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Multiplicity of Interpretation in an Asynchronous Updating Rule: Emergence of Collective Cognition

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Abstract

Many kinds of interactions among individuals construct collective animal behavior, but how to apply this multiplicity of interactions is often unclear when constructing models. We propose multiplicity of interaction in a simple model constructed from three factors: asynchronous updating, learning site patterns, and agent anticipation. We found that the first two contribute to an efficient searching strategy, and that adding agent anticipation enables sign making (avoidance) in heterogeneous environments. Our model surprisingly suggests that searching strategies and territorial behavior such as boundary marking — seemingly contradicting behaviors — emerge from two aspects of our simple interaction rule. We discuss the possibility of collective cognition in animals when heterogeneous environments change. Our study suggests that multiplicity of interaction in asynchronous updating is very important for understanding many aspects of emergent collective behavior in animals.

Introduction

Collective animal behavior results from many kinds of interactions among individuals, and interactions vary flexibly according to the situation encountered by individuals (Couzin, 2009; Sumpter, 2006). In self-organization models, behaviors can emerge by tuning some parameters (Couzin, 2009; Haken, 1983; Sumpter, 2006).However, in most cases these various behaviors are monotonic. For example, flocking behavior can form swarms, tori, or polarized groupings by tuning interaction ranges. This is monotonic behavior because it is a variation of a single aspect, in this case alignment (Couzin, 2009). Self-organization has been discussed from many aspects, but it is increasingly important to consider theoretical aspects of the essential diversity of behavior, which sometime seems mutually irrelevant or even contradictory. The emergence of various behaviors in self-organization traces back to the concept of the "subsumption architecture" proposed by Brooks /citepBrooks. Subsumption architecture can be summarized in two aspects, namely, that agents (or robots) never need representations and that the behavior of subjects in a lower layer such as object avoidance finally emerges in higher-layer behavior such as environment searching (Brooks, 1990). Most importantly, multiple simple behaviors ultimately form qualitatively different collective behavior. To account for the above, we propose a new model of self-organization in collective animal behavior using an asynchronous updating method. In our model, each agent uses scent marking at each site passed. Scent markings are often observed in real systems, such as ants, wolves, and other animals (Cornforth et al., 2005; Giuggioli et al., 2011; Lewis and Murray, 1993).

Although the there is a wide range of scent marking behavior in actual phenomena, we can consider their common properties, namely that scent marking gives collective information about where the animal lives (Giuggioli et al., 2011; Lewis and Murray, 1993). Animals thus demonstrate collective decision-making using this information. However, information about locations in scent marking inevitably has a multiplicity of meanings, because the agent's interpretation of the information would rely heavily on the situation. In our model, asynchronous agent actions give many interpretations, or multiplicities, to this scent marking. When each agent interacts with neighbor sites of locations and other agents, our model divides asynchronous updating of agent states into an active phase and a passive phase. In the passive phase, agents only memorize their environment and obey stochastic rules. In the active phase, agents use the memorized state against their environment. As we will show, these two interaction phases, induced by asynchronous updating, exhibit non-trivial collective searching behavior compared with the normal model. The remainder of this paper is organized as follows: In section II, we describe the algorithm of our asynchronous learning model (ALM) and present the effect of two phases of interaction for collective searching behavior. In section III, we introduce "anticipation" for each agent in ALM to develop an asynchronous learning model with anticipation (ALMA). This anticipation incorporates past and future information with current information when an agent decides its next action. We show the anticipation of each agent contribute to sign making, an avoidance, for heterogeneous environments. Finally, we discuss the possibility of collective cognition for changing environments.

Result

Asynchronous Learning Model (ALM)

Our ALM is mainly divided into two parts, site pattern learning induced by asynchronous updates and asynchronous updating with anticipation. These two parts can be considered as one, but division clarifies the effect of these two factors. We define the neighborhood of each agent as a Moore neighborhood, meaning there are eight sites around an agent of interest. There are three site patterns: occupied, scent marked, and vacant. There are therefore $3^8 - 1 = 6560$ neighborhood patterns.

In this section, we examine the effect of pattern learning using asynchronous updating. First, we divide the method of agent interaction into an active phase and a passive phase. Using asynchronous updating means there are two neighbor site patterns where each agent interacts with its environment (Figure 1). One is the case of no other agent around the agent whose turn it is. This case would be common in early turns, or in a low-density environment. The other case is that there are agents at the neighbor sites of the agent whose turn it is. This case is common in late turns, or in high-density environments. These are called active and passive phases, respectively. These two interaction phases have an important place in our agent interaction model.

We next assign different roles to these two phases. In the passive phase, some agents have already occupied the agent's neighbors on its turn. Assuming the agent cannot move to these occupied sites, it must select from the remaining unoccupied sites, which correspond to vacant sites or scent marked sites. Unoccupied sites are selected stochastically. Our model assigns a high selection probability to scent marked sites and a low selection probability to vacant sites. The ratio between these two probabilities, Prob(Vacant)/Prob(Scent), is represented by a parameter μ ($0 < \mu \leq 1$). We will see that a low value for μ results in agents aggregating through the use of their scent marking. Furthermore, each agent learns (or memorizes) the pattern of neighbor sites (Figure 1, Right).

Next, We set properties in the active phase. Agents used stored information corresponding to the current pattern of neighbor sites to the pattern memorized in the passive phase. The current site pattern is therefore replaced with another site pattern, which almost always includes some occupied (or block) sites. Each agent uses this stored information and stochastically selects one site from among the sites that can be moved to. Using stored information thus affects agent interpretation of the environment.

We divide the roles for the active and passive phases, respectively, as using stored information and learning the site pattern. Because asynchronous movement can create timing conflicts, there is always room for multiple interpretations of a given site pattern in one time interval. Note that the asynchronous update is not a random order scheme, but a density order scheme; high-density agent neighborhoods tend to be selected first, and agents with low density move last. We count as one step all agents being updated using density-dependent asynchronicity.

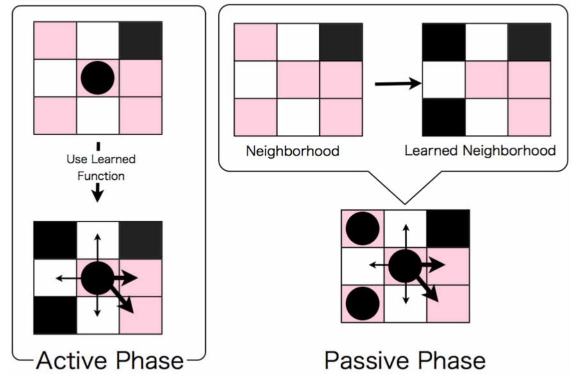

Figure 1: A sketch of the algorithm. Black circles correspond to agents. Each site color shows its state. White corresponds to a void site, pink corresponds to a scent marking site, and black corresponds to an occupied site. Each agent interacts with its neighborhood in an active phase or passive phase. The size of arrows represents the degree of the weight.

Searching Strategy in ALM

Now we examine the effect of multiple site pattern interpretations induced by the asynchronous updating method. To clarify this effect, we introduce a control model constructed using asynchronous updating without site pattern learning in the active phase. In other words, there is no conflict regarding interpretation of the site pattern between the active phase and the passive phase. The parameter $\mu = 0.001$ is the same as in the control model and the number of agents is 150.

Figure 2 shows a distribution of agents on a 35×35 grid for our model and its control. Each color corresponds to the state of the sites. Red corresponds to an agent, pink corresponds to a scent marker, black corresponds to a block (such as an occupied site) and white corresponds to a vacant site. As compared with ALM, agents in the control model clearly form a small aggregation, meaning they are connected by their scent markers. Once agents in the control model form a local group, they hardly move.

In contrast, asynchronously updated agents with learning never broadly cover the space, they instead form aggregations, suggesting that agents with learning connect with each other to effectively search over the entire space, unlike the control model (ALM without learning). Scent markers were originally attractive signs, as we observe in the control model. The asynchronous update with site pattern learning shows an additional ability, collective searching behavior. In fact, Figure 3 shows that mean cover rate of scent marking

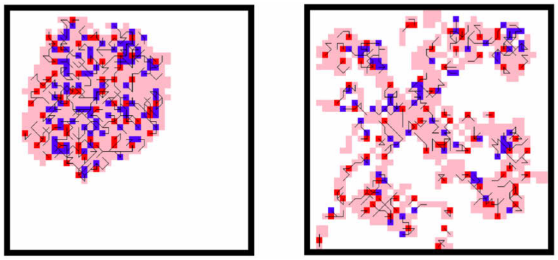

Figure 2: Agent distribution on a 35 × 35 grid. The left figure shows the control model, and the right figure shows ALM. Agent colors correspond to interactions; red agents are in the active phase, and blue agents are in the passive phase. The black tail with each agent represents a trajectory in a few steps.

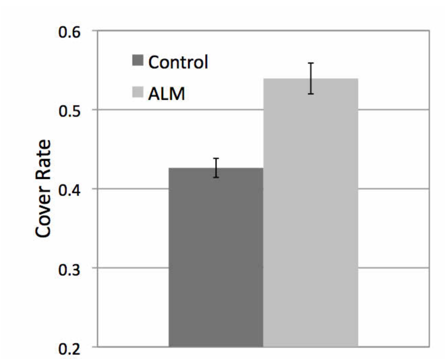

Figure 3: Mean cover rate of scent marking. The ALM value (light gray) is higher than the control model value (gray).

area is larger than the control. This result suggest agent in ALM do more efficient searching than the control model despite the almost same mechanism.

This additional ability is because of the repulsion effect against dense spaces when an agent uses site pattern learning in the active phase. No agent keeps staying in the same group, due to this repulsion effect. In other words, the conflict (or difference) between the active phase and the passive phase allows multiple interpretations of the environment for each agent.

From an ecological perspective, this difference is important. Generally speaking, animals live in groups, which has advantages in reproduction and vigilance against predators(Jackson, 2006; Johnson et al., 2002; Parrish, 1999) . Forming groups is therefore an essential problem for living animals. Group living can also be an issue for individuals, however, because resources are finite and large groups increase the risk of detection by predators. So there is a trade-off between the advantages and risks inherent to group living. Group size optimization is therefore essential for group living (Johnson et al., 2002). Our model suggests that multiple interpretations induced by the asynchronous updating method are a natural connection between group formation and decreasing the risk of large groups. In other words, agents in ALM can decrease their risk of food exhausting because each group never stays the same place. Furthermore, we point out the role of weak connection among small groups. Small groups in weak connections can decrease the risk of detection and group's isolation.

ALMA (ALM with Anticipation)

Now we examine the effect of agent anticipation, defined as incorporating current information with past and future information. The detail of algorithm is listed on **Appendix C**. Current information corresponds to the site pattern of the agent's neighborhood, as discussed concerning the active phase with learning in ALM. To extend this approach, we consider the neighborhood of the neighborhood, in other words all eight neighborhoods of sites reachable in one step. This corresponds to past and future information, because these sites include the site belonged to one step ago, or that which will be belonged to one step from now (see upper left of Figure 4). We can organize these nine site patterns (eight sites for the neighborhood and one for the current agent position). If identical neighborhood patterns exist, we select one of the neighborhoods. These overlapping elements affect the element selection in the next paragraph.

Numbering each neighborhood site (see **Appendix Figure**) enables us to construct a partial order set. Since a partial order set generally never closes with binary operations such as meet and join, this set never constructs a lattice (Davey and Priestelely, 2005). To construct a lattice, we use Dedekind-MacNeille completion(Davey and Priestelely, 2005). Pertinent details from lattice theory are given in **Appendix A and B**. Making a lattice involves constructing a logical relationship, closed under a binary operation, among current, past, and future information.

We then stochastically select an element from the lattice. After selection of an element from the lattice, we next take an ideal from this element and construct a congruence of lattices (Davey and Priestelely, 2005). We applied this method in past research related to topics such as species evolution (Niizato and Gunji, 2013). Roughly speaking, congruence of the lattice means making well-defined groupings on the lattice. This grouping is strongly bound by its lattice structure. We use the group that contains an element of the current information. Since this grouping is also an ordered set, we can certainly pick the top element from this group as a representative element (see upper right of Figure 4). This representative element carries past, current, and future information. Using this method, agents sometimes behave as if they have learned about unlearned situations in the active phase, making the conflict between active and passive phases larger than in ALM.

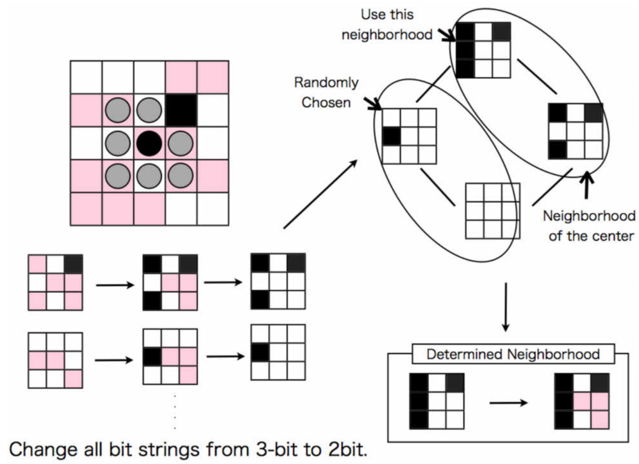

Figure 4: Constructing the lattice. Considering the neighborhood of the neighborhood, we can pick up nine 8-bit strings. The overlapped element will affect the selection of an element, which makes an ideal.

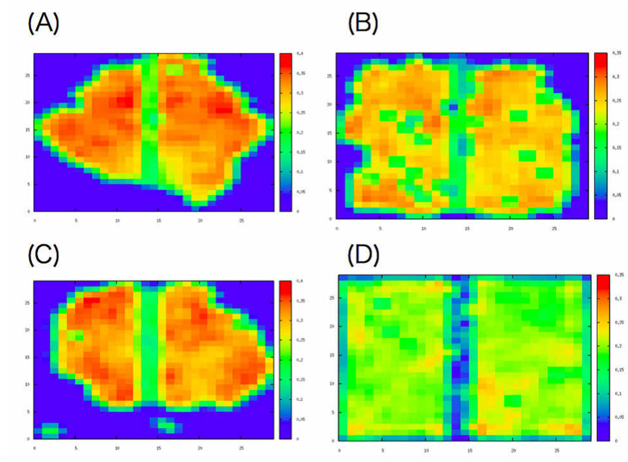

Figure 5: The distribution of density probability for four ways of interactions, the control (upper left), ALM (upper right), the control with anticipation (lower left), and ALMA (lower right). The gird is 30×30 and the number of agent is 150.

Sign Making in ALMA

Anticipation of agents contributes to sign making (avoidance sign). We set the environment as in Figure 6 (A). At the center of the boundary, blocks and scent markings are distributed alternately. The first set of scent markings lasts until the end. The region in a given space is divided as Side I and Side II. These blocks prevent free movement between the two sides, but the existence of scent markings attracts agents around the center. In other words, the centerline of this space provides a contradiction for each agent. Do these factors affect the behavior of agents?

To examine the effect of anticipation in this heterogeneous environment, we compare four interaction patterns, namely, the control model, ALM, the control model with anticipation, and ALMA. Figure 5 shows the distribution of the density probability for these four interaction patterns. Obviously, agents of the both of control models (Figure 5 (A) and Figure 5 (C)) concentrate around the blocks, even after many steps have passed. At first glance the behaviors of ALM (Figure 5 (B)) and ALMA (Figure 5 (D)) seem similar, but there is a radical difference between them; in ALMA the density distribution around the centerline is very low compared with ALM and both the control models. This behavior suggests that each agent in ALMA avoids center blocks, and shows territorial behavior in both regions. In other words, each agent in ALMA recognizes a sign of avoidance.

This result is also supported by statistical results (Figure 6(B)). We compared the mean maximum spending time at Side I or Side II with four patterns of interaction (100 times for 100,000 steps). ALMA contains many agents which spend at one side for long time. It is worth to point out that there is almost no difference between the models without anticipation (ALM and the control model). This result suggests that anticipation helps agents recognize signs of avoidance. Furthermore, Figure 6 (B) also suggests that learning in the active phase in ALMA plays an important role for sign making. Because there are many collisions around center blocks, the neighborhood in ALMA contains many occupied sites when the agent is in the active phase. However, this is insufficient for sign making. There are too many neighborhood patterns (6560 patterns!), and since there are conflicts among the past, current, and future information, the constructed lattice would tend to make a modular lattice, meaning that each element has many complementary elements (two elements satisfy $a \vee b = 1$ and $a \wedge b = 0$). High modularity leads to selection of the top lattice element when we apply the congruence method to the lattice. In this sense, agent's anticipation is driving-force for sign-making in heterogenous environments.

Collective Cognition in ALMA

In this section, we discuss the possibility of collective cognition in changing environments. Collective cognition is defined here as a collective response against a changing environment. Cognition, including human cognition, inevitably involves the surrounding situation. We sometimes find differences even between situations that seem to be the same. One famous example is Rubin's vase, a black and white optical illusion. Focusing on one color in the illustration, one sees a vase. Focus on the other, and one sees opposing faces instead. The stimulus from viewing this picture must be the

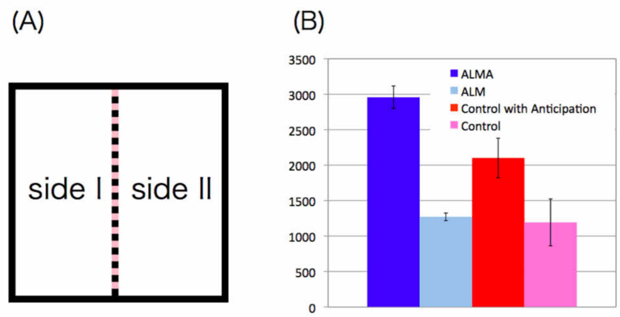

Figure 6: (A) The right side of the center is Side I. The left side of center is Side II. (B) The mean maximum spending time in Side I or Side II. The four bars correspond to the control model (pink), ALM (light blue), the control with anticipation (red), and ALMA (blue).

same, but cognition differs regarding its appearance. Although this is an extreme example, different recognition of the same situation is an essential factor of cognition. Different cognition regarding the same object allows agents multiple interpretations that provoke different reactions against the same object. In this sense, we can discuss an agent's collective cognition in heterogeneous environments.

To examine the possibility of collective cognition in ALMA, we examine the behavior of agents when an environment temporally changes. Here, a changing environment is shown as a changing number of blocks at the centerline, as in the previous section. There are two patterns, additions and subtractions (Figure 7 (A)). Since adding blocks means that blocks suddenly appear in space and removing blocks means gradually removing a wall, these changes are radically different from a cognitive standpoint. We measured the behavioral differences between them as taking the mean density probability around the centerline.

Figure 7 (B) shows the result. The triangles correspond to removed blocks and the rectangles correspond to added blocks. The horizontal line is the rate of blocks in the center, so zero on the horizontal line means there are no blocks in the space. The rate of blocks gradually increases or decreases 0.1 points every 1,000 steps. We performed 100 trials and averaged the value for each block count. Figure 7 (B) suggests that transitions of the density probability are different when adding or removing blocks. The shapes of Figure 7b indicate hysteresis, a property that recalls Harken (1983), a famous study that showed human cognition changes over time.

The density probability around the center gradually decreases with an increased number of blocks. When block rate decreases from 1, however, the density probability drops from 1.0 to 0.5, then increases from 0.5 to 0.0. These differences come from the agent learning in the active phase.

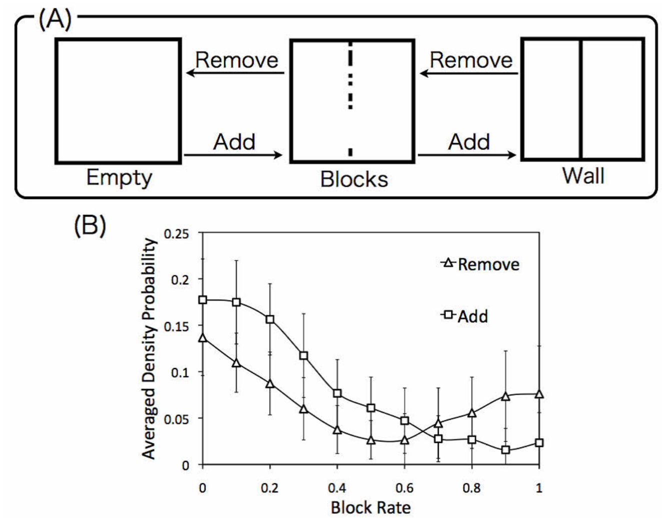

Figure 7: (A) Adding or removing blocks at the center. Three blocks are added or removed every 1,000 steps. (B) The rate of mean density probability around the center with block rates. The triangle corresponds to removing blocks and the rectangle corresponds adding blocks.

Each agent recognizes added blocks as ones that should be interpreted in the active phase. When removing blocks, however, the agent starts to recognize blocks as a part of wall. This wall in itself is not subject to interpretation for each agent, because information about the wall never changes with time. Interpreting blocks as a wall or not a wall changes the agent's behavior.

Discussion

Self-organization has been discussed by many researchers. To briefly summarize the main assertion of self-organization, it is "simple local interaction that creates global behavior." In real systems, however, this is insufficient to understand emergent phenomena. For example, when we consider the evolution of life, it must have started from simple forms and behaviors. We should therefore ask how simple behaviors evolve into complex ones. A related question would ask about the origin of various higher-layer behaviors. It is considered that the concept of self-organization would answer this question.

However, we have to admit that the concept of self-organization contains a serious problem if we seek the origin of various higher-layer behavior. There are examples even in cellular automata. Class IV automata have computational abilities using patterns of time evolution as particles (Wolfram, 2005). Although Class IV automata can exhibit higher-layer behavior, there is a clear distinction between the system as a device and its modelers. In other words, the computational property of a cellular automaton never explicitly emerges unless the modelers (or the observer) set the appropriate initial conditions.Even when one high-layer behavior (universal computation in Class IV) emerges, the theory of the self-organization inevitably contains a kind of this problem.

To solve this division problem between the modelers and the systems, we proposed multiplicity of interaction. Introducing multiplicity to simple interaction allows its own interpretation of situation. We notice that multiple interpretations of situations must be distinct from simple hybrid models, which contain many kinds of interactions because the base interaction is consistently single. Having multiple interpretations of a single interaction, the usage of the interaction would be open to many applications without interference from modelers when agents encounter different situations.

In our model, a single interaction corresponds to the attractiveness of scent markers. The multiplicity of interaction was constructed from three factors: asynchronous updating, site pattern learning in the active phase, and anticipation. The latter two concepts originally come from disagreement of timing among agent actions induced by asynchronous updating. Adding the latter two properties, the discrepancy between the active and passive phases becomes larger than the original one. Our model suggests that the discrepancy, induced by asynchronous updating, becomes the driving force of many applications for single interaction in various environments.

We showed that our model could connect various kinds of behaviors, which are sometimes seen as contradictory or irrelevant to each other. First we observed that agents under ALM could search more efficiently than agents under the control model. We discussed that this result had relevance with the ecological searching trade-off problem. Furthermore, agents under ALMA recognize signs of avoidance for the center blocks. Although the behavioral range of agent covers the entire space, each agent avoids the centerline, behaving as if marking territories where they belong. Searching an area means spreading over its entire space. Marking territories means establishing boundaries. These two collective behaviors are qualitatively conflicting, but can be implemented by introducing multiplicity to the interaction without any contradiction. Collective cognition, which we discussed in the last section, is clearly a kind of behavior different from searching behavior. Collective cognition means that agents in ALMA recognize the difference between two ways of changing the environment (adding or removing blocks from the center). Our result suggested that each agent distinguishes between discrete blocks and a wall of blocks.

In this sense, we can conclude that multiplicity of interaction could be open in its usage with various environments. We have confirmed different kinds of global behaviors—searching, sign marking, and collective cognition—by setting various environments using this multiplicity. In other words, to consider how multiplicity of interaction connects the multiplicity of behaviors. If we take this multiplicity of interaction, the degree of the multiplicity would become one possible measure of self-organization of emergent phenomena. Then we can ask the origin of complex behaviors on living systems.

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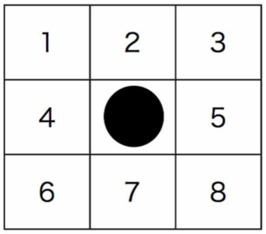
Appendix A

We briefly introduce the lattice theory for unfamiliar readers of the lattice theory.

Definition 1 (*Partial Order*) Let P be a set. An order on P is a binary relation $\leq$ on P such that, for all $x, y, z \in P$.
(i) $x \leq x$



Appendix Figure : The way of numbering the neighborhood.

(ii) $x \leq y$ and $y \leq x$, then $x = y$
(iii) $x \leq y$ and $x \leq y$, then $x \leq z$

We denote a partially ordered set by the pair, $(P, \leq)$. For example, a set of bit (binary) strings can construct a partial order. A bit string $a_1 a_2 \ldots a_n$ is a finite sequence of zero, one ($a_i \in \{0, 1\}$). An order between two bit strings such as $a_1 a_2 \ldots a_n$ and $b_1 b_2 \ldots b_n$ is defined by $a_1 a_2 \ldots a_n \leq b_1 b_2 \ldots b_n$ if $a_i \leq b_i$ for all i. We use a set of bit strings in this study. However, A partial order is not a lattice. Then we define the meet and the join. We define the join " $\vee$ " and the meet " $\wedge$ " of two elements x and y in P. The join can be defined by $x \vee y = sup\{x, y\}$ when it exists. The join can be defined by $x \wedge y = inf\{x, y\}$ when it exists. The notation of $sup(inf)$ means the lowest (greatest) upper bound of $\{x, y\}$ in P.

Definition 2 (*Lattice*) Let $(P, \leq)$ be a non-empty partially ordered set. If $x \vee y$ and $x \wedge y$ exist for all $x, y \in P$, then $(P, \leq)$ is called for a lattice.

Definition 3 (*Ideal*) Let $(L, \leq, \wedge, \vee)$ be a lattice. A non-empty subset of J is called an ideal if
(i) $x, y \in J$ implies $x \vee y \in J$,
(ii) $x \in L, y \in J$ and $x \leq y$ imply $x \in J$.

Definition 4 (*Congruence on a Lattice*) Let $(L, \leq, \wedge, \vee)$ be a lattice. Let an equivalence relation on L be $\theta = \{< x, y >\in L \times L\}$such that any $x, y, z \in L$,
(i) $< x, x >\in \theta$
(ii) $< x, y >\in \theta \Leftrightarrow < y, x >\in \theta$
(iii) $< x, y >\in \theta$ and $< y, z >\in \theta \Rightarrow < x, z >\in \theta$
We also denote $< x, y >\in \theta$ as xy (mod θ). Then an equivalence relation is a congruence on L, if for any $x, y, z, w \in L$, ($x \equiv y$ (mod θ) and $z \equiv w$ (mod θ)) $\Rightarrow$ ($x \vee z \equiv y \vee w$(mod θ) and $x \wedge z \equiv y \wedge w$ (mod θ)).

More detail in Davey and Priestelely in 2005.

Theorem (*Reconstruction of a Lattice from a Quotient Lattice*) Let L be a lattice and$f$ be a natural quotient map such as $f : L \rightarrow L/\theta$. For the binary relation derived from an ideal $J \subseteq L$, there exists a filter $K \subseteq L$ such that $[x]_{\theta(J)} = f^{-1}(x)$, where for any $x \in K$, $f^{-1}(x) :=\downarrow x - \cup_{y \in K, y<x} \downarrow y$
Proof) See in (Gunji et al., 2006; Niizato and Gunji, 2013).

Appendix B

To construct the lattice from a partial order set, we use Dedekind-MacNeille completion, as follows: In the partial order set P, we can take a lower and upper closed set, which we represent as P^u (upper) and P^l (lower). The upper set is $P^u = \{x | y \leq x$ for any $y \in S\}$. The lower set P^l is the dual of P^u.

Theorem (*Dedekind-MacNeille completion*) For any order set P, we can construct the set as follows:
$\mathbf{DM}(P) = \{A \subseteq P | A^{ul} = A\}$

Generally, a set that satisfies $A^{ul} = A$, is called a "cut". To understand the meaning of Dedekind-MacNeille completion, the following lemma is useful.

Lemma For any ordered set P, the following statements are satisfied.
(i) For any $x \in P$, $(\downarrow x)^{ul} =\downarrow x$.
(ii) For any $A \subseteq P$ and $\bigvee A \in P$, then $A^{ul} =\downarrow (\bigvee A)$.

Appendix C

Here we describe the detail of the algorithm of ALM and ALMA. This model consists of N agents moving in discrete time and in a grid space. Numbering neighbor sites as **Appendix Figure**, we can define 3-bit strings as $\bar{a} = a_1 a_2, \ldots, a_n$. We also define the neighborhood of the neighborhood in ALMA as eight 3-bit strings $\bar{b_1}, \bar{b_2}, \ldots, \bar{b_8}$. First, agents are randomly distributed in a given space. The algorithm is constructed as follows. i is a tag of the agent. Each agent is chosen asynchronously in density dependent.

For($\exists j \in \{1, 2, \ldots, 8\}$ $a^i_j$is occupied){
    $Memo^i(\bar{a^i}) = \bar{a'^i}$

    Select the moving site from a set S.
    S :=$\{j | a'^i_j = 0$ or $a'^i_j = 1\}$
    The selection probabilities are;
    Prob($a'^i_j = 0$) = $\mu / \sum_j Weight(a'^i_j)$
    Prob($a'^i_j = 1$) = $1 / \sum_j Weight(a'^i_j)$
}
For($\forall j \in \{1, 2, \ldots, 8\}$ $a^i_j$is not occupied){

    If(Using ALM){
        $\bar{a'^i} \Leftarrow Memo^i(\bar{a^i})$

        Select the moving site from a set S.
        S :=$\{j | a'^i_j = 0$ or $a'^i_j = 1\}$
        The selection probabilities are;
        Prob($a'^i_j = 0$) = $\mu / \sum_j Weight(a'^i_j)$
        Prob($a'^i_j = 1$) = $1 / \sum_j Weight(^{\prime i}_j)$
    }

    If(Using ALMA){
        $\bar{b^i_0} \Leftarrow Memo^i(\bar{a^i})$

        Pick up the neighborhood of neighborhood of i; $\bar{b^i_1}, \bar{b^i_2}, \ldots, \bar{b^i_8}$ respectively.
        For($0 \leq j \leq 8$){
            For($1 \leq k \leq 8$){
                If($\bar{b^i_{j,k}} = 1$)$\bar{b^i_{j,k}} = 0$
                Else If($\bar{b^i_{j,k}} = 2$) $\bar{b^i_{j,k}} = 1$
            }
        }

        Make a partial order set P from $\{b^i_0, b^i_1, \ldots, b^i_8\}$.
        $P := \{s_1, s_2, \ldots, s_m\}$
        $m (\leq 9)$ is the number of elements of P.

Count the overlapping elements.
$Num(s_1) = n_1, \ldots, Num(s_m) = n_m$

Make a lattice L from P
Using **DM** completion.
$L = \{s_1, s_2, \ldots, s_m, s_{m+1}, \ldots, s_{m+l}\}$
l is the number of adding elements.

Select one element from L.
The selection probability of each element is,
Prob(s_k)=$Num(s_k)/\sum_j Num(s_j)$

Make a congruence by using selected element s_k.
Then select the top element in the congruence.
Mathmatically, when J is an ideal,
$s''^i \Leftarrow \bigvee\{s_j \in L | s_j \in [s_0]_{\theta(J)}\}$

For$(1 \leq j \leq 8)\{$
If$(s''_j = 1$ and $b^0_j = 1)$ $a'^i_j = 2$
Else If$(s''_j = 0$ and $b^0_j = 0)$ $a'^i_j = 0$
Else $a'^i_j = 1$
$\}$

Select the moving site from a set *S*.
$S := \{j | a'^i_j = 0$ or $a'^i_j = 1\}$
The selection probabilities are;
Prob($a'^i_j = 0$) = $\mu/\sum_j Weight(a'^i_j)$
Prob($a'^i_j = 1$) = $1/\sum_j Weight(a'^i_j)$
$\}$
$\}$

$Memo^i(-)$ means the store information about site pattern for each agent i. $Num(-)$ means the number of overlapping element in $\{b^i_0, b^i_1, \ldots, b^i_8\}$.