

## Robust Optimization of Adjustable Control Factors Using Particle Swarm Optimization

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### Abstract

Most conventional robust design methods assume design solutions are fixed values. Using these methods, designers set each control factor to a fixed value to maximize the robustness of objective characteristics. However, fluctuations in the objective characteristic often exceed the allowable range in a design problem. Consequently, obtaining sufficient robustness is difficult using conventional methods.

This research defines adjustable control factors whose values can be adjusted within a given range to increase robustness and proposes a method to calculate robustness, including factors to adjust the objective characteristic and to derive optimum ranges of the factors. The robustness index, which indicates the feasibility that the objective characteristic values are within the tolerance by the adjustment, is calculated by the Monte Carlo method, while the range of adjustable control factors is optimized using the Vector evaluated particle swarm optimization. Finally, an engineering example is presented to demonstrate the applicability of the proposed method.

### Introduction

Robust design aims to ensure product performance robustness against fluctuant factors, such as user characteristics and material properties, by deriving the optimum (unique) value of the design parameter (design solution). Due to globalized markets and material procurement, robust design has received much attention, and many robust design methods have been proposed (Matsuoka 2010). Some methods evaluate robustness of the objective characteristic using an orthogonal array for efficiency (Sundaresan et al. 1991; Taguchi, 1993; Yu and Ishii, 1998ab), while others derive robustness using the objective characteristic values calculated via a Taylor series approximation (Arakawa and Yamakawa, 1995; Belegundu and Zhang, 1992; Emch and Parkinson 1994; Parkinson et al. 1993, 1995; Ramakrishnan and Rao, 1996; Zhu and Ting, 2001). Additionally robustness has been calculated as the feasibility of the objective characteristic being within the tolerance to consider the objective characteristic distribution (Eggert and Mayne, 1993; Watai et al. 2009).

In most conventional methods, designers set control factors to fixed values to maximize the robustness. In cases where the objective characteristic distribution is smaller than the tolerance (Figure 1a), these methods can derive a design solution (optimized control factor values)  $x_0$  with sufficient

robustness. However, in cases where the objective characteristic distribution is larger than the tolerance (Figure 1b), a solution to sufficiently maximize robustness cannot be obtained. In such cases, the control factors must be adjusted to ensure robustness. In other words, as the values of the control factors are varied, the whole of the objective characteristic distribution should be located within the tolerance (Figure 1c).

The concept of adjusting the factors originates from Taguchi's method (Taguchi, 1993). In this method, the control factors are set to minimize the objective characteristic fluctuation, and then the designer selects a tuning factor, which has a negligible effect on the fluctuation, to minimize the difference between the nominal value of the objective characteristic and its target value. Otto (Otto and Antonsson, 1993) assumed the tuning factor is adjusted after the fluctuation of objective characteristic. Hence, after the objective characteristic fluctuates, the factors are altered to minimize the difference between the fluctuated objective characteristic values and their target values. Otto proposed a method to evaluate the robustness using the expected value of the objective characteristic adjusted by the tuning factors. This method provided a new concept of robustness in which an adjustment improves the robustness and it helps relax the design requirements such as dimension tolerance and material property tolerance. However, Otto's method is not applicable to design problems where the designer chooses the tuning factors and their adjustable ranges because these parameters are preliminarily set in Otto's method. This is a common problem in mechanical design. For example, to design a seat, the designer must determine which adjustable mechanism, such as seat reclining mechanism, and its adjustable range to apply to the seat. Because a method to address such a design problem has yet to be proposed, the designer must determine these parameters using his/her personal design experience.

This research proposes a method to derive the optimum range for the adjustable factors which are chosen adequately to improve the robustness of the objective characteristics. This paper is divided as follows. Section 2 presents definitions and terminologies. The proposed robustness index, calculation method, and range optimization of the factors using the Vector evaluated particle swarm optimization (VEPSO) are described in Section 3. Section 4 illustrates an application of the proposed method to a seat design problem, while Section 5 provides conclusions and the future research direction.

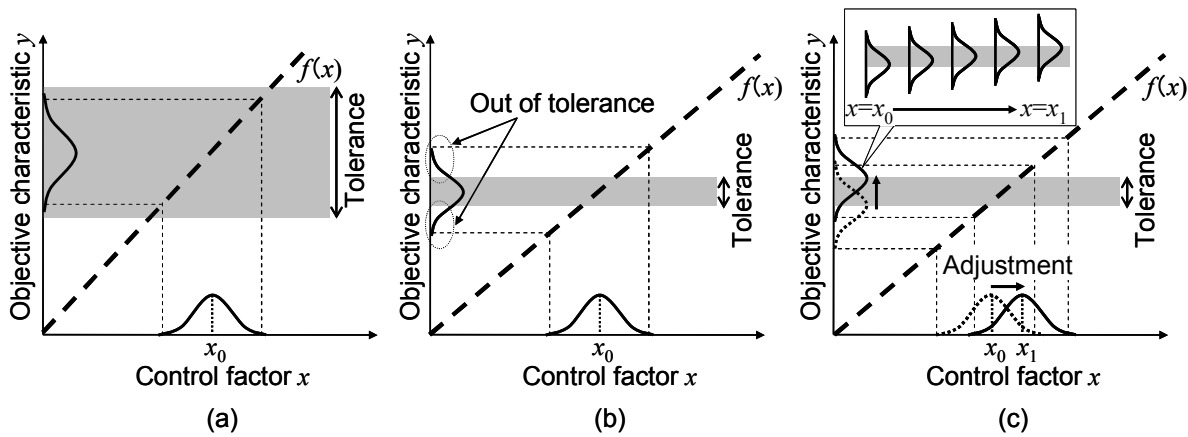


Figure 1: Conceptual illustration of a design problem that includes an factor whose value is adjustable elements)

### Definitions and terminologies

In robust design, objective characteristic (product performance)  $y$  fluctuates according to fluctuant factors (control factors  $x$  and noise factors  $z$ ). Although the values of control factors fluctuate, designers can set their nominal values, but not those of noise factors. In this paper, adjustable control factors (hereinafter called ACFs) whose values  $t$  can be adjusted in the adjustable range  $[t_l, t_u]$  anytime while using or manufacturing the product to maintain the objective characteristic are newly defined.  $t_l$  and  $t_u$  are the lower and upper values of ACFs, respectively. ACFs are similar to the tuning factors defined in Otto's method (Otto and Antonsson, 1993) with respect to their adjustment, but the adjustable ranges of ACFs and tuning factors differ (i.e., the designer defines the ranges of the ACFs).

The concept of robustness in this research is defined below. If the tolerance of objective characteristic  $[y_l, y_u]$  exists as shown in Figure 1, then ACFs can be adjusted to locate each fluctuation value of the objective characteristic within the tolerance. Hence, the robustness index for ACFs ( $R_A$ ) is defined as the feasibility that the objective characteristic values are within the tolerance at least once, by the adjustment of ACFs. Using ACFs and  $R_A$ , design problems in this research are expressed as:

$$\begin{aligned} &\text{Find } [t_l, t_u], \mathbf{x} \\ &\text{to maximize } R_A( y(= f(\mathbf{x}, \mathbf{z}, \mathbf{t})) ) \\ &\text{to minimize } |t_u - t_l| \end{aligned} \tag{1}$$

where  $f$  is the objective function. To prevent an unnecessary expansion of the ACF ranges, which increases the production costs and failure rate, this formulation does not only maximize  $R_A$ . Minimizing the size of the range described in Equation (1) is an example of preventing an unnecessary expansion because other factors (e.g., the form and location of the range) can lead to the aforementioned issues.

Below are definitions and descriptions of the terminologies used in this paper.

*Objective characteristic* ( $y = f(x, z, t)$ ): The characteristic to express the function of the design objective, and is calculated by objective function  $f$ .

*Control factors* ( $\mathbf{x} = \{x_i\}, i=1, 2, \dots, n_x$ ): Factors whose nominal values are set by the designer, but fluctuate the objective characteristic.  $n_x$  represents the total number of control factors.

*Noise factors* ( $\mathbf{z} = \{z_i\}, i=1, 2, \dots, n_z$ ): Factors that fluctuate the objective characteristic, but their nominal values cannot be set by designers.  $n_z$  expresses the total number of noise factors.

*ACFs* ( $\mathbf{t} = \{t_i\}, i=1, 2, \dots, n_t$ ): Control factors with nominal values that can be adjusted within their adjustable ranges.  $n_t$  denotes the total number of ACFs.

*Adjustable range of ACFs* ( $[t_u, t_l]$ ): The range, determined by the designer, where the ACFs are adjustable.

*Assignable points of ACFs* ( $\{t_j\}, j=1, 2, \dots, n_{ap}$ ): The combinations of ACFs' values that can be varied to be within the adjustable range.  $n_{ap}$  denotes the number of ACFs' assignable points.

*Robustness index ( $R_A$ )*: Index to evaluate the robustness of the objective characteristics with regards to the ACFs adjustment.

### Robust design method for ACFs

#### A robustness index for ACFs

In conventional robust design methods, robustness indices are approximated to improve the calculated efficiency. For example, the index in Ramakrishnan's method (Ramakrishnan and Rao, 1996), which is the weighted sum of the mean value and standard deviation of the objective characteristic, is calculated using a Taylor series approximation. However, approximated values significantly differ from the actual values or cannot be derived in the cases where the followings are not satisfied: (1) the objective characteristics monotonically increase or decrease with respect to the factors, (2) the objective function is differentiable, (3) the fluctuations in the factors are sufficiently small, and (4) the factors are independent from each other.

The proposed robustness index must be calculated accurately because the adjustable range must be minimized, as shown in

Equation (1). In other words, using an accurate robustness index, the designers must set the adjustable range as small as possible. Consequently, the Monte Carlo method, which derives accurate values but is time consuming, is applied to calculate the index. The calculation methods is described below.

All ACFs' assignable point values should be used to calculate  $R_A$ . Specifically, sets of objective characteristic fluctuations that satisfy the tolerance are derived with respect to all assignable point values  $t_j$ , as shown in Figure 2.  $R_A$  is calculated as the ratio of the sum of the sets of fluctuant combinations of  $x$  and  $z$  where at least one of the objective characteristic values  $y_j$  derived from  $t_j$  is within the tolerance as shown in Equation (2).

$$R_A = P \left[ \bigcup_{j=1}^n \{ C(x, z) | y_1 \leq f(x, z, t_j) \leq y_u \} \right], \quad (2)$$

where the square bracket expresses a set of  $C(x, z)$  where the objective characteristic is located within the tolerance by adjusting ACFs. This means  $R_A$  is the rate of the set and the entire set. The assignable point values are expressed as a finite number of discontinuous values  $t_j$  because  $R_A$  is calculated using the Monte Carlo method. The number of the assignable (discontinuous) values should be sufficient to assume the ACF is continuous. However, the number should be decreased if the calculation amount is too large. To calculate  $R_A$ , first,  $s$  random combinations of the control and noise factors are generated based on their probability density functions. Second, objective characteristic  $y_i$  is calculated using the generated random combinations  $\{x_i, z_i\}$  ( $i=1, 2, \dots, s$ ) and all the assignable point values. That is, the number of calculating objective characteristic values is the product of the random combination number  $s$  and the assignable points numbers of ACFs. Finally, the values calculated from each random combinations of  $x_i$  and  $z_i$  are assessed to determine whether at least one of the calculated values is within the tolerance (i.e., at least one assignable point which consists an objective characteristic value that satisfies the tolerance). Then  $R_A$  is calculated as:

$$R_A = \frac{1}{s} \sum_{i=1}^s M_i$$

$$\left( M_i = \begin{cases} 1 & (\exists t \in \{t_j\}; y_1 \leq f(x_i, z_i, t) \leq y_u) \\ 0 & (\text{otherwise}) \end{cases} \right) \quad (3)$$

$$j = 1, 2, \dots, n_{ap}$$

**Adjustable range optimization**

This study proposed a optimization algorithm using the VEPSO in order to solve the design problem of Equation 1. An outline of the VEPSO and the algorithm using it are described below.

**Outline of VEPSO.** The VEPSO (Vlachogiannis and Lee, 2005, 2009) is an improved method of the PSO (Kennedy and Eberhart, 1995) that is one of the representative

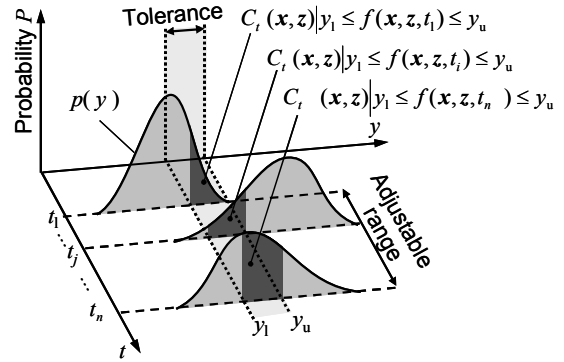


Figure 2: Set of the objective characteristic fluctuations used to calculate the robustness index

metaheuristics, in order to handle the multi objective optimization problems. The PSO imitates the movement of organisms in a bird flock or fish school and searches a solution using the information both from the individuals (particles) and their swarm. The VEPSO assigns an objective to each of swarms and searches a solution using the information inside or between swarms. The location vector (i.e. design variables) of the  $i$  th particle in the  $j$  th swarm  $x_i^{[j]}$  is updated as follows:

$$x_i^{[j]}(T+1) = x_i^{[j]}(T) + v_i^{[j]}(T), \quad (4)$$

where,  $T$  is the number of iterations.  $v$  is the velocity vector to direct the particles to the updated locations and is calculated as:

$$v_i^{[j]}(T+1) = k [ w v_i^{[j]}(T) + c_1 r_1 \{ x_{pb,i}^{[j]}(T) - x_i^{[j]}(T) \} + c_2 r_2 \{ x_{gb,i}^{[s]}(T) - x_i^{[j]}(T) \} ]$$

$$\left( s = \begin{cases} M & \text{if } j = 1, \\ j-1 & \text{if } j = 2, 3, \dots, M \end{cases} \right), \quad (5)$$

where,  $M$  is the number of swarms;  $c_1$  and  $c_2$  are the parameters to express the degree of incidence of the private best location of each particle  $x_{pb}$  and the global best location  $x_{gb}$ , respectively;  $r_1$  and  $r_2$  denote the random numbers uniformly distributed in  $[0, 1]$ .  $w$  is the parameter to define the effect of the current velocity vector and decreases based on  $T$  as shown in the following equation:

$$w(T) = w_{max} - \frac{w_{max} - w_{min}}{T_{max}} \cdot T, \quad (6)$$

where,  $w_{max}$  and  $w_{min}$  are the maximum and minimum value of  $w$ .  $T_{max}$  is the maximum number of the iterations.  $k$  denotes the parameter relating the convergence performance and expressed as follows:

$$k = \frac{2}{\left| 2 - \varphi - \sqrt{\varphi^2 - 4\varphi} \right|} \quad (\varphi = c_1 + c_2). \quad (7)$$

As shown in Equation (5), the velocity vectors are defined using the global best locations of the different swarms. This enables the solution search based on the information from the other swarms and the global locations (solutions) of the swarms to approach each other. Therefore, the solution search

of the VEPSO has the following features: assigning a objective to each of swarms and searching the solutions located close to each other. These features generate the following merits to solve the robust design problem (Equation (1)): 1) the design solution (assignable points) to ensure the robustness can be efficiently derived by assigning the robustness regarding the part of the fluctuation of the factors to each of the swarm; 2) the distance between assignable points (the adjustable range) can be smaller.

**Procedure to derive optimum adjustable range using VEPSO.** This study amended the robustness index (Equation (2)) in order to evaluate each assignable point that assures the robustness regarding the part of the fluctuation of the factors. The robustness assured by the  $j$  th assignable point is expressed as follows:

$$R_j = P[C(x, z)_j | y_l \leq f(x, z, t_j) \leq y_u] \tag{8}$$

where,  $C(x, z)_j$  ( $\in C(x, z)$ ) is a part of the fluctuation of the factors assigned to the  $j$  th assignable point and holds the following equation:

$$\sum_j C(x, z)_j = C(x, z) \tag{9}$$

When considering the four assignable points, the four swarms are defined and search for the adjustable range (assignable points) based on the VEPSO procedure. The optimization algorithm using the VEPSO is described in Figure 3. In this algorithm, the parameters of the VEPSO (e.g.  $c$ ,  $w$ ,  $T_{max}$ , etc) are firstly set. Next, the number of the assignable points is decided and the same number of the swarms are set. The locations of the particles are updated based on the objective (robustness)  $R_j$  ( $j=1, 2, \dots, n_{ad}$ ). The update of the locations iterates until  $T = T_{max}$ , and the global best location of the swarms are derived as an design solution (adjustable range).

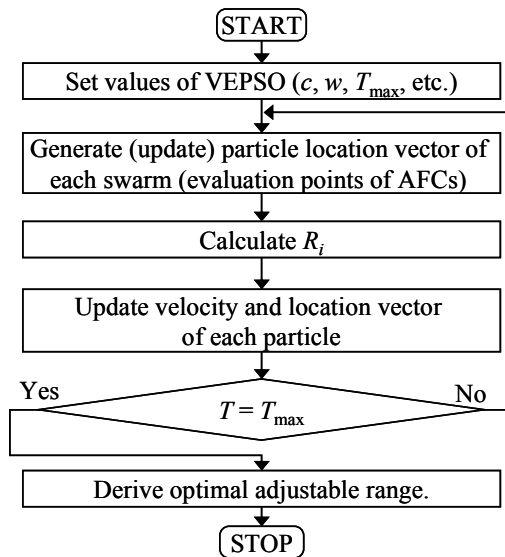


Figure 3: Proposed algorithm of robust design method

## Illustrative example

### Problem description

To demonstrate the proposed robust design method, we applied it to a seat design for railway vehicles because numerous people with diverse physiques and sitting postures use these seats. However, the conventional seat design typically assumes an average physique and posture. Thus, designing a seat that is robust for various physiques and postures is desirable. Herein the design objective focused on the hip-sliding force, which is generated on the buttocks by the static instability of the upper and lower body masses, causing discomfort when sitting (Matsuoka 2000). Therefore, the design objective is to inhibit the hip-sliding force for various physiques and postures.

Table 1 defines the objective characteristic and factors of this design. The control factors, seat cushion angle (hereafter called C.A.), seat back angle (B.A.), and forward tilt angle of the upper seat back (F.A.), can be adjusted by the mechanisms for the seat cushion forefront lifting function, reclining function, and forward tilt function, respectively. Previous research (Matsuoka 1988) has demonstrated the influence of these angles on the hip-sliding force. Therefore, these angles are considered ACFs. Noise factors include users' physiques and sitting postures. The physiques are defined based on actual measurements of Japanese citizens (National Institute of Bioscience and Human Technology, 1996). Additionally, we considered three sitting postures: a standard sitting posture where the lumbar region is in contact with the seat back, a stretched waist sitting posture where the waist is stretched and slid forward from the standard sitting posture, and a bent waist sitting posture where the waist is bent and slid forward from the standard sitting posture. The ratio of these sitting postures is 3:1:6 (Matsuoka, 2000).

Modeling the sagittal plane of the human body and the seat was used to derive the objective function (Figure 4). The human model assumes that the movements of the low momentum joints are zero and is consequently, composed of four high momentum joints. On the other hand, the seat model, constructed based on the existing seat found on a 485 train (Hatsukari), is divided into three parts: seat cushion, upper seat back, and lower seat back, which are rigidly linked. Figure 5 shows the objective functions (the formula to compute the hip-sliding force for each posture) derived based on these models. The robustness index ( $R_A$ ) is derived as the weighted sum of the indices calculated using these formulae.

Additionally, in the PSO, the parameters (e.g.  $c$ ,  $w$ , etc) are important for the convergence or the computational efficiency. Therefore, this study implemented some optimizations regarding the recommended values of the parameters in the conventional studies and compared the results to clarify the proper values of them. This study focused both on  $c_1$  and  $c_2$  and conducted the four analyses using the parameter combinations: ( $c_1=2.80$ ,  $c_2=1.30$ ) recommended by Carlisle (Carlisle and Dozier, 2001); (2.05, 2.05) suggested by Kennedy (Kennedy, 1998); (1.55, 2.55) and (1.05, 3.05) that are smaller values of  $c_1$ . This study also implemented the optimization using the traditional genetic algorithm (GA). The definition of the parameters is summarized in Table 2.



Items	Definition
Hip-sliding force (as objective characteristic)	Tolerance of Hip-sliding force: -10 to 20 N.
Seat cushion angle $\theta_C$ Seat back angle $\theta_B$ Forward tilt angle $\theta_F$ (as ACFs)	$10 \leq \theta_C \leq 25$ $20 \leq \theta_B \leq 35, \theta_C + 10 \leq \theta_B$ $0 \leq \theta_F \leq 30$
Body height $L$ Body height $M$ Sitting posture (as noise factors)	$L$ and $M$ are normal distributions Mean value of $L$ : 1.65m , standard deviation of $L$ : 0.08m Mean value of $M$ : 58.1kg , standard deviation of $M$ : 9.09kg Ratio of standard, stretched waist, and bent waist sitting postures: 3:1:6

Table 1: Definition of objective characteristic and factors

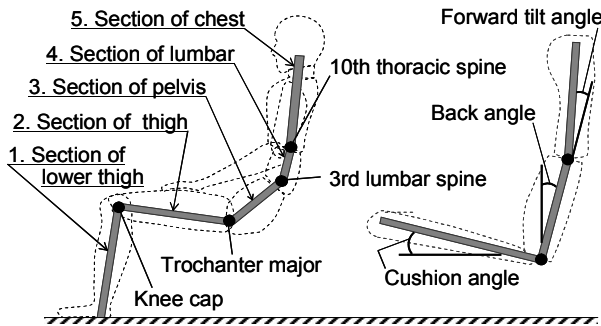


Figure 4: Model of the human body and seat

Items	Set value				
	GA	Proposed method (Analysis)			
		1	2	3	4
Tolerance of $y$		$-10 \leq y \leq 20$			
Feasible area of $t_1$		$10 \leq \theta_C \leq 25$			
Feasible area of $t_2$		$20 \leq \theta_B \leq 35, \theta_B \geq \theta_C + 10$			
Feasible area of $t_3$		$0 \leq \theta_F \leq 30$			
Max iteration number $T_{max}$	10000	100			
$c_1$	—	2.8	2.05	1.55	1.05
$c_2$	—	1.3	2.05	2.55	3.05
$w_{min}$	—	0.4			
$w_{max}$	—	0.9			
Number of assignable points	2	2 (number of swarms)			
Swarm size	—	20			
Solution number		5			

Table 2: Definition of parameters

**Result**

The design solutions (adjustable ranges) derived by the proposed method using the different parameters and by the GA are shown in Figure 6. Additionally, the Euclidean distance between assignable points  $D$  is calculated in order to compare the size of the adjustable range. The average and the standard deviation of the distance are indicated as shown in Table 3. Figure 6 and Table 3 show the adjustable ranges derived by the proposed method are smaller than and assure as same robustness as those by the GA. Particularly, the mean value and the standard deviation of the adjustable ranges derived in analysis 2 are small. This means the values of the parameter ( $c_1 = c_2 = 2.05$ ) are suitable to minimize the adjustable range. This is caused by the two features of the VEPSO: 1) the larger  $c_2$  prevents the global solution search same as the PSO; 2) the larger  $c_2$  encourages the swarms to search the area close to each other. Because of the trade-off relationship between the two features, the same degree of incidence is compromised to be an optimum value in the proposed method.

**Conclusions**

In this research, ACFs, which can be adjusted within a given range to increase the robustness, were defined. Additionally, a method to calculate the robustness  $R_A$ , including the objective characteristics adjustment by the factors and to derive an optimum range of the factors is proposed.  $R_A$  indicates the feasibility that the objective characteristic values are within the tolerance at least once by the adjustment of ACFs. A calculation methods for the index, which uses the Monte Carlo method, are proposed. In contrast, the range of ACFs is optimized by the Vector evaluated particle swarm optimization. In the procedure,  $R_A$  is used to evaluate the particles in several swarms, and each particle searches for the optimum adjustable range of ACFs.

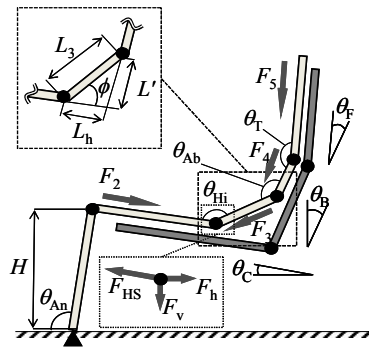
The proposed method was applied to an engineering example (seat design problem). In this application, it was confirmed that the proposed method can derive the design solution with high robustness and small adjustable range.

**Acknowledgement**

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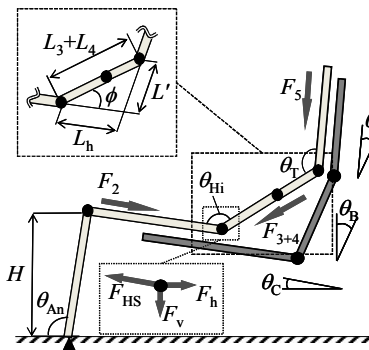
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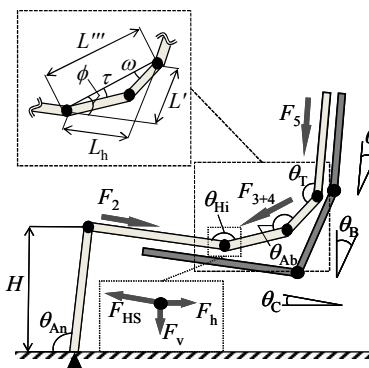
$$\begin{aligned}
 F_{HS} &= -F_h \cos \theta_C - F_v \sin \theta_C - \kappa(-F_h \sin \theta_C + F_v \cos \theta_C) \\
 \left. \begin{aligned}
 F_h &= F_2 \cos \theta_C - F_3 \sin(\theta_{Hi} + \theta_C) \\
 F_v &= F_2 \sin \theta_C + F_3 \sin(\theta_{Hi} - \theta_C) + M_2 l_{2b} g + M_3 l_{3a} g \\
 F_2 &= \frac{M_1 l_{1b} g + M_2 l_{2a} g}{\sin \theta_C - \cos \theta_C \tan \theta_{An}}, \quad F_3 = \frac{F_{4-5} + (M_4 l_{4a} g + M_3 l_{3b} g)(\cos \theta_B - \kappa \sin \theta_B)}{-\cos \theta_{Ab} - \kappa \sin \theta_{Ab}} \\
 F_4 &= (M_5 l_{5a} g + M_4 l_{4b} g)(\cos \theta_B - \kappa \sin \theta_B) + F_5(\cos \theta_F - \kappa \sin \theta_F), \\
 F_5 &= M_5 l_{5b} g \{\cos(\theta_B - \theta_F) - \kappa \sin(\theta_B - \theta_F)\} \\
 \theta_{An} &= \sin^{-1}(H/L_1), \quad \theta_{Hi} = 180^\circ - \phi, \quad \theta_{Ab} = \phi + 90^\circ - \theta_C + \theta_B - \theta_F, \\
 \phi &= \sin^{-1}\{(L'/L_3) \sin(90^\circ + \theta_B - \theta_C)\} \\
 L' &= L_h \cos(90^\circ + \theta_B - \theta_C) + \sqrt{L_3^2 - L_h^2 \sin^2(90^\circ + \theta_B - \theta_C)}
 \end{aligned} \right\}
 \end{aligned}$$

(a)



$$\begin{aligned}
 F_{HS} &= -F_h \cos \theta_C - F_v \sin \theta_C - \kappa(-F_h \sin \theta_C + F_v \cos \theta_C) \\
 \left. \begin{aligned}
 F_h &= F_2 \cos \theta_C + F_{3+4} \cos(\theta_{Hi} + \theta_C), \\
 F_v &= F_2 \sin \theta_C + F_{3+4} \sin(\theta_{Hi} + \theta_C) + M_2 l_{2b} g + (M_3 + M_4) l_{ma} g \\
 F_2 &= \frac{M_1 l_{1b} g + M_2 l_{2a} g}{\sin \theta_C - \cos \theta_C \tan \theta_{An}}, \quad F_{3+4} = \frac{F_5 + (M_5 l_{5a} g + (M_3 + M_4) l_{mb} g)(\cos(\theta_B - \theta_F) - \kappa \sin(\theta_B - \theta_F))}{-\cos \theta_T + \kappa \sin \theta_T} \\
 F_5 &= M_5 l_{5b} g (\cos(\theta_B - \theta_F) - \kappa \sin(\theta_B - \theta_F)), \\
 \theta_{An} &= \sin^{-1}(H/L_1), \quad \theta_{Hi} = 180^\circ - \phi, \quad \theta_T = \phi + 90^\circ - \theta_C + \theta_B - \theta_F, \\
 \phi &= \sin^{-1}\{(L'/(L_3 + L_4)) \sin(90^\circ + \theta_B - \theta_C)\} \\
 L' &= L_h \cos(90^\circ + \theta_B - \theta_C) + \sqrt{(L_3 + L_4)^2 - L_h^2 \sin^2(90^\circ + \theta_B - \theta_C)}
 \end{aligned} \right\}
 \end{aligned}$$

(b)



$$\begin{aligned}
 F_{HS} &= -F_h \cos \theta_C - F_v \sin \theta_C - \kappa(-F_h \sin \theta_C + F_v \cos \theta_C) \\
 \left. \begin{aligned}
 F_h &= F_2 \cos \theta_C + F_{3+4} \cos(\theta_{Hi} + \theta_C - \tau), \\
 F_v &= F_2 \sin \theta_C + F_{3+4} \sin(\theta_{Hi} + \theta_C - \tau) + M_2 l_{2b} g + (M_3 + M_4) l_{m'a} g \\
 F_2 &= \frac{M_1 l_{1b} g + M_2 l_{2a} g}{\sin \theta_C - \cos \theta_C \tan \theta_{An}}, \quad F_{3+4} = \frac{F_5 + (M_5 l_{5a} g + (M_3 + M_4) l_{m'b} g)(\cos(\theta_B - \theta_F) - \kappa \sin(\theta_B - \theta_F))}{-\cos(\theta_T - \omega) + \kappa \sin(\theta_T - \omega)} \\
 F_5 &= M_5 l_{5b} g (\cos(\theta_B - \theta_F) - \kappa \sin(\theta_B - \theta_F)), \\
 \theta_{An} &= \sin^{-1}(H/L_1), \quad \theta_{Hi} = 180^\circ - \phi + \tau, \quad \theta_T = \phi + 90^\circ - \theta_C + \theta_B - \theta_F + \omega, \\
 \phi &= \sin^{-1}\{(L'/L''') \sin(90^\circ + \theta_B - \theta_C)\} \\
 L' &= L_h \cos(90^\circ + \theta_B - \theta_C) + \sqrt{(L''')^2 - L_h^2 \sin^2(90^\circ + \theta_B - \theta_C)}, \\
 L''' &= \sqrt{L_3^2 + L_4^2 - 2L_3 L_4 \cos(180^\circ - 24^\circ)}
 \end{aligned} \right\}
 \end{aligned}$$

(c)

$\theta_C$  : Seat cushion angle,  $\theta_B$  : Seat back angle,  $\theta_T$  : Forward tilt angle,  $\theta_{Hi}$  : Hip angle,  $\theta_{Ab}$  : Abdomen angle,  $\theta_{An}$  : Ankle angle,  $\kappa$  : Coefficient of frictional resistance,  $H$  : Seat cushion height (400mm),  $F_i$  : Force on  $i$  th human body section,  $F_{HS}$  : Hip sliding force,  $F_h$  : Horizontal force on trochanter major,  $F_v$  : Vertical force on trochanter major,  $L$  : Body height,  $L_i$  : Length of  $i$ th body section,  $L_h$  : Buttock-trochanterion length,  $M$  : Body weight,  $M_i$  : Weight of  $i$ th body section,  $l_{ia}$  : Ratio of  $L_i$  and the length from  $i$ th body section upper-edge to gravity-center,  $l_{ib} : 1 - l_{ia}$ ,  $l_{ma}$  : Composite ratio of 3rd and 4th body section in stretched waist sitting posture,  $l_{m'a} : l_{ma}$  in bent waist sitting posture

$$\left. \begin{aligned}
 (L_1 = 0.2880L - 0.0424, \quad L_2 = 0.0027L + 0.4057, \quad L_3 = 0.3274L - 0.2908, \quad L_4 = 0.0609L + 0.0356, \quad L_5 = 0.0930L - 0.0549, \\
 L_h = 0.3118L - 0.4113, \quad M_1 = 0.12M, \quad M_2 = 0.2M, \quad M_3 = 0.14M, \quad M_4 = 0.18M, \quad M_5 = 0.36M \\
 l_{1a} = 0.61, \quad l_{2a} = 0.43, \quad l_{3a} = 0.11, \quad l_{4a} = 0.11, \quad l_{5a} = 0.35, \quad l_{ma} = 0.329, \quad l_{m'a} = -0.608(L_3 + L_4) + 0.579)
 \end{aligned} \right\}$$

Figure 5: Hip-sliding force estimation equations

Method	$c_1, c_2$	$R_A$	$D$	$\sigma_D$
Conventional method	—	0.996	17.7	4.07
Proposed method (Analysis)	1 (2.8, 1.3)	0.998	3.92	9.66
	2 (2.05, 2.05)	0.999	0.23	0.79
	3 (1.55, 2.55)	0.999	0.49	1.41
	4 (1.05, 3.05)	0.999	1.21	4.36

Table 3 Result of analyses

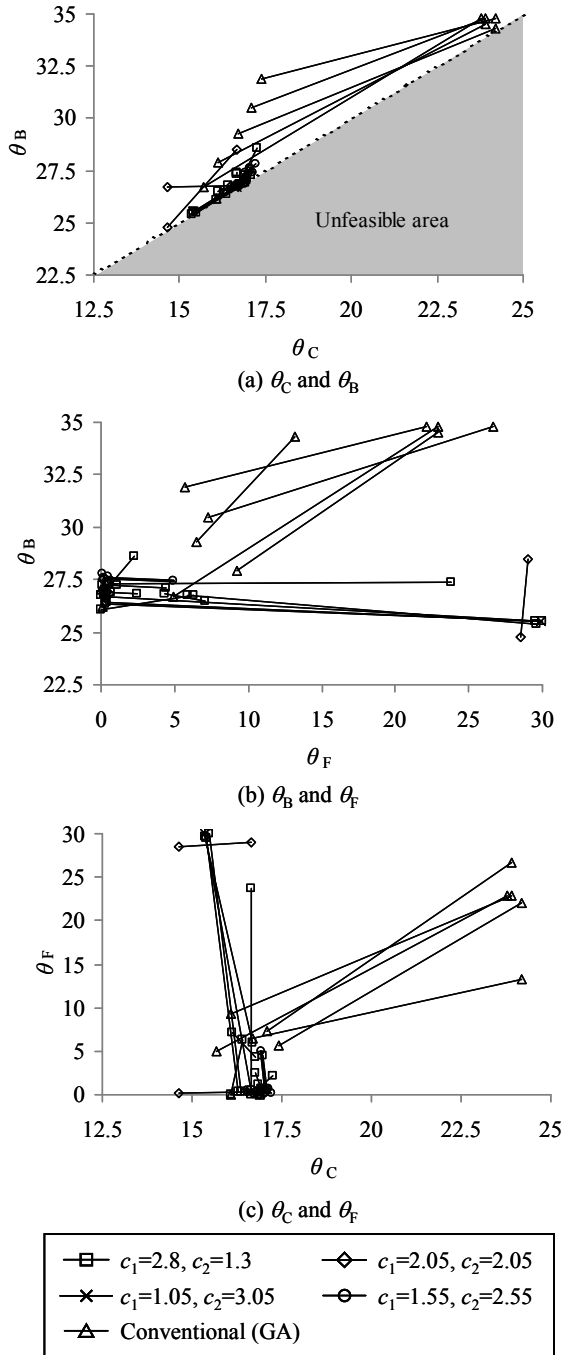


Figure 6: Solution candidates of each analysis

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