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Artificial Causal Space-Time

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Abstract

If a space-time with a causal relationship is viewed from an observer in space-time, the interaction between space-time and the observer has to be implemented. We describe this interaction using a pair of causal sets and its semantics or using a pair of Point and Open Logic. We here propose an artificial causal space-time called an evolutionary topological system, which is based on the changeability between logical operations (disjunction and conjunction) and logical elements (join and meet) of a causal set. The conflict resulting from the interaction is locally and temporally removed by replacing disjunction and join (or conjunction and meet), and a causal set is verified to evolve to a particular logical structure based on the simple summation. We also show this model can design an abnormal space-time feeling, such as an out-of-body experience.

Introduction

One of the most intriguing and important models for subjective and/or cognitive time was proposed by a philosopher, McTaggart (1908). He evaluated two model types, called the A series and B series. The B series consists of events that are linearly ordered and designated by “before” and “after”. The A series consists of past, present and future events that cannot co-exist and are exclusive of each other. Although McTaggart himself argued that neither the A nor B series can be a model for time, the A and/or B series are still used as models for time in philosophy (Grey, 1997; Mellor, 1998; Gunji et al., 2009).

Although the original A and B series appear to be too speculative to be considered models for time, a pair of the A and B series can be utilized as a causal set (Bombelli et al., 1987) and its semantics in the field of quantum mechanics (Markopoulou, 2000), independent of philosophy. The B series corresponds to a causal set defined by a partially ordered set. The A series corresponds to a sieve in the semantics of a causal set. Thus, the idea of the A and B series is taken as a causal relationship and can be argued in quantum gravity (Klugry and Sepanina, 2011).

A causal set that serves as a model for causal relationships in space-time is a given for an observer. It is assumed that an observer living in a space-time passively observes, memorizes and recalls a series of events and calls that set of events the past, present or future, depending on the location of the observer. Although Markopoulou introduced the stance of an observer who observers a space-time internally, the interaction between an observer and space-time was not described. The role of an observer still remains.

We here propose an artificial causal space-time in which an observer moving in a space-time can interact with space-time itself. Why does the interaction occur? Independent of the idea of a causal set, Vickers (1996) proposed the generalized idea of a causal set and its semantics in the form of a topological system. In this framework, logical operations are defined in a causal set, called Point Logic, and in its semantics, called Open Logic. Because a generalized causal set and its semantics are related to each other by a particular binary relationship, they are restricted with respect to each other. It entails a conflict between Point and Open Logic that has to be resolved, which is why the interaction between Point Logic (in a causal set) and Open Logic (in its semantics) can occur.

Against a conflict, Vickers introduced a limited logical operation that is found as ubiquitously as Scott topology (Scott, 1976). In other words, his solution to resolve a conflict results from the stance of an observer who sees a space-time externally. Once his solution is introduced, a conflict never occurs. Therefore, in principle, there is no interaction between an observer and space-time.

Our artificial causal space-time is a dynamic causal set equipped with a particular rule by which perpetually generated conflicts between a causal set and its semantics are discarded locally and temporally. Even if a local consistency is generated, removing the local conflict can generate another local conflict. This dynamic evolves a causal set toward a particular system in which the cause-effect relationship can be calculated by summing up the causes. We also show that the interaction of Point and Open logic can generate a particular subjective sensation called an “out-of-body experience (OBE)” that seems be differ from the OBE previously reported (Ehrsson, 2007; Lenggenhager et al., 2007). Our model suggests how to create new sensations and emotion in subjective space-time.

Causal sets and Topological System

Imagine a series of events in time, such as $\ldots\rightarrow x_1\rightarrow x_2\rightarrow x_3\rightarrow\ldots$, and you are at event x_2. The future and past of x_2 can be expressed as the set of $\{.., x_1, x_2\}$ and $\{x_2, x_3, \ldots\}$, respectively, and the present at event x_2 is expressed as the intersection of the future and past as $\{x_2\}$. This idea is the essence of the B and A series presented by McTaggart (Gunji et al., 2010).

A causal set is a set of events defined as a partially ordered set. Its semantics is a collection of, for example, all possible futures. This pair can be a generalized pair of the B and A series. In addition, the introduction of the relationship between a causal set and its semantics derives motivation for the interaction between a causal set and its semantics.

Causal set and its semantics

Causal set and space-time. A causal set consists of separable events. Each event can be connected to another event via a directed edge without loops. If two events are connected by two edges that have different directions, they are equivalent to each other. Thus, this particular directed network can be expressed as a partially ordered set (POS) (Davey and Priestley, 2002). If an event and a directed edge are expressed as an alphabet and $\leq$, respectively, the POS satisfies the following: (i) $a \leq a$, (ii) $a \leq b$ and $b \leq a$ implies $a=b$, and (iii) $a \leq b$ and $b \leq c$ implies $a \leq c$.

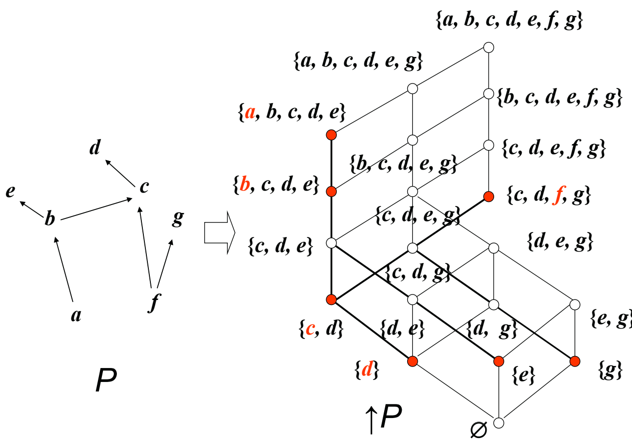

Figure 1. A causal set, P, and its semantics, $\uparrow P$. In a causal set, partial order, $\leq$, is expressed as an arrow. In $\uparrow P$ an element of $\uparrow P$ (i.e., a subset of P) is expressed as a circle. If $A \subseteq B$, they are connected by a line with upper B and lower A. A filled circle represents $\uparrow p$ with p in P.

Some terminologies are also added. Any elements a and b in a POS, P, are anti-chain with each other if neither $a \leq b$ nor $b \leq a$ holds. For any subsets, $Q \subseteq P$ join of Q, denoted by $\vee Q$, is defined such that for any $q \in Q$, $q \leq \vee Q$ and if $q \leq s$, then $\vee Q \leq s$. In particular, if Q is a two-element set such as $\{a, b\}$, $\vee\{a, b\}$ is represented by $a \vee b$. Similarly, the meet of Q, denoted by $\wedge Q$, is defined such that for any $q \in Q$, $q \geq \wedge Q$ and if $q \geq s$, then $\wedge Q \geq s$. If Q is a two-element set such as $\{a, b\}$, $\wedge\{a, b\}$ is represented by $a \wedge b$. Given a partially ordered set, P, if for any $x, y \in P$, $x \wedge y$, $x \vee y \in P$, then P is called a lattice.

Given a POS, P, for any $p \in P$, the future of P is defined by $\uparrow p=\{x \in P \mid p \leq x\}$. The semantics of P is a collection of the possible unions of all $\uparrow p$ for any element p in P. Thus, it is defined by $\uparrow P=\{\uparrow Q \mid Q \subseteq P\}$ where $\uparrow Q=\{y \in P \mid (\exists x \in Q) y \geq x\}$. Fig. 1 shows an example of a causal set, P, and $\uparrow P$.
In P, the future of b is expressed as $\uparrow b=\{b, c, d, e\}$. Similarly, $\uparrow c=\{c, d\}$, and then $\uparrow c \subseteq \uparrow d$. Any elements other than $\uparrow p$ (with p in P) in $\uparrow P$ are expressed as a union of the $\uparrow p$'s (with p in P), such as $\{b, c, d, e, f, g\}=\uparrow b \cup \uparrow f$. Tracing the filled circle in $\uparrow P$, one can see that the ordered structure of P is embedded in $\uparrow P$.

In the context of a causal set, there is no discussion about the relationship between a causal set and its semantics. The relationship is introduced in the context of formal logic, independent of the idea of a causal set. It is called a topological system.

Topological system. Given a set S, if a collection of subsets of S satisfies an axiom of opens, it is called a topology or topological space. An axiom of opens is the following: (i) S and empty set are opens, (ii) a finite intersection of opens is an open, and (iii) a union of opens is an open. A power set that is a collection of all subsets of S is the densest topology, and a collection consisting only of S and empty is the sparsest topology. Topology is a type of metric that can be used to recognize a set, S.

Because topology is constrained under a particular axiom of opens, Vickers (1996) attempted to generalize a topology in the form of a binary relationship between a collection of points and a collection of opens, which he called a topological system. A collection of points and opens is defined by a set and a locale. A topological system is defined by a triplet, $\langle P, L, R \rangle$, where P is a set, L is a locale and R is a binary relationship between P and L. A locale is a partially ordered set that is closed with respect to union (disjunction), $\cup$, and finite intersection (conjunction), $\cap$, and that satisfies the distributive law, such that for any a, b, and $c \in L$, $a \cap (b \cup c)=(a \cap b) \cup (a \cap c)$. It is trivially true that a triplet $\langle P, \uparrow P, \in \rangle$ also satisfies the definition of a topological system.

Because a locale contains logical operations, we can define logical operations for opens. What about points? Imagine an observer moving in a causal set, P. He can manipulate logical operations for an element of $\uparrow P$, which is derived from a basic assumption in which an observer discriminates an event from other events and recognizes a set created from P. Thus, he can also manipulate a logical operation for events. It is, therefore, reasonable that a logical operation for points is also introduced.

Point Logic and Open Logic

Definition of Point and Open Logic. Given a causal set, its semantics and the binary relationship between them, such as the triplet $\langle P, L, R \rangle$, Vickers (1996) introduced logical operations in both P and L. For S, a subset of a local, logical operation, conjunction (AND) $\cap$ and disjunction (OR) are defined by

$$xR\cap S :\Leftrightarrow \quad (\forall a \in S)\, xRa \qquad (1)$$
$$xR\cup S :\Leftrightarrow \quad (\exists a \in S)\, xRa\ . \qquad (2)$$

Because a locale is a generalization of an open set in topological space, logic in a locale is called Open Logic.

Similarly, for T, a subset of P, conjunction and disjunction are defined by

$$\cap TRa :\Leftrightarrow \quad (\forall x \in T)\ xRa \qquad (3)$$
$$\cup TRa :\Leftrightarrow \quad (\exists x \in T)\ xRa\ . \qquad (4)$$

Because an element of a set is a point, logic in P is called Point Logic. These logical operations can also be defined in $<P, \uparrow P, \in>$ by using $\in$ instead of R.

Conflict between Point and Open Logic. Although Point Logic is defined for a point and Open Logic is defined for a set, logical operations are defined in the same manner, and they are related by a binary relationship, which entails a conflict between Point and Open Logic. The question arises as to how we can resolve this conflict.

Let us consider a causal system, $<P, \uparrow P, \in>$. Given a partially ordered set P, the followings two are trivially true:

$$a \cup b \in \{a\}, \qquad a \cup b \in \{b\}. \tag{5}$$

Because the left-hand term means a or b even if the right-hand set contains either a or b, then the statement holds. Thus, statement (5) holds because of definition (4). Statement (5), $a \cup b \in \{a\}$ and $a \cup b \in \{b\}$, means that $a \cup b$ belongs to both $\{a\}$ and $\{b\}$. In other words, $(\forall a \in S)\ x \in a$ where $x = a \cup b$ and $S = \{\{a\}, \{b\}\}$. Due to definition (1), we obtain

$$a \cup b \in \{a\} \cap \{b\}. \tag{6}$$

However, statement (6) also means that ($a \in \{a\}$ and $a \in \{b\}$) or ($b \in \{a\}$ and $b \in \{b\}$); this statement thus never holds. This type of statement results from the conflict between Point and Open Logic.

A solution proposed by Vickers is to restrict an operation of disjunction. In statement (6), disjunction is applied to a set $\{a, b\}$, which results in $\cup\{a, b\} = a \cup b$ in the left-hand term. That result entails a conflict. In Vicker's solution, disjunction can be operated only to a directed set, D, which is defined by the following: for any $x, y \in D$, there exists $z \in D$ such that $x \leq z$ and $y \leq z$. Note that $\{a, b\}$ is not a directed set because a and b are anti-chains for each other. Why are they anti-chains? If $a \leq b$ or $b \leq a$, the right-hand set can be replaced by $\{a, b\} \cap \{b\}$ in the case of $a \leq b$, or $\{a\} \cap \{a, b\}$ in the case of $b \leq a$, because any sets in the right side of $\in$ have to be an element of $\uparrow P$.

Because $a \leq a \vee b$ and $b \leq a \vee b$, $\{a, b, a \vee b\}$ is a directed set. In considering the case in which a join $a \vee b$ exists and $\cup\{a, b, a \vee b\}$, the right-hand term in (6) has to be replaced by $\{a, a \vee b\} \cap \{b, a \vee b\}$. Therefore, we obtain

$$a \cup b \cup a \vee b \in \{a, a \vee b\} \cap \{b, a \vee b\}. \tag{7}$$

Thus, at least one of a, b or $a \vee b$ belongs to both $\{a, a \vee b\}$ and $\{b, a \vee b\}$, which is why statement (7) holds. When disjunction is applied only to a directed set, this particular disjunction is called a directed disjunction and is represented by $\cup^{\uparrow}$. Thus, definition (4) in a causal system $<P, \uparrow P, \in>$ is replaced by

$$\cup^{\uparrow} T \in a :\Leftrightarrow \quad (\exists x \in T)\ x \in a, \tag{8}$$

where T is a directed set. Inversely, conjunction in Point Logic related to disjunction in Open logic entails a conflict that can be resolved by directed disjunction in Open Logic.

This type of solution to resolve a conflict is the construction of a one-to-one correspondence between Point and Open Logic by discarding parts that cannot be correspondent with each other. Although one-to-one correspondence is achieved, logical operations are restricted and used incompletely. Alternatively, we here intend to propose a solution to resolve the conflict in which logical operation is not restricted.

Causal dynamics with local consistency

How can we resolve the conflict between Point and Open Logic? Instead of introducing directed disjunction, we here introduce the changeability of join and disjunction and of meet and conjunction. Actually, if we can replace disjunction with join in P whenever we use disjunction, the conflict between Point and Open logic can be resolved. For example, considering statement (6), one can obtain

$$a \vee b \in \{a, a \vee b\} \cap \{b, a \vee b\}. \tag{9}$$

Thus, by manipulating a and b, there exists a join of a and b, and $a \cup b$ can be replaced by $a \vee b$. The conflict can be resolved by this changeability of disjunction and join and of conjunction and meet, which is why the changeability can be interpreted as a local consistency between Point and Open Logic.

The changeability of conjunction and meet can be verified, but that of disjunction and join cannot be proved. To create a causal system P that satisfies the changeability of disjunction and join, P is locally modified by a particular rule that is a local modification based on the dynamics of a causal system. First, we show the verification of the changeability.

Changeability of logical operation and element in *P*

Conjunction and meet. Because each element of $\uparrow P$ is expressed as an upper set of P, the changeability of conjunction and meet is expressed via $\uparrow x$ with $x \in P$.

Proposition 1 (Changeability of conjunction and meet)
Given a topological system $<P, \uparrow P, \in>$, for any $a, b, x \in P$, $a \cap b \in \uparrow x \Leftrightarrow a \wedge b \in \uparrow x$ if there exists $a \wedge b$ for a and b.

Proof. (i) Assume $a \cap b \in \uparrow x$. It means that $a \geq x$ and $b \geq x$ and then x is a lower bound for $\{a, b\}$. Because of the meet, the greatest lower bound $a \wedge b$ is larger than x, $a \wedge b \in \uparrow x$. We verified $a \cap b \in \uparrow x \Rightarrow a \wedge b \in \uparrow x$.
(ii) Assume $a \wedge b \in \uparrow x$. We obtain $a \wedge b \geq x$. Because $a \geq a \wedge b$ and $b \geq a \wedge b$, we obtain $a \geq x$ and $b \geq x$, which means $a \cap b \in \uparrow x$.

Proposition 2 (Semi-changeability of disjunction and join)
Given a topological system $<P, \uparrow P, \in>$, for any $a, b, x \in P$, $a \cup b \in \uparrow x \Rightarrow a \vee b \in \uparrow x$ if there exists $a \wedge b$ for a and b.

Proof. Assume $a \cup b \in \uparrow x$. It means that $a \geq x$ or $b \geq x$. Because $a \vee b \geq a$ and $a \vee b \geq b$, $a \vee b \geq x$ always holds. Thus, we obtain $a \vee b \in \uparrow x$.

The inverse of proposition 2, in which $a \vee b \in \uparrow x \Rightarrow a \cup b \in \uparrow x$, never holds. A counterexample is given by a partially ordered set, $\{a, b, x, a \vee b\}$, where $a \vee b \geq x$ and a, b and x are anti-chain with each other. Although $a \vee b \in \uparrow x$ holds in this partially ordered set, $a \geq x$ or $b \geq x$ never holds because they are anti-chains. Thus, $a \vee b \in \uparrow x \Rightarrow a \cup b \in \uparrow x$ does not hold in general. Therefore, the changeability of disjunction and join never holds in any partially ordered set.

Thus, we define a particular dynamic by which the changeability of disjoin and join is locally implemented.

Dynamical system for local consistency

We implement a dynamic for the changeability of disjunction and join and simulate the time development of an evolutionary causal set. For this purpose, we define a causal set consisting of binary sequences.

Definition 3 (Causal set of binary sequences) A causal set, P, of binary sequences consists of n bits of sequence, $\boldsymbol{a}$=<$a_1a_2\ldots a_n$> where each a_k=0 or 1 for any $k\in\{1, 2, .., n\}$. The order is $\boldsymbol{a}\leq\boldsymbol{b}$ if $a_k\leq b_k$ for any $k\in\{1, 2, .., n\}$.

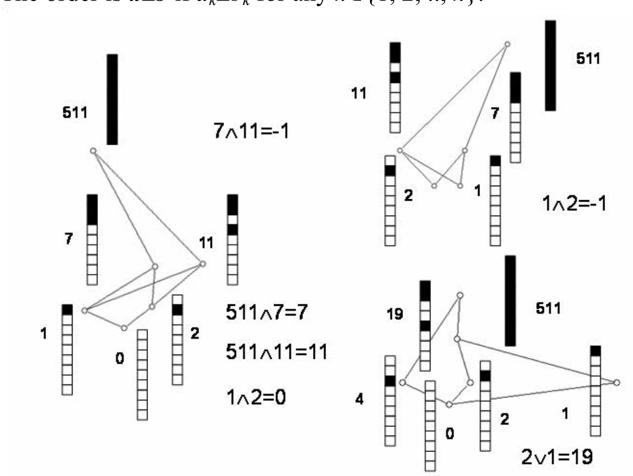

Figure 2. Some examples of a causal set of binary sequences. Each element represented by a binary column is expressed as a decimal number. If there is no join or meet, -1 is returned.

Fig. 2 shows three examples of a causal set of binary sequences. Only the right lower set is a lattice, and the others are not. A binary column represents a binary sequence, where the black and white squares represent 1 and 0 digits, respectively. A decimal number is representation of a binary sequence. Join and meet for a two-element set are shown as an example.

Notice that join is not a union of binary sequences and that meet is not intersection. Here, we denote a union by the symbol $\oplus$. For a pair of binary sequences, $\boldsymbol{a}$ and $\boldsymbol{b}$, $\boldsymbol{a}\oplus\boldsymbol{b}$=<$\ldots a_k+b_k\ldots$> where 0+0=0, 0+1=1+0=1+1=1. If a union $\oplus$ is applied to {1, 2} in the casual set in the lower right of Fig. 2, we obtain $1\oplus2=3$. Because there is no binary sequence represented by 3 in this causal set, we obtain $1\vee2=19$. Similarly, meet is not an intersection. Here, we denote intersection by the symbol $\otimes$. For $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{a}\otimes\boldsymbol{b}$=<$\ldots a_k\times b_k\ldots$> where 0×0= 0×1=1×0=0, 1×1=1. If an intersection is applied to {11, 7} in the upper right of Fig. 2, $11\otimes7=3$ is obtained. However, because there is no 3, we then obtain $11\wedge7=-1$ (i.e., there is no meet).

Given a causal set of binary sequences, we introduce the dynamics of a causal set and define an evolutionary topological system.

Definition 4 (Evolutionary topological system) An evolutionary topological system is defined by <P^t, $\uparrow P^t$, $\in$, F>, where P^0 is an initial causal set of n bit sequences that are randomly given. The time development of a causal system is defined by

$$P^{t+1}=F(P^t) \tag{10}$$

where $F(P^t)$ is defined by the following:

(i) Randomly choose two elements $\boldsymbol{a}$ and $\boldsymbol{b}$ from P^t and obtain $\uparrow\boldsymbol{a}$ and $\uparrow\boldsymbol{b}$ from $\uparrow P^t$.

(ii) If the statement $\boldsymbol{a}\cap\boldsymbol{b}\in\uparrow\boldsymbol{a}\cup\uparrow\boldsymbol{b}$ does not hold, then calculate $\boldsymbol{a}\wedge\boldsymbol{b}$.

(iii) If $\boldsymbol{a}\wedge\boldsymbol{b}$ does not exist, calculate $\boldsymbol{a}\otimes\boldsymbol{b}$, and then each bit with a 1 value is replaced by 0 or nothing happened with an equal probability. When this replacement is denoted by RAND_1, we add the new element of

$$\boldsymbol{a}\wedge\boldsymbol{b}=<\ldots\mathrm{RAND}_1(a_k\times b_k)\ldots> \tag{11}$$

to P^t.

(iv) If the statement $\boldsymbol{a}\cup\boldsymbol{b}\in\uparrow\boldsymbol{a}\cap\uparrow\boldsymbol{b}$ does not hold, then calculate $\boldsymbol{a}\vee\boldsymbol{b}$.

(v) If $\boldsymbol{a}\vee\boldsymbol{b}$ does not exist, calculate $\boldsymbol{a}\oplus\boldsymbol{b}$, and then each bit with a 0 value is replaced by 1 or nothing happened with an equal probability. When this replacement is denoted by RAND_0, we add the new element of

$$\boldsymbol{a}\vee\boldsymbol{b}=<\ldots\mathrm{RAND}_0(a_k+b_k)\ldots> \tag{12}$$

to P^t.

(vi) Choose $\boldsymbol{c}$ from P^t

(vii) If ($\boldsymbol{a}\vee\boldsymbol{b}\geq\boldsymbol{c}$){
  if ($\boldsymbol{a}\geq\boldsymbol{c}$ or $\boldsymbol{b}\geq\boldsymbol{c}$){
  }else{
    if ($\boldsymbol{c}\wedge\boldsymbol{a}$ and $\boldsymbol{c}\wedge\boldsymbol{b}$ are anti-chain){
    }else{
      Remove $\boldsymbol{c}$ from P^t
      Add $\boldsymbol{x}$ such that $\boldsymbol{a}\geq\boldsymbol{x}$
    }
  }
}

The dynamics of an evolutionary topological system are based on the changeability of conjunction and meet and of disjunction and join. According to Proposition 1, if the meet of $\boldsymbol{a}$ and $\boldsymbol{b}$ exists, then it can be replaced by the conjunction of $\boldsymbol{a}$ and $\boldsymbol{b}$. Thus, the meet of $\boldsymbol{a}$ and $\boldsymbol{b}$ is generated for the changeability of meet and conjunction. Notice that, due to RAND_1, a generated meet is not an intersection.

In contrast, given Proposition 2, $\boldsymbol{a}\vee\boldsymbol{b}\in\uparrow\boldsymbol{x}\Rightarrow\boldsymbol{a}\cup\boldsymbol{b}\in\uparrow\boldsymbol{x}$ does not hold even if the join of $\boldsymbol{a}$ and $\boldsymbol{b}$ exists. As mentioned previously, in a causal set {$\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$, $\boldsymbol{a}\vee\boldsymbol{b}$} where $\boldsymbol{a}\vee\boldsymbol{b}\geq\boldsymbol{c}$ and $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are anti-chain with each other, $\boldsymbol{a}\vee\boldsymbol{b}$ cannot be replaced by $\boldsymbol{a}\cup\boldsymbol{b}$. Thus, this type of case is removed. This procedure is implemented by "Remove $\boldsymbol{c}$ from P". However, in the case of P^t={$\boldsymbol{a}\leq\boldsymbol{c}\leq\boldsymbol{a}\vee\boldsymbol{b}$, $\boldsymbol{b}$}, if $\boldsymbol{c}$ is removed and $\boldsymbol{x}$ is added such that $\boldsymbol{x}\leq\boldsymbol{a}$, a causal set with the same structure, $P^{t+1}=\{\boldsymbol{x}\leq\boldsymbol{a}\leq\boldsymbol{x}\vee\boldsymbol{b}, \boldsymbol{b}\}$, is obtained. Thus, this process has fallen into infinite regression. Actually, in a causal set of binary sequences with a finite length, this process turns a causal set degenerate into a one-point set of the least element, **0**, which is why the procedure (vii) discards the case of the subset, $\{\boldsymbol{a}\leq\boldsymbol{c}\leq\boldsymbol{a}\vee\boldsymbol{b}, \boldsymbol{b}\}$, from removing the ***c*** procedure

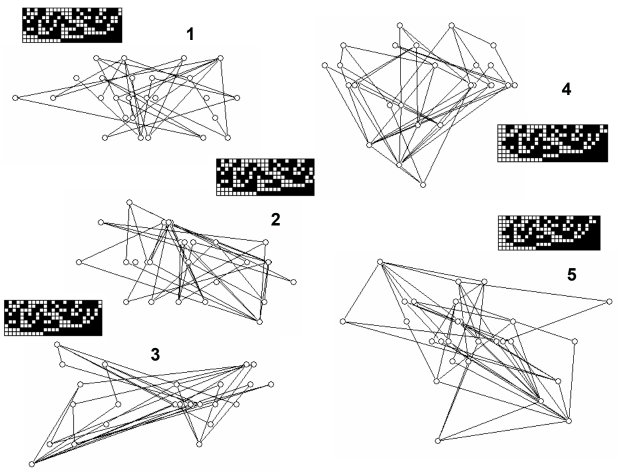

Figure 3. Time development of an evolutionary topological system. P^1, P^2, …, P^5 of 9-bit sequences are shown in the form of a Hasse diagram. All elements of P^t are represented by binary columns above P^t.

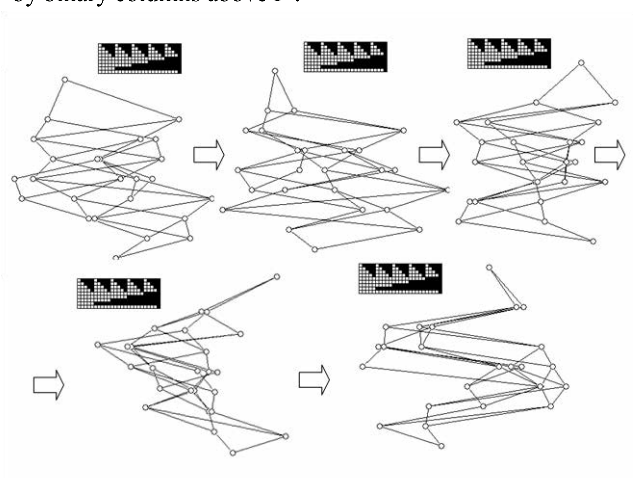

Figure 4. A causal set of an evolutionary topological system has evolved into a distributive lattice.

Fig. 3 shows the time development of an evolutionary topological system. Each causal set, P^t, is represented in the form of a Hasse diagram (i.e., if $\boldsymbol{a}\leq\boldsymbol{b}$ and there is no other element between ***a*** and ***b***, ***a*** and ***b*** are connected by a line). The initial causal set is randomly given, and $P^{t+1}=F(P^t)$ is iterated.

Various time developments suggest that the dynamics defined in definition 4 create a causal set with a particular structure, a distributive lattice. If there is no meet or join, they are added to the causal set. Thus, a POS that is not a lattice is changed into a lattice. In particular, once a causal set becomes a distributive lattice, it is not changed again, and the structure is maintained (Fig. 4).

A distributive lattice is verified to be a lattice that contains no M_3 and N_5 as a sub-lattice (a subset of a lattice that is closed with respect to a join and meet). The structure of M_3 and N_5 are shown in Fig. 5. Given M_3 and N_5 as the initial causal sets, dynamics modify the causal set into a lattice without M_3 and N_5. As a result, the causal set evolves into a distributive lattice.

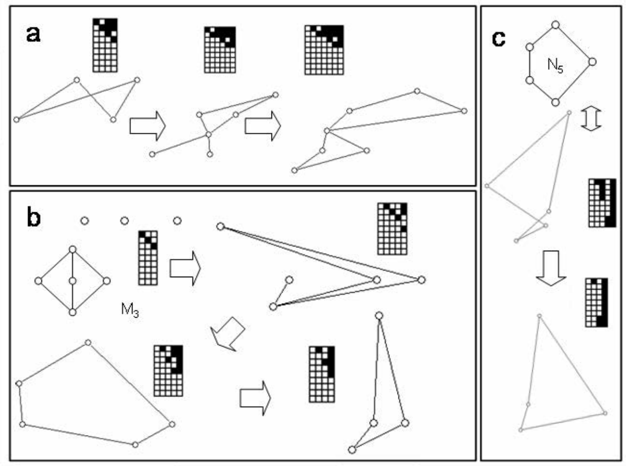

Figure 5. a. The initial POS is developed into a distributive lattice (DL). b. M_3 has also fallen into DL. The arrow represents time development.

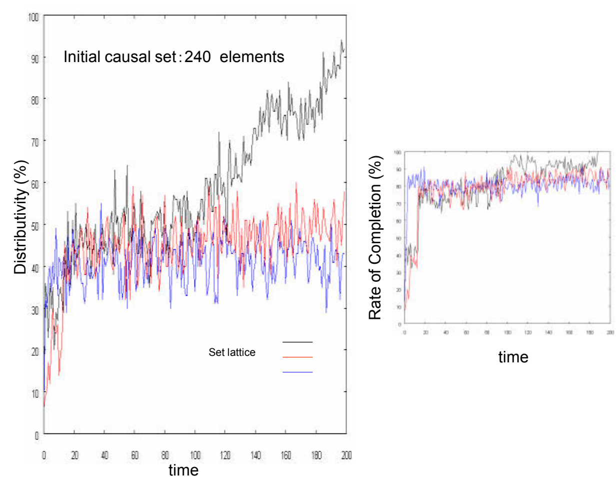

Figure 6. Distributivity plotted against time for a causal set (left) and the rate of completion against time (right). The black line corresponds to an evolutionary topological system, and the red and blue lines correspond to the control experiments. The initial set consists of 240 binary sequences.

To estimate how a causal set converges into a distributive lattice, we define distributivity for a causal set P^t: three elements ***a, b*** and ***c***, are randomly chosen, and whether $\boldsymbol{a}\wedge(\boldsymbol{b}\vee\boldsymbol{c})=(\boldsymbol{a}\wedge\boldsymbol{b})\vee(\boldsymbol{a}\wedge\boldsymbol{c})$ is evaluated, as long as the required meet and join exist, for K times. Distributivity is defined by the number of equality divided by K. We also define two control experiments to compare with the evolutionary topological system. The first control dynamic is only the application of completion (i.e., the procedure of adding meet and join) to a causal set, P^t. It does not contain the procedure of (vii) in definition 4. The second control dynamic also does not contain (vii), and the completion is indeed defined by a union and intersection. Thus, the second control dynamic does not contain equations (11) and (12); therefore, $\boldsymbol{a}\wedge\boldsymbol{b}=\boldsymbol{a}\otimes\boldsymbol{b}$, and $\boldsymbol{a}\vee\boldsymbol{b}=\boldsymbol{a}\oplus\boldsymbol{b}$. If completion is achieved by these procedures, a causal set can become a lattice of sets, which is well known as a distributive lattice.

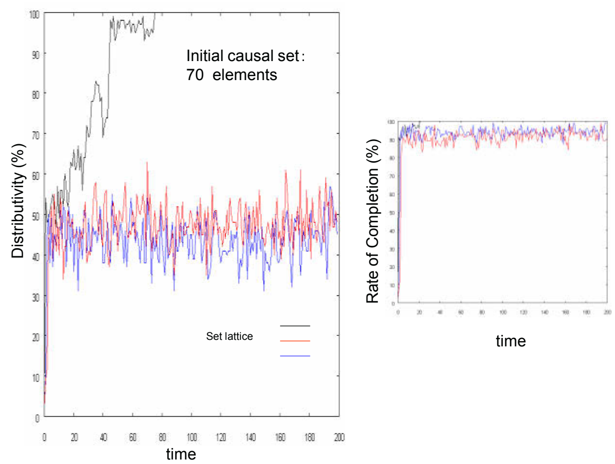

Figure 7. Distributivity plotted against time for a causal set (left) and the rate of completion against time (right). The black line corresponds to an evolutionary topological system, and the red and blue lines correspond to the control experiments. The initial set consists of 70 binary sequences.

As shown in Fig. 6, the distributivity of a causal set of an evolutionary topological system increases towards a distributive lattice. A causal set consists of 9-bit sequences and 240 elements (sequences) initially. Adding the meet and join and removing elements that do not satisfy the changeability increases the distributivity. Compared with the evolutionary topological system, the two control experiments never increase their distributivity. Even if a meet and join are added in the form of an intersection and union, respectively, in the second control dynamic, adding a new element as the join and/or meet entails another requirement to create the join and/or meet. Thus, completion cannot be achieved, and the distributivity is not increased. If the number of initial causal sets is small, the tendency of increasing distributivity is also found (Fig. 7).

Distributivity of a causal set

An evolutionary topological system can converge into a distributive lattice. Because of the restricted changeability of join and disjunction and of meet and conjunction, when the system reaches a distributive lattice can be verified.

Proposition 5 (Evolutionary topological system) An evolutionary topological system defined by $<P^t, \uparrow P^t, \in, F>$ will converge into a distributive lattice.

Proof. (i) The case is that $\boldsymbol{b}\vee\boldsymbol{c}\geq\boldsymbol{a}$ and $\boldsymbol{a}\wedge\boldsymbol{b}$ is an anti-chain of $\boldsymbol{a}\wedge\boldsymbol{c}$ but not $\boldsymbol{b}\geq\boldsymbol{a}$ or $\boldsymbol{c}\geq\boldsymbol{a}$, $\boldsymbol{b}$ (i.e., $\boldsymbol{b}\vee\boldsymbol{c}\in\uparrow\boldsymbol{a}\Rightarrow\boldsymbol{b}\cup\boldsymbol{c}\in\uparrow\boldsymbol{a}$ does not hold, but procedure (vii) in definition 4 is not applied). This case is shown in Fig. 8. This case leads to a distributive sub-lattice by adding the meet, $\boldsymbol{b}\wedge\boldsymbol{c}$., which can be achieved at any point.

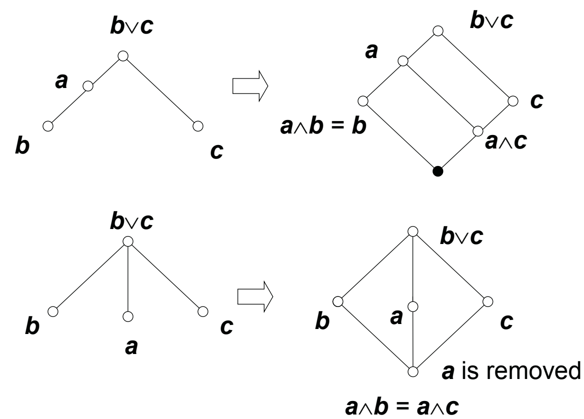

Figure 8. The case that $\boldsymbol{b}\vee\boldsymbol{c}\geq\boldsymbol{a}$ and that $\boldsymbol{a}\wedge\boldsymbol{b}$ is an anti-chain of $\boldsymbol{a}\wedge\boldsymbol{c}$ but not $\boldsymbol{b}\geq\boldsymbol{a}$ or $\boldsymbol{c}\geq\boldsymbol{a}$, $\boldsymbol{b}$ (above). The case that $\boldsymbol{b}\vee\boldsymbol{c}\geq\boldsymbol{a}$ and that $\boldsymbol{a}\wedge\boldsymbol{b}$ is not an anti-chain of $\boldsymbol{a}\wedge\boldsymbol{c}$, $\boldsymbol{b}\geq\boldsymbol{a}$ or $\boldsymbol{c}\geq\boldsymbol{a}$ (below).

(ii) The case that $\boldsymbol{b}\vee\boldsymbol{c}\geq\boldsymbol{a}$ *and that* $\boldsymbol{a}\wedge\boldsymbol{b}$ is not an anti-chain of $\boldsymbol{a}\wedge\boldsymbol{c}$, and not that $\boldsymbol{b}\geq\boldsymbol{a}$ or $\boldsymbol{c}\geq\boldsymbol{a}$, $\boldsymbol{b}$. Although this case also allows that $\boldsymbol{b}\vee\boldsymbol{c}\in\uparrow\boldsymbol{a}\Rightarrow\boldsymbol{b}\cup\boldsymbol{c}\in\uparrow\boldsymbol{a}$ does not hold, the procedure (vii) in definition 4 can be applied, so $\boldsymbol{a}$ is removed and a distributive sub lattice is then obtained, as shown in Fig. 8 below.

(iii) Another case is from (i) and (ii). Because the statement $\boldsymbol{a}\wedge(\boldsymbol{b}\vee\boldsymbol{c})\geq(\boldsymbol{a}\wedge\boldsymbol{b})\vee(\boldsymbol{a}\wedge\boldsymbol{c})$ holds in any lattice, we will prove that $\boldsymbol{a}\wedge(\boldsymbol{b}\vee\boldsymbol{c})\leq(\boldsymbol{a}\wedge\boldsymbol{b})\vee(\boldsymbol{a}\wedge\boldsymbol{c})$. Because $\boldsymbol{a}\wedge(\boldsymbol{b}\vee\boldsymbol{c})$ is a lower bound for $\{\boldsymbol{a}, \boldsymbol{b}\vee\boldsymbol{c}\}$, we obtain $\boldsymbol{a}\wedge(\boldsymbol{b}\vee\boldsymbol{c})\leq\boldsymbol{a}$ and $\boldsymbol{a}\wedge(\boldsymbol{b}\vee\boldsymbol{c})\leq\boldsymbol{b}\vee\boldsymbol{c}$. In a topological system, it means that

$$\boldsymbol{a}\in\uparrow(\boldsymbol{a}\wedge(\boldsymbol{b}\vee\boldsymbol{c})) \text{ and } \boldsymbol{b}\vee\boldsymbol{c}\in\uparrow(\boldsymbol{a}\wedge(\boldsymbol{b}\vee\boldsymbol{c})). \quad (13)$$

Because of the changeability of disjunction and join, we can replace this statement with

$$\boldsymbol{a}\in\uparrow(\boldsymbol{a}\wedge(\boldsymbol{b}\vee\boldsymbol{c})) \text{ and } (\boldsymbol{b}\in\uparrow(\boldsymbol{a}\wedge(\boldsymbol{b}\vee\boldsymbol{c})) \text{ or } \boldsymbol{c}\in\uparrow(\boldsymbol{a}\wedge(\boldsymbol{b}\vee\boldsymbol{c})). \quad (14)$$

Because a logical statement satisfies the distributive law, this equation can be rewritten as $(\boldsymbol{a}\in\uparrow(\boldsymbol{a}\wedge(\boldsymbol{b}\vee\boldsymbol{c}))$ and $\boldsymbol{b}\in\uparrow(\boldsymbol{a}\wedge(\boldsymbol{b}\vee\boldsymbol{c})))$ or $(\boldsymbol{a}\in\uparrow(\boldsymbol{a}\wedge(\boldsymbol{b}\vee\boldsymbol{c}))$ and $\boldsymbol{c}\in\uparrow(\boldsymbol{a}\wedge(\boldsymbol{b}\vee\boldsymbol{c})))$.

Additionally, due to the changeability of conjunction and meet, we obtain

$$\boldsymbol{a}\wedge\boldsymbol{b}\in\uparrow(\boldsymbol{a}\wedge(\boldsymbol{b}\vee\boldsymbol{c})) \text{ or } \boldsymbol{a}\wedge\boldsymbol{c}\in\uparrow(\boldsymbol{a}\wedge(\boldsymbol{b}\vee\boldsymbol{c})). \quad (15)$$

By replacing the disjunction with join,

$$(\boldsymbol{a}\wedge\boldsymbol{b})\vee(\boldsymbol{a}\wedge\boldsymbol{c})\in\uparrow(\boldsymbol{a}\wedge(\boldsymbol{b}\vee\boldsymbol{c})). \quad (16)$$

Thus, we finally obtain

$$a \wedge (b \vee c) \leq (a \wedge b) \vee (a \wedge c). \qquad (17)$$

Significance of Distributivity. What is the significance of a distributive lattice? It is an abstract expression of a way of thinking in which anything can be considered as the result of summation, which is also known as a representation theorem for a distributive lattice (Davey and Priestley, 2002). The theorem states that any distributive lattice can be expressed as a lattice of sets consisting of sets equipped with a join that is defined by a union and meet, which are defined by an intersection.

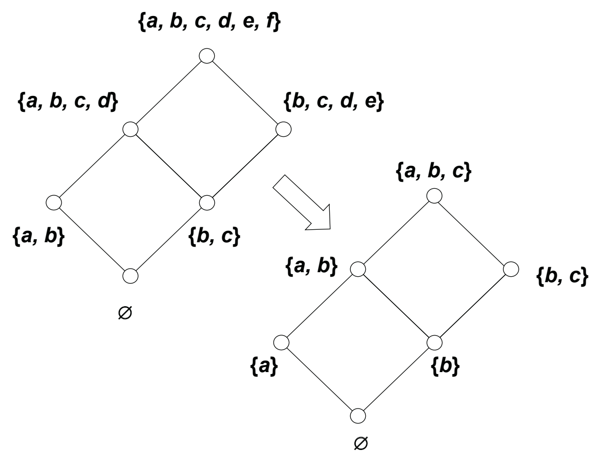

Figure 9. A distributive lattice (above) can be a lattice of sets (below) (i.e., a join and meet can be defined by a union and intersection, respectively).

Fig. 9 shows a representative example. The above Hasse diagram exemplifies a distributive lattice containing elements that are sets. Notice that join does not equal union and that meet does not equal intersection while meet is $\{a, b\} \wedge \{b, c\} = \emptyset$ and intersection is $\{a, b\} \otimes \{b, c\} = \{b\}$ in Fig. 9 (above). Similarly, while $\{a, b\} \vee \{b, c\} = \{a, b, c, d\}$, $\{a, b\} \oplus \{b, c\} = \{a, b, c\}$, which explains why the lattice in Fig. 9 (above) is not a lattice of sets. However, this lattice can be represented by a set of lattices by replacing elements with other elements represented in Fig. 9 (below). In this lattice, any meet is defined by an intersection, and any join is defined by a union.

Thus, a distributive lattice is an abstract expression of set-based thinking: a whole system can be reduced to elements, and summing up the elements can create a whole system. There is no non-linear interaction among the elements. Our results, in which an evolutionary topological system evolves towards a distributive lattice, indicate that a cause-effect relationship in space-time can be developed as the simplest logical structure. Although the changeability of disjunction and union (conjunction and meet) can be erroneous, space-time appears to be constructed as an operationally simple cause-effect relationship.

Out of body experience resulting from the changeability of disjunction and join

An evolutionary topological system is based on the changeability of join and disjunction and of meet and conjunction. What is the real implementation of the changeability? Although a disjunction or conjunction is a type of distribution or a set of elements in a causal set, join and meet are single elements of a causal set. However, they differ with respect to logical status and can be replaced with each other to improve the conflict between Point and Open Logic. We believe that this changeability plays an essential role in our cognitive system.

Body image. The generation of a body image can be one of the examples resulting from the changeability of disjunction and join. In brain science, the relationship between body schema (operational body) and body image (body owned by oneself) is investigated. Even a hermit crab can detect a sudden change in the carried shell in terms of size, thereby changing its method of walking based on the shell size (Sonoda et al., 2013). This observation implies that body schema appears to affect body image and that both always interact with each other.

Although a body schema is based on controlling a point, a body image is a collection of parts as a whole. The former is related to Point Logic, and the latter is related to Open Logic. Thus, the interaction between body image and schema is also faced with the conflict between Point and Open Logic.

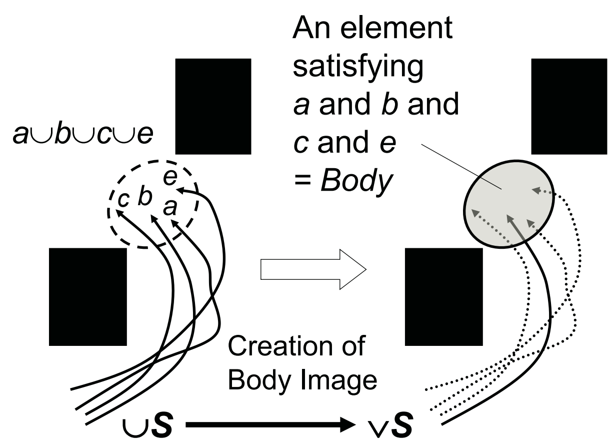

Figure 10. A model for the creation of a body image resulting from the changeability of disjunction and join. The black squares are pillars or obstacles.

Imagine an infant who operates his own body. He first assumes that he is just a point and has to control this point to walk between pillars in his room (Fig. 10 left). He never controls this point in a strict sense. However, he attempts to move this point to a central point between the pillars, and the point is occasionally at a location *a* and occasionally at *b* (because he ignores his body size). As a result, a body can be at *a* or *b*. However, if an idea occurs to him that the possibility of *a* or *b* itself can be a “big” point, then a point has to be something big that is at *a* and *b* at the same time (Fig. 10 right). Thus, the body image was created as a join resulting from disjunction.

Out of body experience. According to our evolutionary topological system, two events that are exclusive of each other can be replaced with a single event, which can satisfy two exclusive events in a causal set. Thus, we can design an artificial space-time event appearing from the changeability of disjunction and join. Using a variation of the Substitutional Reality (SR) system (Suzuki et al., 2012), we construct the sensation of an out-of-body experience, as shown in Fig. 11.

The system consists of a head-mounted display (HMD) fitted with a video camera at the front center (subjective-eye camera), a panoramic video camera (objective-eye camera) and a control computer. In our preliminary experiment, a participant sitting in a room first sees an experimenter in front of him with his naked eyes. Then he wears the HMD and see the experimenter through the subjective-eye camera. This causal relationship is shown as an event, $a \le a'$ (Fig. 11). Then, the scene pre-recorded by the objective eye camera set in front of the participant is projected in the HMD. Thus, the participant sees himself appearing and wearing the HMD, corresponding a causal relationship of $b \le b'$ (Fig. 11). With several virtual-reality-inspired tricks, even in the objective view he is able to look around freely as in the subjective view.

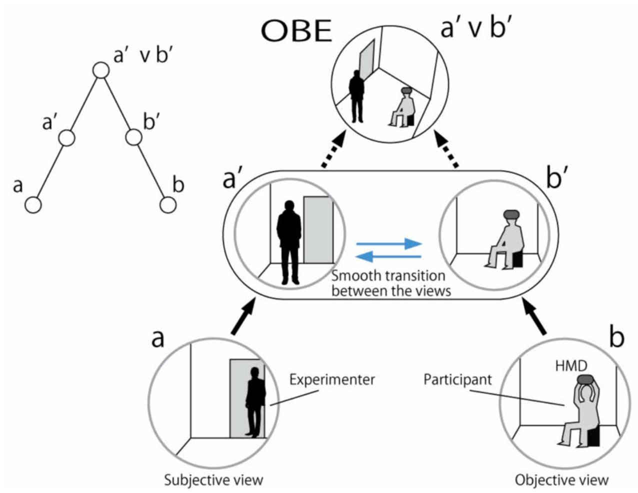

Figure 11. Design for the "Out-of-Body Experience (OBE)" in an evolutionary topological system.

The subjective view, a', and the objective view, b', are exclusive of each other, although they are both sides of the same coin –"now". They are not united by a single event in this situation. However, if the subject experiences a smooth transition between objective and subjective view (by changing the objective camera position and using several video effects), represented by the blue arrows in Fig. 11, he feels as if he is seeing himself in his subjective view. That feeling corresponds to the replacement of $a' \cup b'$ by a single event, $a' \vee b'$ According to his verbal report, he feels an OBE which is not just an experience of seeing himself. Instead, he feels as if he has created another perspective by his imagination (Fig. 11). Therefore, in this feeling, exclusive subjective and objective scenes can be considered to be united as a single event, which is different from the feeling experienced in a previous experiment (Lenggenhager et al., 2007).

Conclusion

A causal set developed in quantum physics attempts to describe a space-time from an observer's view. If so, we have to pay attention to the interaction between the causal relationship and an observer that is a computation and/or logical operation in a space-time equipped with a causal relationship. For this purpose, we describe a causal set and its semantics as a topological system consisting of Point and Open logic.

The binary relationship between Point and Open Logic can derive a conflict that can be improved by restricting logical operations. We here, however, propose an evolutionary topological system in which a conflict between Point and Open Logic is locally and temporally improved that can generate an artificial causal relationship. This local improvement is implemented by the changeability between join and disjunction and between meet and conjunction represented by the replacement of a set with an element to keep non-restricted logical operations.

We show and verify that a causal set of the evolutionary topological system can converge to a distributive lattice that is an abstract expression of the simplest logical operation for summation. We also show that the changeability of disjunction and join can generate abnormal space-time feelings, such as an out-of-body experience. We can design both normal and abnormal artificial space-time based on an evolutionary topological system.

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