning of a body using positional information, Wolpert (1968) proposed a French-flag model. The model is composed of three stripes with different colours along the body and is used by many researchers with different approaches of evolving systems (e.g., Miller (2003); Bowers (2005); Cussat-Blanc et al. (2011)).

In the field of artificial life and evolutionary computation, various models are inspired by genetic and chemical systems in biological organisms. Gene Regulatory Networks (GRNs) and reaction-diffusion models are two examples of these systems that have drawn attention in the recent years. They consist of a number of different underlying processes that control their dynamics. Although the source of inspiration and the details are different for these two models, but there are some similarities between the models. In this work, a reaction-diffusion model and a GRN model are investigated in the context of pattern formation.

GRNs are inspired by internal interactions between genes and proteins in cells. Various models of computational GRNs have been defined and investigated from different perspectives, e.g. studying dynamics (Banzhaf (2003)), applying for morphology development (Eggenberger (1997); Roggen and Federici (2004)), developing both morphology and controller of robots (Bongard and Pfeifer (2001)).

Fractal Gene Regulatory Network (FGRN) (Bentley (2004b)) is an example of GRN models. It is originally designed as a single-unit of control and successfully implemented for different tasks, i.e., controlling conventional robots (Bentley (2004a)) and pole-balancing (Krohn and Gorse (2010)). Since no explicit communication mechanism between different units is defined in FGRN, environmental feedback is used to coordinate modules in multi-modular robotic applications of FGRN (Zahadat et al. (2010, 2012)).

Reaction-diffusion models are inspired by intracellular signaling in biological organisms. The models contain both a process of local reaction between substances and diffusion of substances across the organism. Artificial Homeostatic Hormone System (AHHS) is an example of these models which is originally introduced in Schmickl and Crailsheim (2009) and has been used successfully in robotic applica-

tions for both single and multi-modular robots (Stradner et al. (2009); Schmickl et al. (2010); Hamann et al. (2010)).

In this paper, FGRN and AHHS are evolved and investigated for generating target patterns with fixed maternal morphogen gradients as well as generating a memory such that the target pattern is preserved after elimination of the maternal gradients. Behaviours of different evolved solutions are discussed and a sample evolved controller is tested for its reaction against an instant reset in a single unit in order to have an evaluation of robustness of the produced pattern.

While the two systems are similar in terms of having mechanisms to produce various mappings between input and output as well as forming internal feedback loops, they are different in their complexity and details of mechanisms, structure, and communication abilities. For instance, FGRN model provides more complicated interaction network between local substances in comparison with AHHS. A prominent difference between the two models is the lack of intra-unit communication in FGRN. Due to that, the spatial patterns generated by FGRN lay solely on the maternal gradients and internal interactions of each particular unit. On the other hand, in AHHS, along with the maternal gradients and internal interactions, the pattern formation can benefit from the diffusion of substances over the units.

In this work, in addition to FGRN and AHHS with their standard underlying processes, a diffusion-free version of AHHS has been also implemented in order to investigate the importance of diffusion in the observed differences between the behaviours of FGRN and AHHS.

Short Summary of FGRN

FGRN (Bentley (2004b)) is a GRN model that uses an abstract model of proteins, called fractal proteins, as the means of interaction between genes. These means of interactions are encoded in the genome and evolved by a version of Genetic Algorithm (GA) (see Bentley (2004b) for details).

The genome consists of a number of genes and parameters. Every gene in the system belongs to a type of genes: input genes, output genes, regulatory genes and receptor genes. Input, regulatory and receptor genes encode corresponding fractal proteins. A fractal protein has a shape and a concentration level. The shape is encoded in a gene by three real values. These values determine a square window on Mandelbrot fractal set. A protein's concentration level is a variable value. The changes in concentration level is controlled by the other proteins' concentration levels, shapes of the fractal proteins, and other parameters of the genome. To every sensory input into an FGRN system, a set of input proteins are associated. The input value determines the concentration levels of the corresponding input proteins and consequently participate in driving the dynamics of the system. Receptor proteins act as filters over inputs by manipulating shapes of input proteins. Regulatory proteins participate in driving the internal dynamics. Their concentration levels are both controlled by and also participate in controlling the dynamics of the system. In fact, they make regulatory connections in the network of proteins and are potentially capable of establishing recurrent loops and act as a sort of memory in the system. Output genes determine the influence of the concentration levels of the proteins on the output of the system. (For a detailed introduction of FGRN see Bentley (2004b))

FGRN can be seen as several systems of Difference Equations ($O\Delta E$s) where each $O\Delta E$ system controls the internal dynamics over time in a particular part of the state space. Concentration levels of proteins are state variables of the system. When the value of a state variable changes from a positive value to zero or vice versa, the system switches between different parts of the state space and its behaviour changes due to activation of a different system of $O\Delta E$s (For a detailed description of this representation of FGRN see Zahadat and Støy (2012)). Table 1 demonstrates an example FGRN represented as conditional sets of $O\Delta E$s.

Table 1: An example of a simple FGRN as several conditional sets of $O\Delta E$s. The state-space of this system is divided into four parts. S represents the set of proteins with positive concentration levels indicating a division of the state-space. $P1$ and $P2$ correspond to an input and a regulatory protein respectively and $p1$ and $p2$ are their corresponding concentration levels. out is the output of the system.

condition	equation set
if $S = \{P_1, P_2\}$	$p_2 \leftarrow 0.8p_2 - (0.2p_1 + 0.5p_2)$ $\times tanh(0.6p_1 + 1.5p_2 - 3.6) - 0.2$ $out \leftarrow 0.15p_1 + 0.2p_2 + 4$
if $S = \{P_1\}$	$p_2 \leftarrow 0.4p_1 + 0.5$ $out \leftarrow 0.32p_1 + 2$
if $S = \{P_2\}$	$p_2 \leftarrow 0.8p_2 - 0.25p_2$ $\times tanh(0.5p_2 - 0.4) - 0.2$ $out \leftarrow 0$
if $S = \{\}$	$p_2 \leftarrow 0.4$ $out \leftarrow 0.25$

Due to the lack of any intra-unit communication in FGRN, symmetry breaking and differentiation of units is achievable by providing different inputs for different units.

Short Summary of AHHS

AHHS (Artificial Homeostatic Hormone System) (Schmickl and Crailsheim (2009)) is a reaction-diffusion-based system inspired by Turing process (Turing (1952)) that describes processes of natural pattern formation and growth.

An AHHS is defined by a set of artificial hormones and a set of rules. The rules define how sensory input and hormone concentrations participate in changing the concentrations and outputs of the system. Both hormones and rules are evolved by using a standard real-valued GA.

An AHHS can be represented as a dynamical system consisting of several state variables (hormone concentrations) and a system of $O\Delta E$ that governs their dynamics. The key feature of AHHS is how the parameters of this $O\Delta E$ are encoded and determined. Concentrations of the hormones are

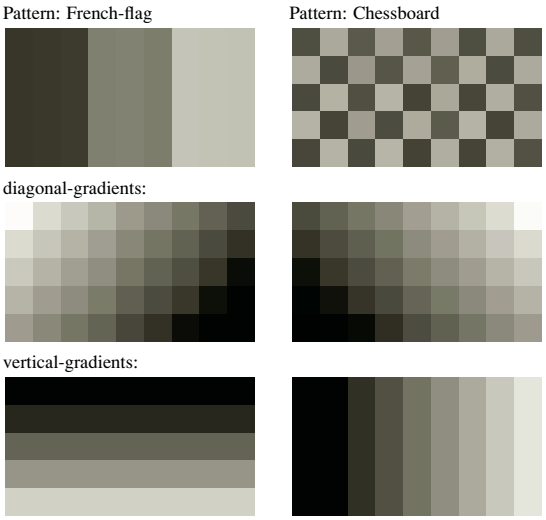

Figure 1: Target patterns (first row), diagonal gradients (second row), and vertical gradients (third row).

allowed to increase independently by a base production rate and are also subject to a certain decay.

The dynamics of hormone concentration H at time t is defined for hormone h as follows:

$$\frac{\Delta H_h}{\Delta t} = \alpha_h + D_h \nabla^2 H_h(t) - \mu_h H_h(t) + \sum_i \mathcal{L}_i(t), \quad (1)$$

where α_h, D_h, and μ_h are base production rate, diffusion rate, and decay rate of hormone h respectively. $L_i(t)$ is the influence of rule i on hormone h and is defined as:

$$\mathcal{L}_i(t) = \theta(H_k(t))(H_k(t)\lambda_i + \kappa_i), \quad (2)$$

$$\theta(x) = \begin{cases} 1 & \text{if } min_i < x < max_i \\ 0 & \text{else} \end{cases}, \quad (3)$$

where λ_i, κ_i, min_i and max_i are parameters of the rule. $\theta(H_k(t))$ determines if the rule is triggered or not. If the rule is triggered, the concentration of hormone h changes linearly based on concentration of hormone k.

In the current implementation, sensory inputs are scaled in the range of hormone concentrations ([0,1]) and directly set to the concentration level of particular hormones. In the same way, concentration level of a particular hormone is considered output of the AHHS unit.

Experiments

In this work, the systems are evolved to produce patterns in a 5×9 rectangular grid. Every cell of the grid has a colour which is determined by the output of its controller. The controllers are genetically identical all over the grid. Two maternal gradients are provided over the grid in every experiment.

Two types of target patterns and two types of maternal gradients are considered with different degrees of difficulty (see Figure 1 for both target patterns and maternal gradients). For the FGRN controllers, the two maternal gradients enter a unit in form of two sensory inputs that in turn influence the concentration levels of input proteins of the unit. In AHHS system, the values of the maternal gradients directly set the concentration levels of two particular hormones. The output from each unit (either a FGRN or AHHS controller) is mapped into one of the three predefined colours regardless of the number of colours in a particular target pattern.

The controllers are evolved for the following tasks:

- Since French-flag pattern as suggested in Wolpert (1968) is a benchmark in evolving for pattern formation, in the first task, the target pattern is a French-flag with vertical maternal gradients (third row of Figure 1). In this case we suspect that the controllers make a direct mapping between one of the maternal gradients and the output.

- In the second task, the target pattern is again a French-flag but this time the maternal gradients are diagonal, as in Cussat-Blanc et al. (2011) (second row of Figure 1).

- The third task aims at producing a chessboard pattern with diagonal maternal gradients. In the first three tasks, the system runs for 100 time-steps while the gradients are stable during the run-time.

- In the forth task, the controllers are supposed to produce a sort of memory. The gradients are presented for the first few time-steps when the target pattern is generated. Then the gradients are removed while the target pattern is expected to be preserved by the system. The simplest combination of target pattern and gradients (French-flag with vertical gradients) is chosen for this task in order to keep the focus on the memory capability.

Evolving for the target patterns

Populations of 50 random individuals are evolved for each task for FGRN, standard AHHS, and diffusion-free AHHS controllers. Every experiment is repeated for 10 independent runs with 1500 generations. Table 2 represents the controller and evolutionary settings.

Table 2: Controller and evolutionary settings

FGRN:

population-size	50	#generations	1500	#recomb.	0.4
mut. prob.	1	#receptors	2	#inputs	2
#regulatories	2	#outputs	2		

AHHS:

population-size	50	#generations	1500	recomb. prob.	0.01
mut. prob.	0.4	#hormone	6	#rules	30

In the first three tasks, fitness is defined as the number of correct coloured cells in the final pattern. In the case of evolving for memory, two fitness factors are required. A fitness factor is required to direct the evolution towards generating target pattern and the second factor is needed for evolution of the memorizing ability. Considering these factors, fitness is defined as a combination of the number of correct colours in the last time step with maternal gradient (time-step 10) and the average number of correct colours in all the next time steps:

$$fitness = 3 \times C_{T_p} + \frac{1}{T - T_p} \sum_{t=T_p+1}^{T} C_t$$

where T is the number of time-steps of the experiment and in the first T_p time-steps the gradient is present, C_{T_p} is the number of correct colours in time-step T_p, and C_t is the number of correct colours in time-step t.

Comparisons between FGRN, standard AHHS, and diffusion-free AHHS controllers for all the tasks are represented in Figure 2. The figure represents that FGRN and AHHS are evolvable to produce the pattern for all the tasks while none of the runs of diffusion-free AHHS produces a perfect French-flag and chessboard with diagonal gradient. Figure 3 demonstrates the median fitness-progression of the controllers over generations. The figure represents that the French-flag is easy-to-produce for all the controllers when the gradients are vertical. In the case of diagonal gradients for both French-flag and chessboard, diffusion-free AHHS has the lowest fitness indicating that diffusion has a positive effect for AHHS in these tasks. In the case of evolution for memory, diffusion-free AHHS makes higher fitness than standard AHHS while FGRN represents the highest value. It implies a negative effect for diffusion in keeping a pattern without external clue of maternal gradients.

Looking at the behaviours

In order to make an impression of the solutions of each controller type, we will have a look at the behaviours of the evolved controllers in the following sections. Representative examples of different observed behaviours in each task are displayed in Figure 4. Each curve in the figure represents the development of fitness achieved by an example controller over time (Note that the fitness in the last time-step is considered the actual fitness of the controller).

French-flag with stable vertical gradients All the three types of controllers are able to produce perfect target pattern. The difference between the number of successful runs is not statistically significant (Figure 2).

In FGRN, 9 runs out of 10 produce target pattern perfectly. In six runs out of 10, the pattern is perfectly generated from the first time-step that represents that static controllers are found that simply threshold the maternal gradients to produce the output that is mapped to the respective colours. The other three runs reach the perfect pattern after less than 20 time-steps and then the pattern stays stable.

In AHHS, 9 out of 10 runs produce perfect pattern. In four runs, the output oscillates between different patterns such that in time-step of observation (time-step 100) the perfect pattern is represented. In the other five runs, fitness increases gradually and perfect pattern is produced after several steps and stays stable until time-step 100. Generation of perfect pattern is slower than similar cases in FGRN.

In diffusion-free AHHS, 9 runs out of 10 generate the pattern. Five runs generate oscillatory patterns such that the target pattern is represented in time-step 100. In the other four runs, the pattern is slowly produced during time and stays stable. The increase in fitness is again slower than FGRN but no significant difference with standard AHHS is observed.

French-flag with stable diagonal gradients In FGRN, four runs out of 10 reach the perfect pattern. Three of them make the correct pattern from the first step indicating thresholding of the maternal gradients. The other run, makes the pattern after about 10 steps. The unsuccessful runs mainly produce changing patterns with chaotic oscillation in the fitness curve, although an static pattern is observed in one run.

In AHHS, four runs produce the perfect target pattern. All the four successful runs generate the correct pattern very slowly and gradually in last time-steps. In all the other runs the fitness gradually increases during time although short deceases are also not impossible. No oscillation is observed.

In diffusion-free AHHS, none of the runs are successful to produce perfect target pattern. Changing patterns with both chaotic and ordered oscillations and also patterns with gradual increase in the fitness are observed.

Chessboard with stable diagonal gradients In FGRN, three runs reach the perfect target pattern and two other runs are correct except for one cell of the grid. Although we suspected that the success of FGRN in this case might be a result of precise thresholding but the patterns oscillate over time such that the perfect pattern is presented in the last time-step. In fact all the ten runs make oscillatory patterns mostly with big differences from one step to the next.

In AHHS, one run reaches the perfect pattern. This pattern is not stable and oscillates between the correct and the inverse patterns such that in the last time-step the perfect pattern is presented. The oscillation is more ordered comparing the FGRN runs. Most of the other runs produce oscillating patterns but stable patterns are also observed.

In diffusion-free AHHS, no run produces the perfect pattern. The runs generate different patterns and although some stable patterns are generated but most of the runs make oscillatory patterns. In comparison with AHHS, ordered oscillations with high frequencies are not common and oscillations are more chaotic and in comparison with FGRN the oscillations have lower frequencies.

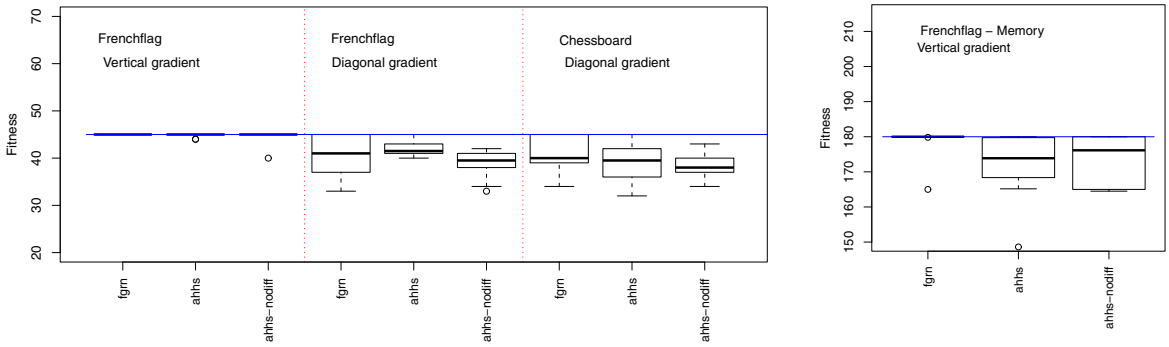

Figure 2: Fitnesses of the best individuals in the last evolutionary generation for FGRN, AHHS, and diffusion-free AHHS controllers for the four tasks. Box-plots indicate median and quartiles, whiskers indicate minimum and maximum, circles indicate outliers (values are collected from 10 independent runs).

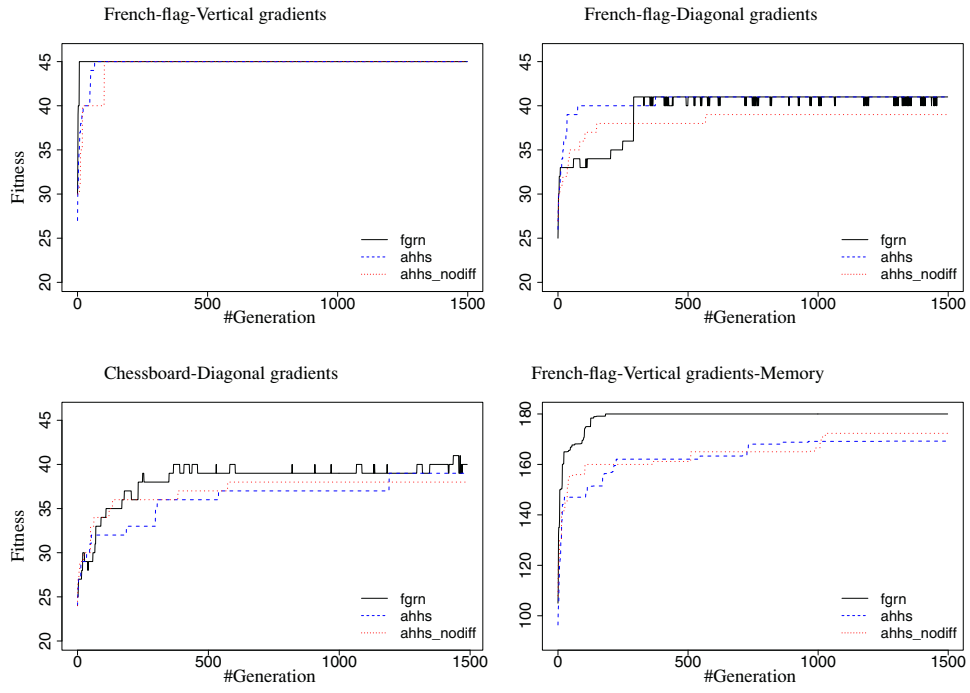

Figure 3: Comparison of fitness trajectories for FGRN, AHHS, and diffusion-free AHHS controllers in the four tasks. The values are medians of the best fitnesses in the 10 independent evolutionary runs. (y-axis starts from 20 due to the space-limitation and since there is no data below 20.)

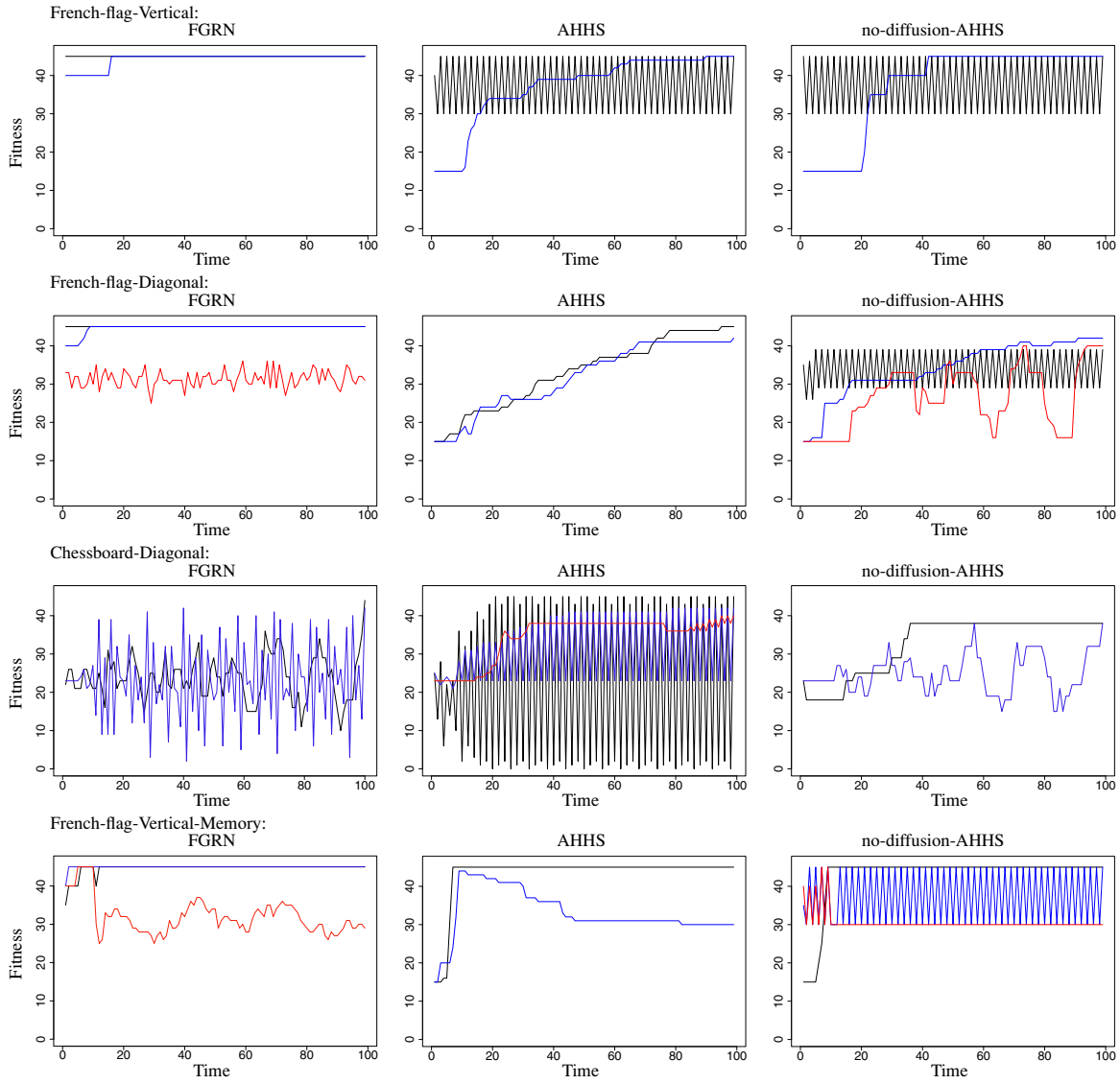

Figure 4: Fitness development over time for representative examples of the evolved solutions for the three controller types.

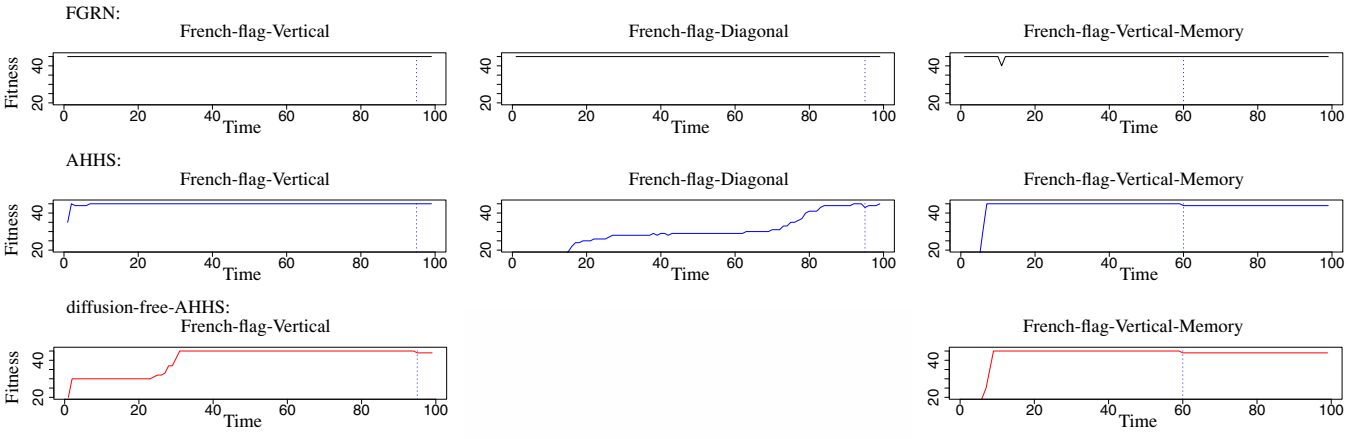

Figure 5: Behaviour of example controllers when all the state variables of the cell in the middle of the grid is reset to zero in a particular time-step that is represented by the vertical dashed-lines. The diagrams are squeezed due to space-limitation.

Memory in French-flag with vertical gradients In FGRN, four runs are successful in producing the pattern at time-step 10 (last time-step with gradients) and keeping it for the next 90 time-steps (with no gradient). In three other runs, only for one step after vanishing the gradient, the pattern is disturbed and then is regenerated. In the other three runs, the target pattern is produced in time-step 10 but then it changes and the fitness oscillates chaotically or ordered.

In AHHS, in two runs the pattern is produced until time-step 10 and is kept until the end. In one run, the pattern is represented in time-step 10, but when the gradients are vanished the pattern slightly deviates from target for few time steps but again is generated and kept until the end. In another run, the perfect pattern in time-step 10 is kept for about 60 time-steps and then deviates from target in a single cell of the grid until the end. In all the rest except one, the target pattern is represented in time-step 10, but then it deviates from target when the gradients are vanished.

In diffusion-free AHHS, five runs produce and keep the perfect target pattern from time step 10 to 100. In one run, the produced pattern in time-step 10 is perfect, then it changes in the next time step when the gradients are vanished but it is produced again after one step and is kept until the end. This is the effect that is also observed in FGRN. In one run, the pattern is switched between the target and a simpler pattern in every step. The other three runs produce the target pattern in time-step 10 and then the pattern changes to a simpler pattern and stays stable until the end.

Robustness in example controllers

In the last experiment we aim to evaluate robustness of the controllers by disturbing the internal variables of a cell in the grid in order to see if the correct pattern is regenerated after few time-steps. Experiments are performed with the French-flag pattern in the three cases of stable vertical gradients, stable diagonal gradients, and memory with vertical gradients, for all the controller types. In each case, we chose a controller from the previous experiments that produces the target pattern long before the end of evaluation period. Then the controller is used to produce the pattern and in a particular time-step all the dynamic values (hormones/proteins) of the controller in the middle of the grid are set to zero. The fitness of the system at the end of the evaluation period is then calculated that represents whether the pattern is regenerated or not. The results are represented in Figure 5. Since diffusion-free AHHS did not evolve for French-flag pattern with diagonal gradients in any of the runs, it is omitted in the figure. The figure demonstrates no change in any of the patterns produced by FGRN controllers indicating robustness of the controller against the reset. In AHHS controller, for the stable vertical gradients the pattern does not change after reset. In the stable diagonal gradients, the pattern changes (fitness decreases) but it is reproduced after few time-steps. In the case of memory as well as the tasks with diffusion-free AHHS, the patterns do not regenerate after reset.

Conclusions

FGRN (Fractal Gene Regulatory Network), standard AHHS (Artificial Homeostatic Hormone System), and a diffusion-free AHHS are evolved and investigated for their capability in pattern formation in presence of maternal gradients.

FGRN is computationally expensive due to encoding of proteins as the intermediate substances that establish interaction connections between genes. By this indirect encoding, a potentially complex interaction network of genes is formed. FGRN defines no communication mechanism between units and all the dynamics of the system are based on the internal interaction network. On the other hand, AHHS is a reaction-diffusion-based model and employs diffusion as a means of communication between units. In AHHS, the interaction connections between hormones are directly encoded which provides a simpler interaction network inside a unit, in comparison with FGRN. The dynamics of the system is based on both the internal interaction network and the diffusion mechanism.

In this work, three sets of evolutionary experiments with different combinations of target patterns and stable gradients are performed. In addition, a memory test experiment is performed where the maternal gradients are provided in the first time-steps and then they are vanished from the environment. The system is expected to produce the target pattern in presence of maternal gradients and keep it intact after elimination of the gradients.

In all the three experiments with stable maternal gradients, both FGRN and standard AHHS are successfully evolved for the perfect solution although the rates of success are different. It has to be mentioned that in a previous work by Cussat-Blanc et al. (2011), successful evolution of another GRN model (Banzhaf (2003)) in the case of French-flag with diagonal gradients was reported where the result of a single evolutionary run was presented. Diffusion-free AHHS is evolved successfully for producing the French-flag pattern in presence of vertical maternal gradients but it did not find the solutions in more complicated tasks of producing chessboard pattern and French-flag with diagonal maternal gradients. It implies that diffusion is an important part of an AHHS system. On the other hand, despite of importance of diffusion in AHHS, FGRN is also successful in all the tasks. It indicates that in principle the investigated patterns are producible even without diffusion. The more complicated nature of FGRN system enables it to be evolved for the proper patterns although there is no communication mechanism implemented in FGRN. In addition, observing the evolved solutions for French-flags (both gradient types) represents that FGRN can produce the pattern in one time-step while AHHS always needs time to build it up. This is also due to the complex structure of FGRN in terms of $O\Delta E$s.

All the three types of controllers were successfully evolved for the memory experiment. The rate of success (number of perfect evolutionary runs) was lower for standard AHHS in comparison with diffusion-free AHHS and FGRN. It can lead to the conclusion that diffusion has a negative effect in keeping the memory. This effect is intuitively expected, since diffusion tends to flatten the pattern and having that in the system requires a compensation mechanism, e.g. elaborating internal feedback loops.

A preliminary experiment for evaluating robustness of the systems has been also performed in this work and represented the highest robustness for the solutions generated by FGRN system. In the future, controllers with different subsets of internal processes will be evolved for series of spatial patterns with increasing levels of complexity and the effects of the internal processes will be investigated in details.

Acknowledgements

This work is supported by: EU-ICT project 'CoCoRo', no. 270382; EU-IST-FET project 'SYMBRION', no. 216342; EU-ICT project 'REPLICATOR', no. 216240; Austrian Federal Ministry of Science and Research (BM.W F); EU-ICT project 'ASSISI_bf', no. 601074.

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