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There can be Only One: Reversible Cellular Automata and the Conservation of Genki

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Abstract

Reversible cellular automata (RCAs) are a special class of cellular automata with some very distinctive properties. We present a novel observation regarding a certain class of RCAs, a class that includes Norman Margolus' "Critters" rule. From a broad range of initial conditions, this class of cellular automata converge to a state in which all the structures in the system are periodic (the equivalent of blockers and blinkers in Conway's life), with the exception of a *single* glider.

The glider that remains is immortal. It follows from a generic property of RCAs that the last glider in the system cannot be destroyed, and its motion cannot enter a periodic cycle. On colliding with a periodic structure, the last glider may change direction or turn into a different type of glider. Very occasionally it will transform into two gliders for a period of time, but when these collide the result is likely to contain only a single glider again. We give some intuitive explanations for why the system converges to a state with only one glider, rather than many.

It seems relatively easy to construct systems with this single-glider property using block CA rules. We give an example where cells can take a number of different colours, and gliders must contain at least three colours (including the background one). When a glider collides with a periodic structure, the new glider resulting from this collision may be composed of different coloured cells than the original one. Thus, some essential organisational property has been transferred from one set of coloured tiles to another. We call this property "genki", after a Japanese word meaning health or vitality. We speculate on how it might be formally defined and whether it is applicable to RCAs in general.

Some cellular automata have the interesting property of being reversible, meaning that the previous state can always be determined from the current state. Such systems can never "destroy information", which constrains their dynamics in interesting ways. A reasonably well-known example is Norman Margolus' "Critters" (Margolus, 1999), the rules for which are shown in Figure 1. Here we present a slight variation on the Critters rules (also in Figure 1), which we call "Highlander", after a movie in which immortals battle to the death until only one remains (Mulcahy, 1986).

In addition to being reversible, Critters and Highlander share the property that they conserve the number of white and black cells. They have similar dynamics, but with a key difference: gliders emerge less readily from random initial configurations in Highlander than they do in Critters.

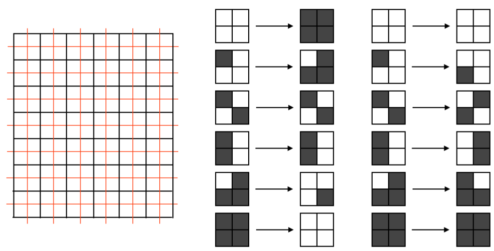

Figure 1: (Left) Critters and Highlander are block cellular automata. On even time steps the grid is divided into blocks as indicated by the black lines, and on odd time steps as indicated by the red lines. (Right) on each time step, the rules are applied to each block independently. Critters restores the original number of white cells after two time steps, whereas Highlander preserves the number of white cells directly. Both sets of rules are reversible because, for each block, the previous state can be uniquely determined given the current state.

As pointed out by Margolus, reversible CAs have the property that if two gliders (or a glider and a blinker) collide, the resulting pattern cannot remain spatially bounded. If it did, it would have to go into a periodic orbit, violating reversibility. Thus, at least one glider must always emerge from a collision. We call the property of being able to produce one glider "genki". Gliders transfer genki from one region to another, and genki is always preserved in collisions, although the number of gliders is not.

Because gliders are difficult to produce in Highlander, there is a tendency for only one glider to be created from the debris of a two-glider collision. An example of this is shown in Figure 2. This has a curious consequence: Starting from an initial condition with only a small proportion of white pixels, which are initially concentrated in one place, the system converges to a state in which only a single glider exists.

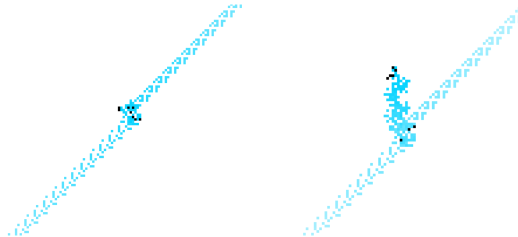

Figure 2: A head-on collision between two gliders. White pixels that were black a multiple of 60 time steps ago are shown in shades of blue. (Left) after the collision, a somewhat chaotic state is produced. However, since this state cannot be periodic, it must eventually emit at least one glider. (Right) 3360 time steps later, a glider is visible moving upwards from the point of the collision, with some non-moving debris left behind.

This glider bounces off stationary pixels in a pseudo-random way, gradually dispersing them throughout the space. An example of these dynamics (on a 400×400 grid with periodic boundary conditions and an initial square of 11×11 black pixels) is shown in Figure 3.

We can make sense of this from the point of view of statistical mechanics. The reversibility property means that a finite system must eventually return to its initial state, but this may take a huge number of iterations. The initial state we choose is a highly atypical one, but the system must spend the vast majority of its time in much more typical states. Our initial conditions have a proportion of black cells of about 0.75%. A typical configuration with this proportion of black cells will not contain a single glider, because gliders require several black cells to be near each other. (We have not observed any gliders with fewer than 4 black cells.) However, the conservation of genki prevents these states from being reached: the final glider cannot be destroyed. Thus, we hypothesise, the system will spend the vast majority of its time in the most typical type of state that it can access: one where the majority of white cells are scattered uniformly, with the initial genki contained in a single glider.

It seems relatively easy to create RCAs with the Highlander property. We suspect that Critters itself has it, but it takes longer to reach the single-glider state because new gliders are formed more readily in Critters. In particular, we have produced a multi-coloured version of the Highlander rules, where cells can have n different states, and the rule applied depends only on the number of *distinct* states in a given block: if there are two distinct states in a block (including black), it is rotated $90°$; if there are three then it is rotated $180°$, and if there are four then it is rotated $270°$. (Many variations of these rules work equally well.) With these rules a glider may change its constituent colours in a collision, as well as its speed and direction. (Figures illustrating this are omitted for reasons of space.)

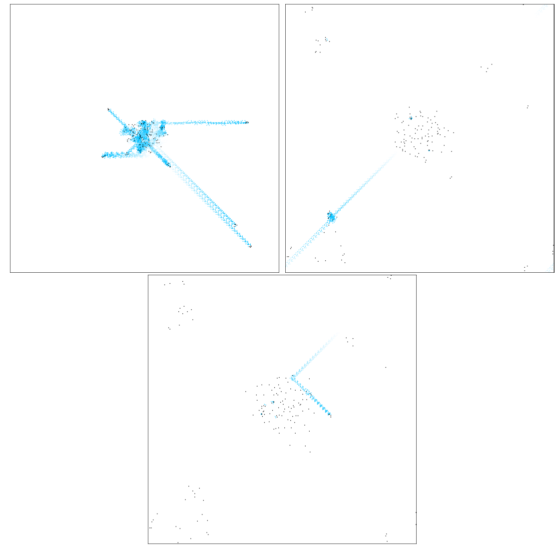

Figure 3: (Top left) $t = 14400$. Note that there are several types of glider, which move at different speeds in different directions. (Top right) $t = 1374240$; the last two gliders have collided. A single glider will result from the collision. (Bottom) at $t = 6100560$ there is still only a single glider in the system. Collisions may transform it into a different glider type and/or change its direction but are very unlikely to produce a second glider. These collisions have moved the black pixels slightly closer to a uniform distribution.

This abstract is not intended as anything other than an observation of an interesting phenomenon that we believe has not been noted before. However, careful study of reversible cellular automata might help us to understand complex phenomena in physics and chemistry. The laws of quantum mechanics obey unitarity, which is a close cousin of reversibility, and we therefore expect properties of reversible cellular automata to be shared by physical phenomena on the microscopic level. It is not clear what physical concept, if any, corresponds to our notion of genki. However, we note a resemblance between gliders in RCAs and particles in quantum field theory. In both cases, collisions may result in a variety of different outcomes, which are constrained by conservation laws. On a slightly larger scale, the gliders in these systems resemble radicals in chemistry, in that a reaction which destroys a radical also tends to create a new one. Continued study of these artificial systems may provide new insights into the physics that underlies living systems.

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