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Population Dynamics of Centipede Game using an Energy Based Evolutionary Algorithm

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Abstract

In the context of Evolutionary Game Theory, we have developed an evolutionary algorithm without an explicit fitness function or selection function. Instead players obtain energy by playing games. Clonal reproduction subject to mutation occurs when a player's energy exceeds some threshold. To avoid exponential growth of the population there is a death event that depends on population size. By tweaking with the relation between payoff and energy and with death event, we create another dilemma that a population must overcome: extinction. We demonstrate this phenomena in the Centipede game. Simulations show that if players can only play one of the two positions of this asymmetric game extinctions are common. If players are versatile and can play both positions there are no extinctions.

Introduction

Game Theory provides a series of tools to model how agents interact (Gintis, 2000; Shoham and Leyton-Brown, 2009; Fudenberg and Tirole, 1991). Evolutionary Game Theory (EGT) studies the population dynamics of individual agents (Maynard Smith, 1982; Hofbauer and Sigmund, 1998). Therefore it is natural to use games as a starting point in any study on interaction dynamics be it at the individual level or at the population level. Nash Equilibrium is one of the most important concepts in Game Theory (Nash, 1951). Its equivalent in population dynamics is Evolutionary Stable States. It posits that no player has incentive to move from its strategy because he would be worse. Since the publication of this result, several games have been proposed that have cooperative dilemmas: if players follow the equilibria they are worse off. Among these games we cite Prisoner's Dilemma (PD), Centipede, Ultimatum and Public Good Provision (PGP). When these games are repeated a known number of times, the theory, through backward induction (Fudenberg and Tirole, 1991; Shoham and Leyton-Brown, 2009), predicts the demise of cooperation. When players have limited resources such as memory, they may lack the ability to count how many stages have passed, so they cannot perform backward induction. In fact, earlier work on using finite state automata to play games has shown the prevalence of cooperation (Neyman, 1985; Axelrod, 1997).

The capability of players finding the equilibria as recently been shown to be a hard problem. Research on the complexity of computing Nash Equilibria has established it as belonging to class PPAD or *polynomial parity argument, directed version* Papadimitriou (1994). This class sits between complexity classes FP and FNP which generalise classes P and NP from decision problems to functions computable in polynomial time. Examples of classes of games in this class include general sum games in normal form with two or more players belong to this class (Chen and Deng, 2006; Daskalakis et al., 2009), repeated games (Borgs et al., 2010) or stochastic games. These results show that if player wants to compute the equilibria he would spend an exponential time. Clearly biological system do not sit still that long. In fact their computational resources are limited and thus they have to use them in some efficient way. Early research on artificial systems used finite-state machines to model the behaviour of agents (Axelrod and Hamilton, 1981). This approach when applied to cooperative dilemma games resulted in agents playing the strategy profile that result in higher social payoff.

Instead of focusing on the (in)capability of computing the Nash Equilibria, one may turn into to the population dynamics. It the population is some evolutionary stable state this is not the result of players computing the best response to their peer in order to become dominant in the following generations. Instead it is a combination of population-level mechanisms such as mutation, selection and stochastic events.

Evidence from experimental play (Camerer, 2003) showed that people do not follow the equilibria and that instead cooperation is prevalent in many places. Therefore it may be counterproductive to discuss the equilibria of some system or the rationality of players (Aaronson, 2013). Instead, one should focus on the mechanisms behind the behaviour of real players and population and how they have overcome a series of dilemmas. Partner selection (Aktipis, 2004) or player connectivity (Santos and Pacheco, 2005) explain the prevalence of cooperation in these type of games.

We should be focusing on how evolution has overcome a series of dilemmas to produce the biodiversity that we can appreciate. One special dilemma is extinction. If evolution cannot surpass it then the model or system is not viable. Therefore it is important to consider a model that explicitly takes into account and allows it and does not have mechanisms to circumvent it by magic repopulation or by keeping a constant flow of energy resources (Yaeger, 2009).

In this paper we address the extinction dilemma. This event can appear in cooperative dilemmas because players choose the lower payoff profile. In order to introduce this event we have to create an Evolutionary Algorithm (EA) with varying population size. This has already been done in artificial ecosystems (Lenski et al., 2003; Ray, 1997) although the focus was not the study of how to avoid extinctions. The model that we propose is generic in that it can be applied to any game, just as the replicator equation (Taylor and Jonker, 1978), the Moran process or an Individual Based Model (IBM) (Grimm et al., 2006; McLane et al., 2011) can be applied to any game. This is done by simply interpreting a game as an energy transfer process and creating a player's life cycle that is driven by the energy acquired by playing games.

The rest of this paper is organised as follows. In the next section, we start by reviewing related work on models that can be used to study population dynamics while focusing on the ability to study extinctions. The following section is the major contribution of this paper as we describe the EA we have developed. The next section presents our characterisation of the Centipede game. Afterwards we present simulation results of our algorithm with the Centipede game. We finished the paper with a discussion section and a conclusions and future work section.

Related Work

The study of evolutionary traits has used models based on differential equations such as the replicator equation (Taylor and Jonker, 1978; Nowak, 2006; Hofbauer and Sigmund, 1998; Maynard Smith, 1982; Gintis, 2000; Hofbauer and Sigmund, 2003) or the Moran process. They have been used to describe the broad behaviour of systems (Meadows et al., 1972).

There are a set of assumptions behind replicator equation (Roca et al., 2009). One assumes a considerably large or infinite population. Another assumes a well mixed-population such that everybody plays with everybody else. A similar approach is randomly pairing players. These are unrealistic assumptions and have led to alternative proposals. Among them are structured populations where players are placed in the nodes of some graph and interactions are restricted to links between nodes (Nowak et al., 1994; Szabó and Hauert, 2002). Despite not allowing varying population size, they have been used to model scenarios that may cause extinctions such as climate change (Santos et al., 2012).

Agent or IBM address the difficulties of creating a formal model of a complex system (Forrest and Jones, 1994). After a series of artificial ecosystems populated with these type of individuals (Ray, 1992; Lenski et al., 2003; Yaeger, 1994) specific protocols to construct such systems have emerged (Grimm et al., 2006).

There are IBMs that analyse the possibility of extinctions but they do that in specific contexts such as model population growth of endangered species (Beissinger and Westphal, 1998), tree mortality (Manusch et al., 2012), impact of logging activities in bird species (Thinh et al., 2012). Some of these models are characterised by using specific differential equations or operate at higher level than the individual. Often they are specific to their case study and their methods are not directly transferable to another scenario.

McLane et al. (2011) provides a review of IBM used in the literature of ecology to address the issues of managing ecosystems. They presented a set of behaviours that individuals can choose in their life cycle: habitat selection, foraging, reproduction, and dispersal. In the papers that they reviewed, some used all the behaviours in the set while others used just one. Such set of behaviours could constitute the set of actions of some generic game played by animals. Moreover we can divide them in two sets, one where an animal obtains energy (foraging) and a second where an animal spends energy (habitat selection, reproduction, and dispersal).

While standard EGT models use either infinite populations or constant finite populations, IBMs have been used to model scenarios where populations could go extinct. This can happen because a player's actions do not provide him enough resources to reproduce. While their models are often used in specific problems it is important to create a general evolutionary algorithm that can be applied to any game and where extinctions can occur independently of game characteristics.

Evolutionary Algorithm Description

The Energy Based Evolutionary Algorithm (EBEA) we have developed is characterised by a game. The concept of a game as an energy transfer is a redefinition of the payoff function. A game G is a tuple (N, A, E) where N is a set of n players, $A = \{A_1, \ldots, A_n\}$, where A_i a set of actions for player i, and $E = \{e_1, \ldots, e_n\}$ is a set of payoff functions, $e_i : A_1 \times \ldots \times A_n \to \mathbb{R}$, which we interpret as an energy flow. In this context, players are characterised by a strategy s and an energy level e. Each iteration of the algorithm has three phases:

play in this phase all players play the game and update their energy. Partners are selected randomly.

reproduction in this phase the players whose energy is above threshold e_R produce one offspring and their energy is decremented by this value. The offspring is mutated with some probability. This is asexual reproduction with mutation.

death in this phase, the entire population goes through a carrying capacity event where the probability to die depends on population size. This contrasts with our previous work (Mariano and Correia, 2011) where the event mixed population size with player's age.

Regarding the relation between the payoff function and the energy function, we have extended our work in Mariano and Correia (2011). In order to compare the evolutionary dynamics of games with different payoff functions we scale the payoff π obtained by a player. This gives the following equation:

$$e \leftarrow e + \frac{\pi}{\overline{\pi} - \underline{\pi}} \quad , \tag{1}$$

where $\overline{\pi}$ and $\underline{\pi}$ are the highest and lowest payoff obtainable in game G respectively, and e is a player's energy. The rationale for this equation is that player's energy increases proportionally to the payoff obtained in the game.

The chromosome codes the strategy to play the game G. The coding may range from direct, where actions are explicitly defined for all possible histories, to compact, where the strategy is represented in a symbolic form, for example by a set of rules to be executed by the phenotype. In both cases the result is either a specific action, for a pure strategy, or a probability distribution over the set of actions, for a mixed strategy. Notice that mutation operator must be adapted to the coding used.

Regarding the phenotype of a player, besides executing the strategy coded in the chromosome, it has associated an energy level. This is used for reproduction. As we have seen, an individual reproduces when his energy is above threshold e_R. This may also be considered as an indirect fitness assessment.

Unlike our previous work, in this paper we use random partner selection. Although this is an unrealistic assumption, we want to present a core evolutionary algorithm. From this core algorithm we can study what mechanisms are necessary to escape extinctions. In the context of PGP we have shown that partner selection can escape extinctions and promote cooperation for some parameter settings (Mariano and Correia, 2011).

The goal of this paper is to show what population dynamics we can observe with our evolutionary algorithm using Centipede as the game. In our current implementation of the algorithm, the random partner selection does not take into consideration if the game is symmetric or not. This means that in asymmetric games incorrect players may be matched. For instance, in the Ultimatum game two dictators may be paired. In this case both get zero energy.

In order to avoid exponential growth, in each iteration of the algorithm all players go through a death event. While in previous work we had a single event that mixed population size with player's age, in this paper we only use population size. Death by old age is optional and is not strictly needed to avoid exponential growth. The probability of a player dying because of population size is:

$$P(\text{death population size}) = \frac{1}{1 + e^{6\frac{K - |\mathcal{P}|}{K}}} \quad , \tag{2}$$

where $|\mathcal{P}|$ is the current population size and K is a parameter that we call carrying capacity. This probability is a sigmoid function. The exponent was chosen because the logistic curve outside the interval $[-6, 6]$ is approximately either zero or one. In the advent of the entire population duplicating size, it will not go from a zero probability of dying to certain extinction.

From the description of our algorithm it is clear that there is no explicit fitness function nor selection function. Instead, the energy update function represented by equation (1) combined with the reproduction threshold induce a process where players that acquire more energy per game are able to reproduce faster (and spread their genes) than a player that gains less energy. A fit player is one whose strategy allows him to obtain more energy in the current population state.

The dynamics of this algorithm can be characterised by a Markov Chain. Supposing a well-mixed population, each state encodes a bag of pairs (s, e), where a pair represents a player with strategy s and energy level e. Due to the carrying capacity we can impose a limit on the number of states, such that the probability of the population size passing this limit is negligible. This limit can be computed from equation (2). The Markov Chain has at least one absorbing state, namely the empty bag which corresponds to an extinction.

If there is a strategy profile of the underlying game that gives the minimum payoff to all players, then by equation (1) players using the corresponding strategies cannot increase their energy. This means that paths from states containing only strategies from this type of profile can only go to the empty bag state. However the time to walk this path may be large because death probability becomes negligible. From a practical standpoint the corresponding Markov states can be characterised as almost absorbing.

If there are no such strategy profile, then from any state other than the empty bag state there is a positive probability to reach any state in the Markov chain. This probability depends on event *death population size*, on event mutation, and on the mutation operator.

Centipede Game

The Centipede game is a sequential game of perfect recall where in each stage a player decides if he keeps a higher share of a pot of money or decides to pass the pot to the other player (Rosenthal, 1981; McKelvey and Palfrey, 1992; Rand and Nowak, 2012). If the player keeps the higher share the game stops. If he passes the pot is increased by some external entity. The game has some fixed number of stages. The payoff structure is constructed such that the payoff the deciding player obtains at stage t is higher than he obtains at stage $t+1$. The game can be characterised by the initial size of the pot, p_0, how the pot is increased, p_i, and the pot share given to the player that decides to stop or not, p_s. We consider two methods to increase the pot: an arithmetic progression with difference d, represented by $p_i = \mathtt{a}(d)$; and a geometric progression with ratio r, represented by $p_i = \mathtt{g}(r)$. The pot size at stage t is given by:

$$p(t) = \begin{cases} p_0 + d(t-1) & \text{if } p_i = \mathtt{a}(d) \\ p_0 r^{t-1} & \text{if } p_i = \mathtt{g}(r) \end{cases} \quad (3)$$

From this equation, the payoffs at stage t are $\pi_D(t) = p(t)p_s$ for the player that stops and $\pi_{\neg D}(t) = p(t)(1-p_s)$ for the other player. The subscript D represents the player that decides to stop.

Figure 1a shows an example of the Centipede game for some parameter settings. Time goes from left to right. Since Centipede is an asymmetric game, we will consider two types of players: *first* represents the players that decide in odd stages; *second* represents the players that decide in even stages. In the extensive form game shown in figure 1a the types are represented by numbers one and two, respectively. In the last stage if player two decides to stop he receives the higher share of the pot. Otherwise the pot is increased but he receives the lower share.

In this paper we use a variant where the pot size is increased and then split as given by p_s. The parameters must obey the following set of conditions:

$$\begin{cases} p_s > 0.5 \\ d > 0 \land (p_0 + d)(1-p_s) < p_0 p_s & \text{if } p_i = \mathtt{a}(d) \\ r > 1 \land p_s > r(1-p_s) & \text{if } p_i = \mathtt{g}(r) \end{cases} \quad (4)$$

The second part in the second and third conditions represents the fact that $\pi_D(t) > \pi_{\neg D}(t+1)$

To decrease the number of parameters, we set the initial size of the pot to one, $p_0 = 1$. The admissible parameters of Centipede given by equation (4) can be represented graphically as shown in figure 1b.

The two methods to increment the pot create different pressures on players during our evolutionary algorithm. The difference in energy obtained per game is higher in the geometric method than the arithmetic variant. This means birth rate are different for both methods thus population viability is higher in the arithmetic variant.

The chromosome contains two genes. The first gene (binary) represents the player type while the second gene (natural number) represents the stage where it decides to stop the game. We will use $t_\mathtt{f}$ to represent an action of the player that moves first and decides to stop the game at stage $t_\mathtt{f}$. Likewise, we will use $t_\mathtt{s}$ for the other player. Recall that in our current implementation of the algorithm, if players of the same type are paired, they obtain zero energy. Otherwise, they play the game.

	number of iterations	10^5
K	carrying capacity	$\{300, 400, 500, 600, 700, 800\}$
	mutation probability	0.1
e_R	reproduction threshold	20
	number of stages	$\{4, 6, 8, 10, 12, 14\}$
p_s	pot share	$\{0.8, 0.9\}$
p_i	arithmetic pot increase	$\{0.1, 0.2, 0.3\}$
	geometric pot increase	$\{1.5, 2, 2.5\}$

Table 1: Parameter values tested

Experimental Analysis

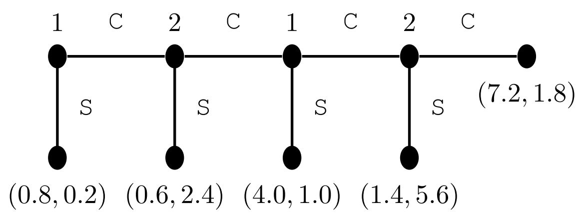
Simulation Settings

The purpose of the experiments is to study what kind of dynamical behaviour we can achieve with this new EA. Since the population size may vary, extinctions may happen because players cannot get enough energy to reproduce and they are slowly killed. We have tested different combinations of parameters. Regarding the parameters of the EA we varied only the carrying capacity in order to assess its impact on the occurrence of extinctions. We opted for using a single value for the mutation probability and reproduction threshold parameters. The number of iterations of the algorithm was 10^5. Regarding the game parameters, we varied the number of stages in order to assess the amount of strategies on the population dynamics. We used high pot share to strengthen the backward induction argument. As for the pot increase method we opted to increase the pot share as given by equations (3) and used different differences or ratios. Table 1 shows the parameter values used in the experiments.

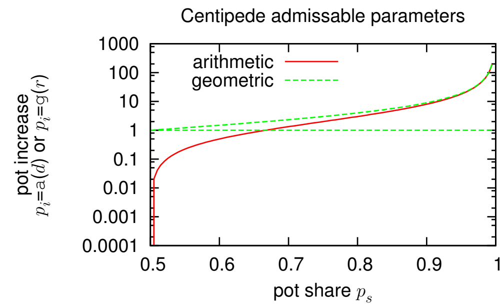
The initial population consisted in 10 players with chromosome type *first* and 10 players with chromosome type *second* both with the highest time to stop the game, $t_\mathtt{f}$ and $t_\mathtt{s}$. Whenever a player exceed the reproduction threshold and produce an offspring, he was subject to mutation with probability 10%. The mutation operator consisted in adding or subtracting to the time to stop the game, $t_\mathtt{f}$ or $t_\mathtt{s}$, a discrete Gaussian distribution with average zero and standard deviation one. The resulting value was constrained in the interval one to the number of stages in the game.

For each parameter combination we performed 10 runs in order to get some statistical information on the population dynamics. For each simulation run, per iteration, we recorded population size, number of births, number of deaths, number of players with each type and average time to stop for each type. If the population size dropped below two we stopped the simulation.

Figure 2 shows the plot of the probability of the event *death population size*, as given by equation (2) for the tested



(a) An example of the Centipede game with $p_0 = 1$, $p_s = 0.8$, $p_i = \mathtt{a}(2)$, and four stages. Time goes from left to right, so player 1 is the first to decide to stop or not the game. Letter $\mathtt{S}$ in the edges represent the stop action, while letter $\mathtt{C}$ represents the continue action.



(b) Admissible parameter region space. Each line represents an equation that results from the inequalities in equation (4). The parameter in the vertical axis is either d or r depending on the method to increase the pot size, $p_i = \mathtt{a}(d) \vee p_i = \mathtt{g}(r)$.

Figure 1: Schematics of the Centipede game used in the experiments.

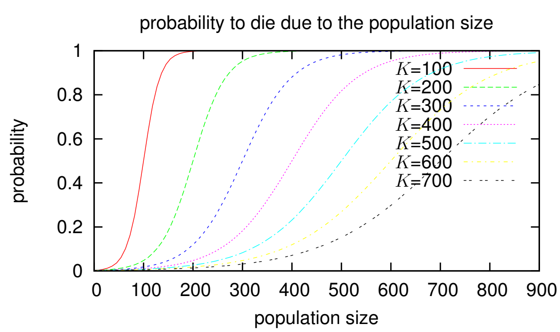


Figure 2: Plot of the probability of event *death population size* as given by equation (2) for different carrying capacity values.

carrying capacity values. As the parameter increases the plot becomes smoother meaning population size is unlikely to remain nearer the value of K. Moreover, as K increases, it becomes increasingly unlikely that the population doubles at specific iterations as more players had to synchronise their reproduction behaviours.

Simulation Results

The major contribution of EBEA is the possibility of extinctions. In the evolutionary model that we have used extinctions can only occur if the population size drops below the number of players of the game. In the work that we report in this paper it is two. Another possibility is one of the player types extinguishing from the population. Regarding simulations using geometric pot increase, Figure 3 shows the number of simulations where some type of extinction occurred as a function of carrying capacity (vertical axis) and number of stages (horizontal axis). There is not a clear trend although a higher number of extinctions of second players compared to first player extinctions is clear and extinctions of first players tend to increase with number of stages. Simulations with the arithmetic pot increase resulted in less extinctions for both types of players with around 45% less for second players and 95% less for first players. This is mainly due to a smaller difference (compared to the geometric increase) between stage payoffs.

We can also analyse what strategies are more common under this evolutionary algorithm. In the simulations that we performed the average time to stop the game, $t_{\mathtt{f}}$ and $t_{\mathtt{s}}$, showed a decreasing trend to the smallest value, meaning players become less cooperative. Rand and Nowak (2012) argue that population size and selection strength affect the prevalence of cooperator strategies, meaning players that stop the game latter. In their paper they have used the geometric pot increase variant. However, their strategies could play both types. Given randomisation over partners, this means that on average a player is better never stopping the game. If we take the example of the Centipede game in figure 1a their players would get on average a payoff of $(7.2 + 1.8)/2 = 4.5$. When they decrease selection strength the weight of a player's payoff in his fitness decreases and selection becomes a random process. They only observe higher levels of cooperation when selection strength is mild, but we suspect that the cause is the ability of players playing both types. We have performed some simulations with our evolutionary algorithm where players could play both types. In these simulations we never observed an extinction. Regarding the geometric pot increase, we went from a scenario where some type of extinction occurred in 78% simulations to a scenario with no extinctions. Moreover players stopped the game at later stages. Switching from specialist players that only played one type to generalist players that played both types solved the problem of extinction.

This fact is very important given that, although our algorithm does not have an explicit fitness function, it has a high selection strength. If we examine the population dynamics of single simulations we often observe variations in population size that are correlated with player type's abundance and their strategies. When they stop the game at latter stages, they are able to acquire energy faster thus produce more offsprings per iteration. Therefore given two populations one that stops earlier and another that stops later and we run our algorithm without mutations, the second population outgrows the first.

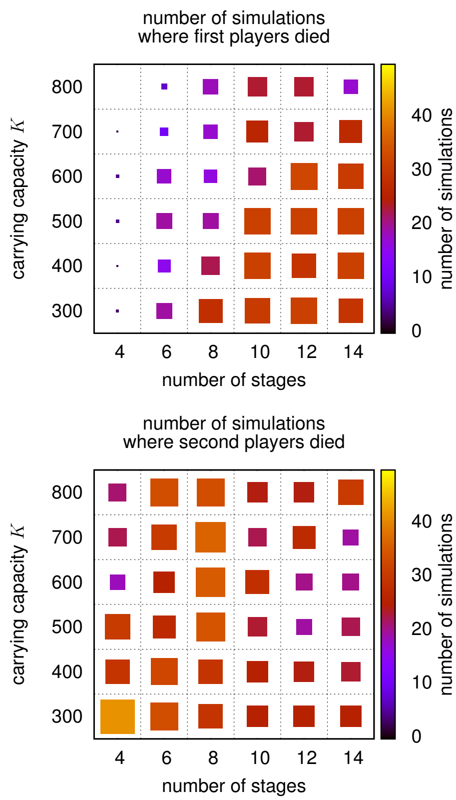


Figure 3: Results from the simulations with geometric pot increase. The plots show the number of simulations where first or second players got extinct – the higher the number of simulations the bigger is the square.

Discussion and Comments

The EA algorithm that we have presented does not have an explicit fitness function nor selection function. Instead players must acquire energy in order to be successful. This is an approach similar to individual-based models such as Echo (Forrest and Jones, 1994) or Avida (Lenski et al., 2003). Their models are ecosystems that use very specific games. Indeed one could formalise the interactions performed by the individuals in those systems as a game. With our approach we interpret games as an energy transfer process. In these systems there is also an energy concept. Individuals must perform some tasks in order to obtain energy tokens that are used to generate offsprings. While offsprings usually replace some stochastic chosen individual, in our approach they are added to the population. This is a similar approach to Yaeger (2009).

Scaling allows us to compare the evolutionary dynamics of games with different payoff functions: consider the number of offsprings per iteration. We could remove scaling, which means that energy range is equal to payoff range.

The EA has at least one absorbing state, namely the one corresponding to an empty population. A state of the corresponding Markov Chain may not be reachable if the population is filled with players that only get the lowest payoff. Therefore the only path is towards extinction, which can take a long time as we have seen in simulations using Centipede. This could be resolved if we had a death by old age event. This would put extra pressure on a population to escape extinction.

This process (energy dynamics and population control) is different from other approaches (Aktipis, 2004; Ray, 1997; Lenski et al., 2003). Even when they use energy, the focus is not the evolutionary algorithm and extinctions are not possible. Interactions between players are mediated by some game, which determines how much energy a player obtains. Therefore it is applicable to any game, either some simple game such as Iterated Prisoner's Dilemma (Aktipis, 2004) or a complex game where strategies are computer programs (Lenski et al., 2003). Population control is also independent of the game as only depends on population size.

Conclusions and Future Work

We presented an Evolutionary Algorithm (EA) that fits in the field of Evolutionary Game Theory (EGT) and of Individual Based Model (IBM). That is to say, players go through the stages of birth, growth, reproduction and death all mediated by some game that is characterised as an energy transfer process. A game instead of having a set of payoff functions, has a set of energy transfer functions. A player's chromosome codes his strategy and a player's phenotype executes the coded strategy and has associated an energy level. Reproduction occurs when a player reaches some energy threshold. In order to avoid exponential population growth, a death event, by population size, is performed in every iteration of the algorithm. This creates a new dilemma that players of a game must face, namely extinction.

Our EA can be compared to other models where energy managing is the focus (Lenski et al., 2003; Yaeger, 2009; Ray, 1997). While in these models extinction is circumvented or is not the focus, in our algorithm it is a dilemma and a danger that a population must keep evading. This is a harder dilemma than cooperation, because while with the latter a population may stay in some non-cooperative state for a long a time, an extinction is a dead end, it is the only absorbing state of our algorithm.

The algorithm is generic has it can be applied to any game, be it a simple one such as Centipede, Prisoner's Dilemma (PD), Ultimatum or PGP or a complex game whose action space is not explicitly given but instead there is some logical description to construct strategies to play the game.

We have applied our EA to the Centipede game. This is an asymmetric game with two types of players. Extinctions occur when players can only play one of the two types. This dilemma can be circumvented if players are able to play both types. Contrasting with Rand and Nowak (2012) we did not have to lower selection strength.

Concerning future work, we have some preliminary results regarding the application of our algorithm to other games such as Ultimatum, PGP or 2-player 2-action games. In the results obtained so far we have seen the occurrence of extinctions. We plan to increase the extinction pressure on players by adding a death by old age. This puts pressure on players to find mechanisms to avoid the fate of extinction. A possible mechanism is partner selection (Mariano and Correia, 2011). Another avenue of research is considering variable player's energy as is done in artificial ecosystems (Yaeger, 2009; Ray, 1997). This means that the fitness function can be interpreted as an energy transfer process not only from the environment to the player but backwards. This puts extra pressure in games that have costs associated to some actions such as PGP or some variants of 2-player 2-action games.

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