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Invasion of cooperation in scale-free networks: Accumulated vs. average payoffs

Genki Ichinose[1,2] and Hiroki Sayama[2]

[1]Anan National College of Technology, Anan, Tokushima, JAPAN
[2]Collective Dynamics of Complex Systems Research Group, Binghamton University, Binghamton, NY, USA
ichinose@anan-nct.ac.jp

Abstract

It is well known that cooperation cannot be an evolutionary stable strategy for a non-iterative game in a well-mixed population. In contrast, structured populations favor cooperation since cooperators can benefit each other by forming local clusters. Especially, previous studies have shown that scale-free networks strongly promote cooperation. However, little is known about the invasion mechanism of cooperation in scale-free networks. Here we conducted evolutionary simulations of the evolution of cooperation in scale-free networks where, starting from all defectors, cooperators can spontaneously emerge by mutation. The purpose of this study is to reveal microscopic and macroscopic behaviors of the cooperators' invasion into the network. Since the evolutionary dynamics are influenced by the definition of fitness, we tested two commonly adopted fitness functions: accumulated payoff and average payoff. The simulation results show that cooperation is strongly enhanced in the case of the accumulated payoff when the temptation for defection is low. However, as the temptation becomes higher, cooperation persists more in the case of the average payoff. Moreover, in the case of the average payoff, lower degree nodes play a more important role in spreading cooperative strategies compared to the case of the accumulated payoff.

The emergence of cooperation is one of the challenging problems in both social and biological sciences. Cooperators benefit others by incurring some costs to themselves while defectors do not pay any costs. Therefore, cooperation cannot be an evolutionary stable strategy for a non-iterative game in a well-mixed population. In contrast, structured populations favor cooperation since cooperators can benefit each other by forming local clusters (Nowak and May (1992)). Especially, it is known that scale-free networks strongly promote cooperation (Santos and Pacheco (2005)). However, little is known about the invasion mechanism of cooperation in scale-free networks.

Here we conducted evolutionary simulations of the evolution of cooperation in scale-free networks where, starting from all defectors, cooperators can spontaneously emerge by mutation. The purpose of this study is to reveal microscopic and macroscopic behaviors of the cooperators' invasion into the network. Since the evolutionary dynamics are influenced by the definition of fitness, we tested two commonly adopted fitness functions: accumulated payoff and average payoff. Barabási-Albert method (Barabási and Albert (1999)) is used for generating initial scale-free networks in simulations. Then, each generated network is substantially randomized by the double-edge swap method while keeping the original degree distributions. Self loops and parallel edges are avoided during the randomization.

A network is made of N nodes occupied by individuals. Each node has its strategy classified as either C (cooperator) or D (defector). Initially, all individuals are defectors. Each node i plays the Prisoner's Dilemma game (PD) with all of its k_i neighbors. The payoffs of the game are calculated as follows. Both individuals obtain R for mutual cooperation and P for mutual defection. If one selects cooperation and the other selects defection, the cooperator obtains S as the sucker of defection, and the defector obtains T as the reward for temptation to defect. The order of the four payoffs is $T > R > P \geq S$ in typical PD. We set $P = 0$, $T = 1+b$, $R = 1$, and $S = 0$, where $b > 0$ is the only control parameter, following previous studies. The payoff of individual i against its k_i neighbors is denoted by p_i. Here we consider two types of p_i: accumulated payoff and average payoff. The average payoff is obtained by dividing the accumulated payoff by k_i.

At the beginning of each simulation, one randomly selected individual x plays PD with the neighbors and obtains payoff p_x. Next, one randomly chosen neighbor of x, denoted by y, also plays PD with its neighbors and obtains payoff p_y. If $p_x < p_y$, individual x imitates individual y's strategy with probability $(p_y - p_x)/[(T-S)k_{\max}]$, where $k_{\max} = \max(k_x, k_y)$. Finally, another randomly selected individual z (z might be the same as x or y) flips his strategy (C will become D and D will become C) by mutation with probability m. These operations consist of one time step. We regard N time steps as one generation, in which all individuals are selected once, on average, for the strategy update and mutation. The parameter sets used in the simulations are $N = 5,000$, $\bar{k}$ (average degree) $= 4$, and $m = 0.005$.

First, we focus on macroscopic behaviors of the cooperators' invasion. Figure 1 shows the fraction of cooperators vs. time (A) and vs. temptation to defect, b (B). As previous studies already revealed, cooperation is promoted in the case of accumulated payoff than the case of average payoff (Fig. 1(A) $b = 0.2$). This is because cooperators on hubs, i.e., highly connected nodes, can gain much greater payoffs than others. However, if the temptation to defect is high ($b = 0.8$), cooperators cannot occupy hub nodes, and therefore the hub effect disappears and cooperation is not promoted, which was not discussed much in previous studies. In contrast, cooperation persists for higher values of b (> 0.6) in the case of average payoff, although the fraction of cooperation always stays at low levels (Fig. 1(B)).

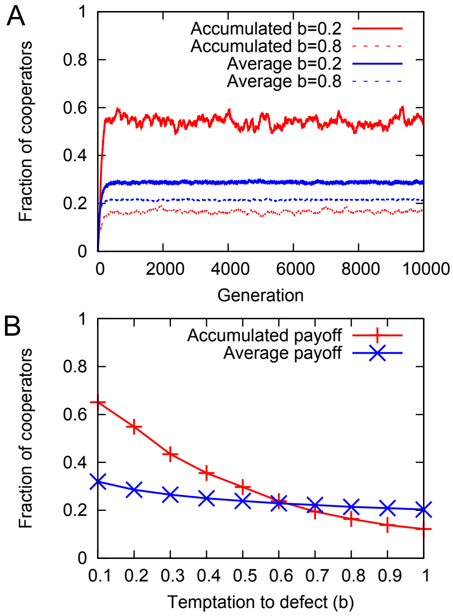

Figure 1: (A) Fraction of cooperators vs. time. Ten independent simulation runs are averaged. (B) Fraction of cooperators vs. temptation to defect (b). Five independent simulation runs (the last 1,000 generations for each) are averaged.

Next, we focus on the microscopic behaviors of cooperator invasion. Figure 2 visualizes simulation results in histograms of propagation events of cooperation, plotted over the degree of the source node of cooperation propagation and its neighbors' state ratio (1 = fully cooperative, 0 = fully defective neighborhood). In the case of the average payoff (Fig. 2(B)), lower degree nodes play a more important role in spreading cooperative strategies compared to the case of the accumulated payoff (Fig. 2(A)), because hubs cannot contribute to the spread of cooperation with the average payoff. Moreover, when cooperators spread, they need a certain fraction of cooperators in the neighbors. However, the fraction of cooperators in the neighbors tends to be low if the degree is high such as hubs. This also makes it difficult for cooperators to spread in the high degree nodes in the case of the average payoff.

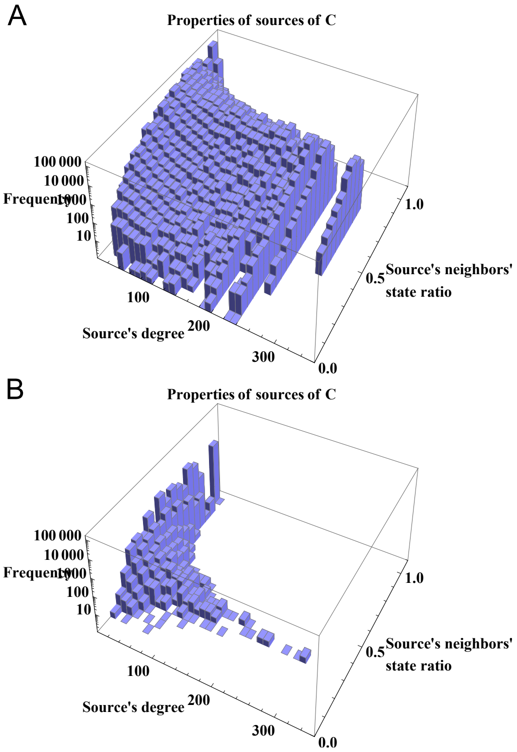

Figure 2: Frequency of propagation events of cooperation, plotted over the source node's degree and its neighbors' state ratio in the accumulated payoff (A) and the average payoff (B) ($b = 0.2$). First 500 generations of 10 simulation runs are accumulated.

In conclusion, we investigated cooperators' invasion in scale-free networks that are initially dominated by defectors. Here we show that cooperation is promoted more in the case of the accumulated payoff only when the temptation to defect is low. This implies that the evolution of cooperation on a network depends significantly on how game players are rewarded through their game play. Moreover, from the in-depth analysis of microscopic behaviors, we show that the relative importance of low degree nodes for the evolution of cooperation is much higher in the case of the average payoff, compared to the previously studied case of the accumulated payoff where hubs are known to play a major role in the propagation of cooperation.

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