

## The Effects of Elitism on Spatial Coevolutionary GAs

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### Abstract

Elitism is a common concept in the standard Genetic Algorithm (GA), but one that has received little to no research when considered on spatial coevolutionary GAs. This paper attempts to address this by implementing and analyzing elitism mechanisms within a spatial coevolutionary system. To this end we develop a new globally selected local elitism and compare it with the more basic strictly non-local elitism generally seen in standard GAs. To aid in this analysis we also create a new simple function, one-max matching, which incorporates the simplicity of one-max yet is highly multimodal. We find that elitism seems to have a massive positive effect on coevolutionary systems. Heat maps show that the globally selected local elite often cluster within in the spatial network. Finally, surprisingly it took two elite per population in a coevolutionary system to see any benefit from elitism; we don't know why.

### Introduction

The Spatial Coevolutionary Genetic Algorithm (Hillis, 1990; Weigand and Sarma, 2004) brings together two paradigms within evolutionary computation: coevolutionary systems and spatially determined mating restrictions. However the spatial coevolutionary system provides the opportunity to introduce a hybrid mechanism not present in either system individually, spatial evaluation, which had been incorporated in both Hillis' and Weigand and Sarma's systems. In coevolution, the evaluation of a member from one population is based on the genotype or phenotype of one or more members from another population. With spatial evaluation the selection of the member, or members, of the other population is restricted to a spatially determined subset. The evaluative spatial structure is usually created such that members from the two populations that have an evaluative edge between them have neighbors that also have evaluative connected edges. Unlike the reproductive spatial structure, the evaluative spatial structure forms a bipartite graph between the populations.

While spatial coevolution has been around as long as coevolution has in the evolutionary algorithm world (both simultaneously introduced in Hillis 1990), only limited analysis has been done on these systems (Pagie and Hogeweg, 1997; Weigand and Sarma, 2004; Williams and Mitchell, 2005; Mitchell et. al. 2006) and none of them investigated the effects that elitism might have in such systems.

Elitism, where the best member from the previous population is copied unchanged into the current population, is a mechanism that has been used in the Genetic Algorithm (GA) almost since its inception (De Jong, 1975). Its purpose is to

prevent a population from losing the best solution it has found up until that point. While the elitism mechanism is not one that maps directly to the natural world of genetics, it has over time become ubiquitous in the GA world because of its effectiveness.

The addition of a spatial structure to reproduction added even more opportunity with the addition of local elitism. First introduced by De Jong and Sarma (1995) to fine-grained parallel GA systems and extended to any spatially structured GA (Sarma and De Jong, 1997), local elitism extends tournament selection, the most common selection technique used for spatial GA systems, to include the population member from the current generation, unaltered by mutation or crossover, in the spatial slot being filled when creating the next generation. With the addition of local elitism, the original "global" elitism becomes unnecessary since if the globally elite member would be the best member in the next generation, it would win its slot in its local tournament selection. If it doesn't win its slot, then it wouldn't be the elite after the generation following, so the effective difference is very short-lived.

Adding elitism to spatial coevolutionary systems is an obvious step to do, but probably to reduce complicating factors in their analyses, it has not been included in any of the spatial coevolutionary systems previously discussed. While elitism is the primary mechanism used in the Potter and De Jong's (1994) cooperative coevolutionary system, where each of the members from a population being evaluated is evaluated against the elite member of the other population, there is no spatial structure used for either the reproductive or evaluative aspects of the algorithm. This is unlike the cooperative coevolutionary system of Weigand and Sarma, 2004, which did use spatial structures but not elitism.

In this paper we will study the effects of elitism within a spatial coevolutionary system demonstrating its importance and utility. To accomplish this we needed to create a technique for elitism that can be used to seamlessly transition between the standard GA's global elitism and the spatial GA's local elitism. To aid in the analysis we also develop a simple test problem based on one-max, but is also multimodal.

In the next section we will define the spatial coevolutionary GA used for our set of experiments. We will then describe the experiments that will be used to determine the effectiveness of elitism in spatial coevolutionary systems. This will be followed by the results of the experiments, analysis and discussion of the results, and the conclusion.

## Implementation

### Elitism

In a regular GA, elitism is important as it prevents the best solution found so far from being accidentally lost through the vagaries of selection. This allows important genetic information to continually be disseminated throughout the population. The effect of the elite is not so clear when dealing with spatial GAs since the genetic material from the elite member can only be of immediate use to its local neighborhood. Local elitism (De Jong and Sarma, 1995) allows for the elite to emerge locally through tournament competition as described previously. However this produces a “ratcheting effect” everywhere, were no solution in the next population can be worse than from the previous population, which is not present in a standard GA. While it is possible to invoke a tournament between parents and children for inclusion in the next generation (which is done in some GA implementations) this is far from being standard, and could easily place too strong a selection pressure on the population. On the other hand, if you treat the “global” elite in a spatial population the same as you would in a standard GA and just copy it unchanged into the next generation, it limits the amount of spatial variation in its locality, which is already limited by spatial reproduction and can easily stall evolutionary progress in the very area that has the most promising solution. Since the effect of elitism is so different between the standard GA and a spatial GA (and even more so a spatial coevolutionary GA), it becomes very difficult to compare elitism directly between the two systems.

To facilitate direct comparison, we introduce an “intermediate” elitism mechanism between the “global” elitism of the standard GA and local elitism of the spatial world. Instead of implementing local elitism everywhere, we choose  $k$  unique elite and for each of those “global” elite, we apply local elites. In other words, we globally select and locally implement.

This globally selected local elitism allows us to smoothly transition between the elitism used in the standard and the spatial GA. When the spatial system is fully connected, the elite member is compared against offspring that come from parents that can be anywhere in the population<sup>1</sup>. The chances are therefore that the elite member will win the tournament and be copied into the next generation as with a standard GA. When  $k$  is set the full population size, the system becomes local elitism. In our experiments  $k$  is set to 1, 2, 10 and 100, where 100 is the full population size.

We will use this version of elitism in all of our experiments except for one. In that experiment we allow the elite member to be copied into the next generation whether or not it won the local tournament. This will allow us to observe whether, as predicted above, the detrimental effects that non-local elitism should have on spatial systems actually transpires.

### General GA

One of the main experimental controls we imposed for this study was to use as simple a GA on as simple a problem as possible without becoming overly trivial and providing sufficient features to differentiate between various properties of

<sup>1</sup> Please note that neither parent need be the elite member itself; see the discussion of how our spatial system is designed in GA Setup.

interest. With this in mind we chose to use a population size of 100, big enough to setup non-trivial spatial networks, yet small enough to be able to study with a fair amount of ease. We have a 0.8 probability of crossing over using uniform crossover with a parameter of 0.3 (i.e. a 0.3 probability of crossing over a bit location). The mutation rate is set to  $2/L$  where  $L$  is the chromosome length used in the problem and which is problem specific. For convenience we applied a max generation of 5000 for all of our simulations to ensure that the majority of runs will complete, but if not that they would still terminate in a timely manner; a simulation which has not completed after 5000 generations is simply considered a ‘failure’, having failed to find the optimal solution. Each experiment will be run through 50 repetitions in order to obtain statistically relevant results.

### Spatial GA

The reproductive spatial GA used for coevolutionary system, while keeping close to a standard spatial system, does have some slight differences introduced to allow for a seamless transition between the spatial GA and the standard GA, which is designed to occur when running the spatial GA on a complete graph. In our spatial system instead of mutating the current individual at a location or having it as one of the two parents for crossover, the neighborhood is determined, which includes the node to be filled, and then one or two members (depending on whether crossover is to be done or not) are selected from the entire neighborhood, reproduced and then stored in the next generation’s node. When crossover is performed only one of the children are kept (randomly determined). This technique was chosen to match the design decision used in Mitchell et. al. (2006): the only spatial coevolutionary paper to both use and fully describe their spatial crossover technique. If elitism is used, and the current node holds one of the elites, the newly created offspring and the elite member have binary tournament selection applied, implementing local elitism.

As we are attempting to study the effect of elitism we will be changing the number of elites used in order to test its effects. In general we are testing with 0, 1, 2, 3, 5, 10, 50 and 100 (or full) numbers of elites, as stated previously. It is important to remember that we are implementing unique elitism when considering the case where  $k = 100$  (i.e. full local elitism), as this means that there are still some nodes which may be not be elite if there are duplicates at other nodes.

At this point we must point out a minor bias in our elitism selection; one that becomes fairly obvious when we examine the movement of elites in our heat map experiment. When we are selecting our unique elites we sort the nodes by fitness and then remove duplicates. However, the sorting algorithm used is a stable sort, so when it comes across a tie it keeps the nodes in the same order as they occur in the population of which we choose the first as the elite member.

The reproductive spatial structure used in all cases is the standard toroidal grid. This topology was chosen as it was the most common one used in the spatial coevolutionary literature (Hillis, 1990; Wiegand and Sarma, 2004). Mitchell et. al. (2006) also used a toroidal grid, but included the diagonal grid nodes in the neighborhood.

## Coevolution and Spatial Evaluation

For all coevolutionary experiments done in this paper all evaluations are performed using a single member from one population paired with a single member from the other population. In all spatial coevolutionary systems the spatial evaluative structure is a simple one-to-one matching on the reproductive grids. In other words, if a member from one population is located at grid location  $\langle 5, 7 \rangle$ , it is evaluated along with the member from the other population also located at  $\langle 5, 7 \rangle$  on its reproductive grid structure.

This method of evaluation has a bleed-over effect on the non-spatial coevolutionary system as implemented. Just as with the standard GA, the non-spatial coevolutionary GA is just the spatial GA run on a complete graph (for both populations). Consequently, our non-spatial coevolutionary system also uses a one-to-one evaluation. This comes about naturally through the implementation, since the exact same coevolutionary system is used, but with the graph being changed from a grid to a complete network. While at first one may think that this is not very much like a standard coevolutionary setup, with some reflection it can be seen that the parents can come from anywhere in the population (since the reproductive structure is completely connected), and so the evaluative pairing is actually random. Thus evaluation has no actual one-to-one pairing of children from the same location within the two populations that, a priori, one might expect with such a setup. The one exception to this comes from the elitism mechanism as elites hold their position independent of where the offspring come from. Therefore, matched elites would still be evaluated against each other and would not shift positions. Thus elites create a stable matching pair through this mechanism, and with many elites, could give the system similar behaviour as the spatial coevolutionary system. We choose to use this elitism style in order to use a cohesive GA for all experiments, spatial and non-spatial; a more general version of elitism will also be explored.

## Experimental Design

We will look at the effects of elitism using a cooperative one-max problem and a cooperative one-max matching problem (described below). Although we do have preliminary results for a related competitive function, a fuller exploration will have to wait for future work.

In all cases the problems will be run on the standard GA, the spatial standard GA, the non-spatial coevolutionary GA, and the spatial coevolutionary GA.

### Elitism on One-Max

The one-max problem is one of the simplest problems used for evolutionary computational analysis; it is a simple summing of the number of ones in a binary chromosome, with the maximum occurring on a chromosome with all ones. While it is obvious that elitism should help improve the GA's performance on the one-max problem as it is a simple unimodal linearly separable problem, which is extremely amenable to hill climbing, it is not obvious as to what degree it will help. It also allows us to compare the effects of elitism on both a coevolutionary and standard GA using a very easily understood function to better observe the behavior of the systems

without complicating factors. The number of elites investigated are 1, 2, 3, 5, 10 and 100. Finally the one-max problem used has a chromosome length of 60.

The cooperative coevolutionary version of the problem is the same as used for the standard GA, but now the 60-bit solution is split into two halves of 30 bits each. During evaluation the two chromosomes, one from each population, are spliced back together into a single 60 bit one max chromosome which is then scored using same function as the original one-max problem.

### Elitism on One-Max Matching

One-max matching is a problem of our own design that combines the one-max problem with a simple matching problem commonly seen in game theory (Chen et. al., 1996). This problem was created in order to produce locals in the unimodal one-max, thus increasing its difficulty in a predictable fashion. Once again, this was done in order to reduce extraneous factors that a more complex problem could present in order to more clearly observe the behaviour of the underlying system. Matching is where one section of a chromosome must match another section in order to gain fitness. Each gene is paired with another gene and if they match the individual gains a fitness point. The result of the matching function is then added to the one-max score that has been calculated across the entire chromosome, though the matching problem's fitness is doubled to get the desired behaviour of matching being equal to one-max (there are twice as many loci to be counted in one-max as there are potential matches to be counted in matching). For our coevolutionary systems, individual members of both populations have "half" the chromosome, each of which is in turn matched within itself (as seen in Figure 1). In other papers in-progress we refer to this function as *inner-matching*. This is to distinguish it with the basic matching function that matches the chromosome from one population with the chromosome from the other. When basic matching is implemented in the standard GA the first half of

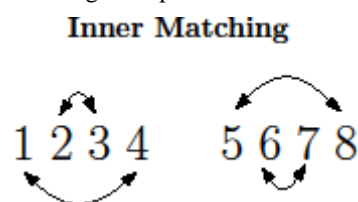


Figure 1: Inner Matching. Genes are matched within the same chromosome.

the chromosome is matched with the second half. As we only implement one-max-inner-matching, we shorten its name to simply one-max matching. All one-max-matching experiments use a chromosome length of 32 and elite sizes of 0, 1, 2, 5, 10, 50 and 100.

The one-max matching problem is interesting as it maintains the characteristic of being a simple problem like unimodal one-max while actually being massively multimodal, albeit with very small locals. Consequently, we believe that any observations made on this function have a better chance of generalizing to other fitness functions. The global optimum occurs at the same location as one-max, with all loci set to one, as both one-max and the matching function separately have maximum values at this location. The matching function however has many other equal global maxima; i.e. any chromosome where the two halves of the matching segments are the same. For example using an 8 bit chromosome with two 4 bit matching segments (separated by a space for easy compar-

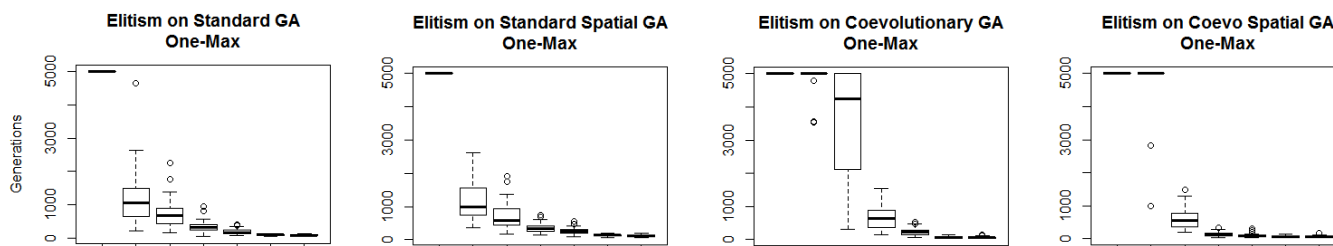


Figure 3: One-max elitism effects on different GA types. Each graph displays the number of generations required to find the solution at each elite size. Notice that the performance improves as we increase the number of elites for all types of GAs.

ison), 1101 1101 will produce a matching value of 4 (and a one-max-matching value of  $6 + 2 \cdot 4 = 14$ ). This produces a local maximum, as shown in Figure 2. Changing any of the 0 bits to a 1 will add 1 to the fitness due to one-max, but subtract 2 from the fitness due to loss of a match (which, remember is doubled). If on the other hand you change a 1 bit to a 0 bit, the function will lose both the one-max point and the matching double point. In either case the fitness will drop leading us to conclude that we are at a local maximum. A chromosome length of 32 will result in  $2^{32}$  possible solutions,  $2^{30}$  of which are local maxima, with the global maximum being one of them.

### Strictly Non-Local Elitism on One-Max

Here we perform a comparison between (globally selected) local elitism and (globally selected) strict elitism where the elite member is passed into the next generation independent of the local offspring's fitness. As noted previously, when implemented using a complete graph for spatial reproduction, local elitism should have almost identical behavior as regular elitism on a standard GA, but with subtle differences that we believe are insignificant.

To test this hypothesis we implement the standard GA style strict elitism and apply it to all GA types: standard, standard spatial, non-spatial coevolutionary and spatial coevolutionary. We will use elite sizes of 1, 2, 5, 10 and 50. An elite size of 0 is not used because when we have no elitism the style used is irrelevant and so the results would be the same as first experiment (see Figure 3). Also, an elite size of 100 (which we refer to as *full elitism*) is not tested as no chromosome in our population could change as every member would be an elite and would be passed to the next generation unchanged. If our hypothesis is correct, we should see little to no difference

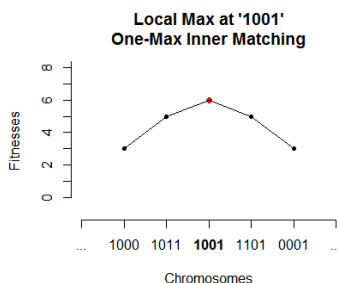


Figure 2: One-max matching's local optimum at '1001'. All chromosomes with a single bit flipped from '1001' are displayed; all cause the fitness to decrease. Note that the maximum fitness for this problem would be 8 at '1111'.

between the two elitism versions on the standard GA. Furthermore, we expect strict elitism to function poorly when compared to local elitism in all systems that have a spatial component. One possible caveat to the above is that the "locked-in" effect that strict elitism might produce in non-spatial coevolutionary systems could lead to unpredictable results.

### Heat Maps

We will use the conceptualization tool known as a heat map to track the relative fitnesses of nodes within our spatial GA at various generation points. Viewing the relative fitnesses of these nodes should help us understand the role elitism is playing on the spatial standard and spatial coevolutionary GAs. We will show heat maps of sample runs at elite levels of 0, 2, 5, 10 and 100. We do not bother using this visualization technique on the non-spatial GAs as the layout of fitness value would be completely random in the absence of a reproductive spatial network.

While the heat map looks like a basic grid it is important to remember that it is a representation of a toroidal grid where the left/right and top/bottom sides are connected with one another. Lighter coloured (white) nodes are the nodes with the highest fitness at that generation and darker (red) nodes are the nodes with the worst fitness in that generation. Five heat maps are shown per run: one at the beginning, one near the end, and 3 approximately evenly spaced throughout. Please note that the number of generations between heat maps is variable because of the stochastic nature of the number of generations it takes to find the solution.

### Experimental Results

We have found that our experimental data is generally non-parametric using a normal Q-Q plot. Therefore, we will be using non-parametric statistical techniques, such as the Wilcoxon Rank Sum test to compare data. The type of box plot we use displays the 'box' around the 25% and 75% quartiles and a bar at the median value. Whiskers are drawn from the box up/down to the most extreme value found within 1.5x of the interquartile range. Dots are then used to display all 'extreme' points that lie beyond the whiskers.

While often you can discern whether a difference is significant or not just by examine the box plots side-by-side, to ensure statistical accuracy, we use a Wilcoxon rank-sum test using a 95% confidence interval along with a Holm-

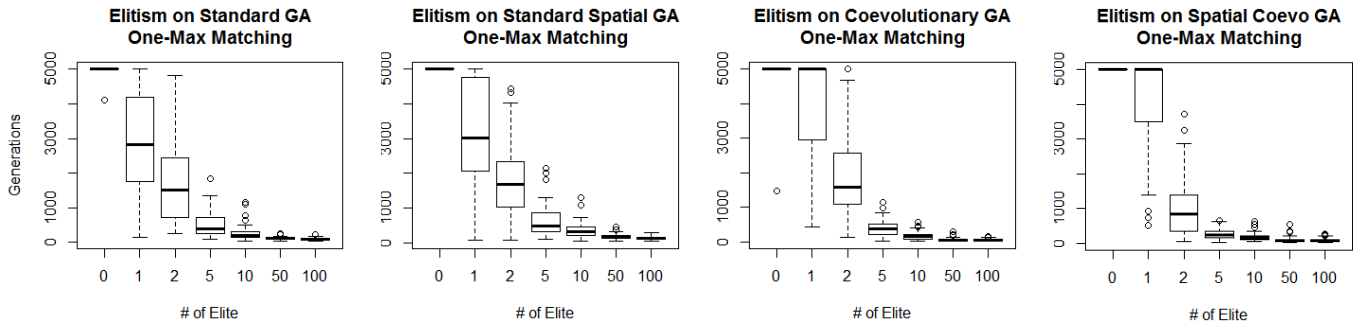


Figure 5: Elitism effect on different GA types solving one-max-matching, a variant of one-max that has local optima. Each graph displays the number of generations required to find the solution at each elite size. Notice that the performance once again improves as we increase the number of elites for all GA types.

Bonferroni post hoc correction. The post hoc correction is applied to ensure that when performing multiple comparisons, all hold simultaneously with a 95 confidence level and not just individually (i.e. controlling the family-wise error rate).

### Elitism on One-Max

The results from this experiment can be seen in Figure 3. Each graph displays the number of generations each of the 50 repetitions at each elite size took to find the completed solution. We can clearly see that elitism provides a significant performance increase to all four of our GA types. We also see that we require at least an elite level of 2 for our coevolutionary GA to be able to compete with the standard GA, although we do not yet understand this effect. Once we have enough elite members, elitism obviously helps all of our GA types; however the jump in performance for both the spatial and non-spatial coevolutionary GAs is astounding. Not only does the coevolutionary GA gain an asymmetric amount of performance from elitism, the spatial coevolutionary GA actually overtakes the standard GA in performance by the time we have 10 elites. In addition, the non-spatial coevolutionary GA has roughly the same performance as the standard GA at this elite level. These results can be seen more clearly in Figure 4 with p-values displayed in Table 1. Once we reach full elitism the coevolutionary GA has achieved better performance than either of the standard GA types, while the spatial coevolutionary GA has maintained its performance lead over all of the other GA types.

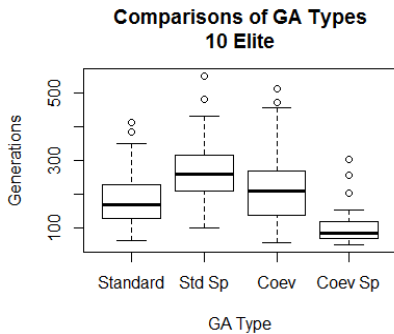


Figure 4: One-max with 10 elites. By 10 elites we see that the coevolutionary spatial GA has surpassed the others in performance (and continues to do so as more elites are added).

GA Type		W	P-Value
Coevo Spatial	Coevolution	259.5	4.39e-11
Coevo Spatial	Std Spatial	96	1.10e-14
Coevo Spatial	Standard	356	2.91e-09
Coevolution	Std Spatial	861	0.0148
Coevolution	Standard	1534.5	0.0502
Std Spatial	Standard	570.5	8.56e-06

Table 1: One-max with 10 elite performance comparisons using a Wilcoxon rank-sum test and a Holm-Bonferroni post hoc correction.

### Elitism on One-Max Matching

The one-max matching problem was being tested with the purpose of testing elitism on a problem with local optima. We're interested in seeing if our elitism mechanism would be hurt by a problem more complex than one max. Viewing the results in Figure 5 we can see that elitism did not hurt the GA systems when solving these problems at all, and in fact full local elitism provided the best performance for this problem on all GA types. We do see, as expected, that the one-max matching problem is a harder problem than standard one-max as we have a shorter chromosome length and yet we are finding worse performance (greater number of generations to find the solution).

We again see that the coevolutionary GAs are receiving a greater gain in performance than the standard GAs for increasing elitism. At an elite value of 2 the coevolutionary spatial GA is already statistically better than any of the other GA types, while the other three are roughly the same. By the time we've reached an elite level of 10, both coevolutionary GA types are better than the standard GA. This is shown in more detail in Figure 6 with p-values displayed in Table 2.

GA Type		W	P-Value
Coevo Spatial	Coevolution	1204	0.754
Coevo Spatial	Std Spatial	1915.5	2.73e-05
Coevo Spatial	Standard	1614.5	0.0443
Coevolution	Std Spatial	588	2.73e-05
Coevolution	Standard	896.5	0.0443
Std Spatial	Standard	1619	0.0443

Table 2: One-max matching with 10 elite performance comparisons using a Wilcoxon rank-sum test and a Holm-Bonferroni post hoc correction.

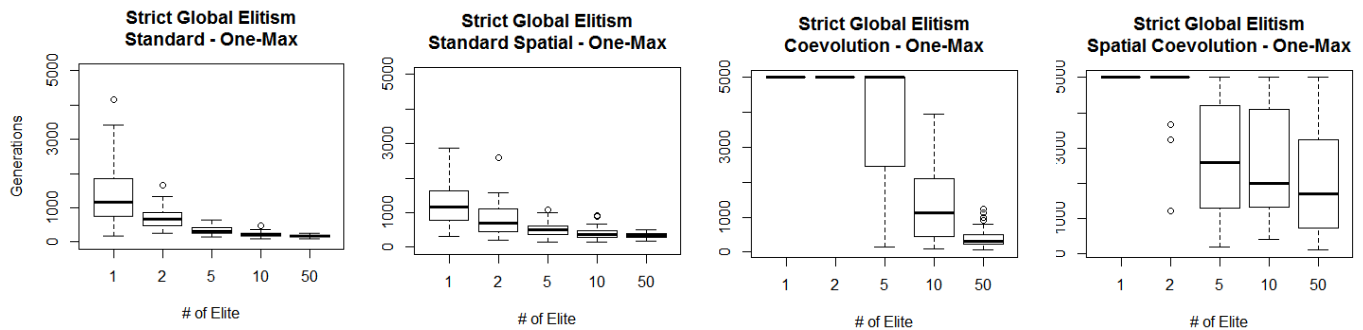


Figure 7: Strictly Non-Local Elitism on different GA types solving one-max. The coevolutionary GAs show an extreme degradation in performance when using this elitism style.

### Strictly Non-Local Elitism on One-Max

Strict elitism had a massive effect on the behaviour of the spatial and non-spatial coevolutionary GAs. Using strict elitism the coevolutionary GA's were unable to solve the one-max problem in any of the 50 trials we tried with up to 5 elites. Once we were using 10 strict elites our coevolutionary GA was able to succeed some percentage of the time, but the performance was still up to 10x worse than the standard GA; these results can be seen in Figure 7.

To contrast the massive failure of strict elitism on coevolution, the spatial and non-spatial standard GAs show little to no degradation in performance on the strict elitism style. This is very interesting as we had predicted that our local elites would be an improvement for the spatial GAs, and while there was a minor increase in performance for our standard spatial GA, it was much more so for both of the coevolutionary GAs.

This result somewhat undermines our 'ratcheting' hypothesis, as while this hypothesis may still be true, there is clearly at least something else at play here. We believe that the local elitism may be affecting coevolution due to more of a synergistic effect between the two populations. The elites in the two populations will end up in the same location as both organisms share the same fitness. This allows the population to keep two strong individuals (the elites) 'linked up', and allows them to work together (coevolve) and eventually find the optimum. When we are using strict elitism the linked up indi-

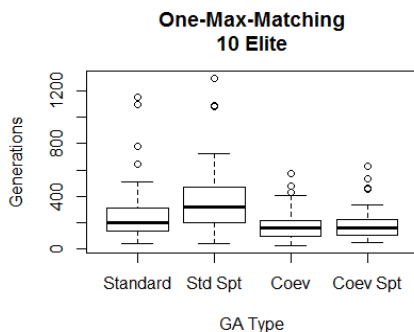


Figure 6: One-max-matching with 10 elites. Again, after 10 elites we see that the coevolutionary GA's performance has improved

viduals are unable to continue evolving as they are 'locked in' to their current set of chromosomes; they are both unable to continue evolving. Instead of the current elites eventually getting more highly fit (or possibly chromosomes nearby), the GA must instead wait until the other individuals in the two populations find their respective optimums at the same reproductive location in the same generation.

### Heat Maps

The heat maps are shown in Figure 8 for the coevolutionary and standard GA respectively. We can see a few interesting things from them. The first thing to mention is that we see very little difference when looking at the heat maps of the standard GA versus those of the coevolutionary GA. This implies that the movement of fitness values, and therefore elites, have similar behaviour in both GA types. This is interesting considering the fact that we have shown in the earlier experiments that elites are much more important to the success of the coevolutionary GA than the standard one. This increased success clearly has nothing to do the elites' movement within the population.

Another point of interest is that we see a significant difference in the fitness structure of the GA as we increase the number of elites. We can see that our elites are grouped near each other to form clusters of highly fit nodes. We see evidence of this in all graphs shown, but it is most readily apparent looking at the 3<sup>rd</sup> and 4<sup>th</sup> graphs of the 10 elite coevolutionary GA. We can also see this effect looking at the standard GA's graph with 5 elites, though they are focused around the bottom left corner which involves them wrapping around our toroidal structure which can be somewhat hard to visualize on our 2D grid.

The final point to note is the difference between the 0 elite heat maps versus full elite; one might think that these graphs would be the same as we have no elite clustering, however this is not quite true. While the early generations show little to differentiate them, in the later generations the nodes in the full elitism heat maps have a much more average set of data than in the non-elite heat maps. Full elitism appears to not only help the GA find the optimum faster, but also increases the average fitness of the population as well.

Fig 8a: Spatial coevolutionary GA

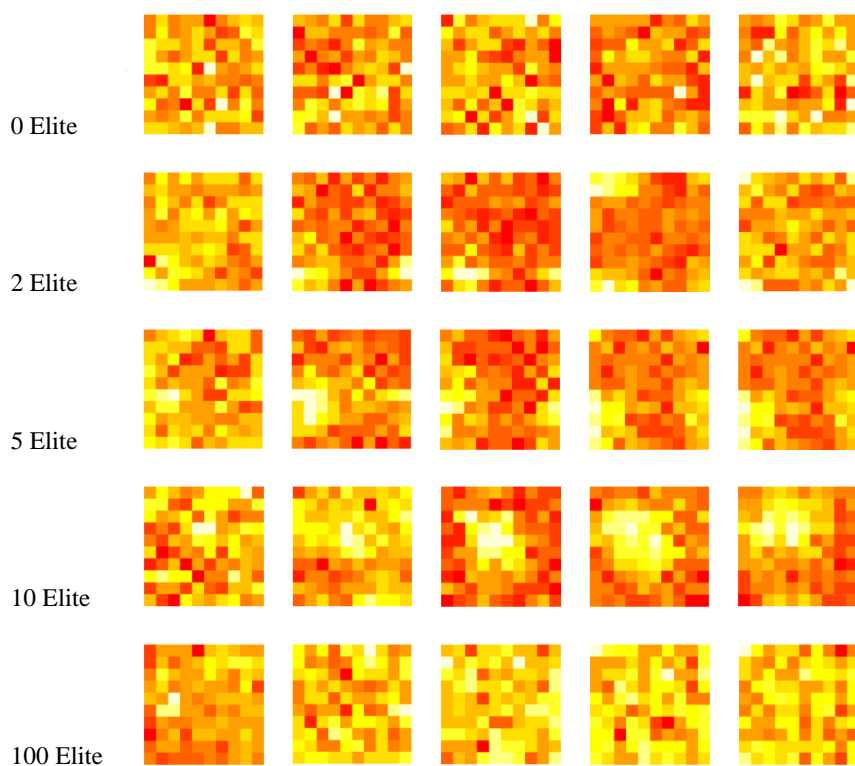


Fig 8b: Spatial standard GA

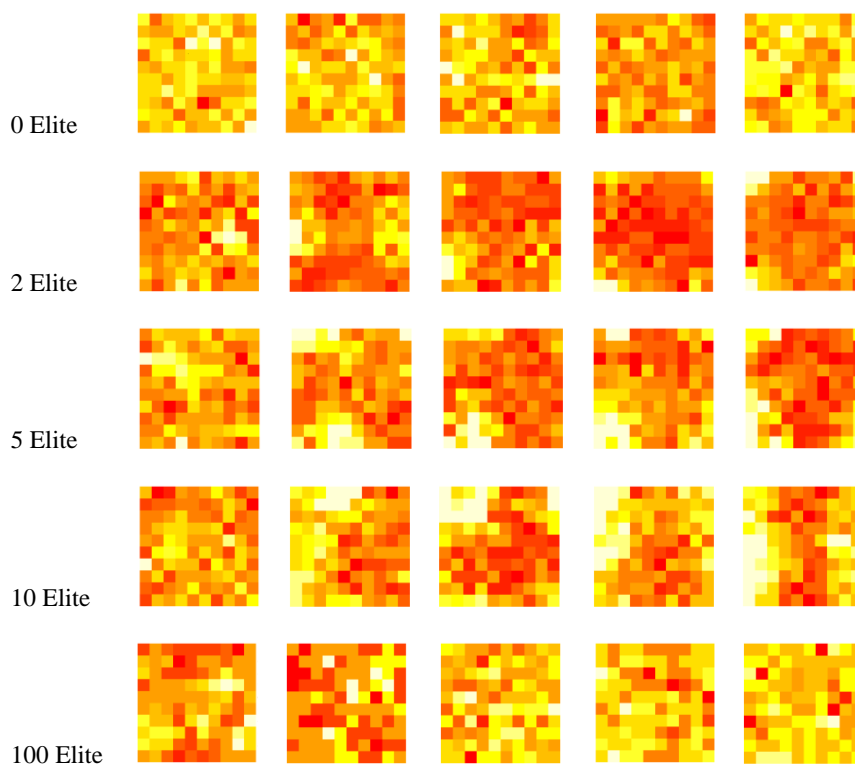


Figure 8: Heat map of fitness values with varying levels of elitism on coevolutionary (Fig 8a) and standard (Fig 8b) GAs. In each of the sub-graphs, the first graph displays the 10<sup>th</sup> generation (the 1<sup>st</sup> generation would just be a random seed) and the last graph shows the final generation. The other three graphs are evenly spaced between the first and max generation. The colours in each generation are relative to that generation. The lighter (white) nodes are the most highly fit and the darker (red) nodes are the least fit.

## Conclusion

In this paper we analyzed the effects of elitism on a spatial coevolutionary system and found it to be extremely, almost ubiquitously effective. To help in this analysis we developed a globally selected local elitism for the spatial component of the spatial coevolutionary system in order to transition between the strictly global behavior of the elitism mechanism of the standard GA and the local elitism of the spatial GA. This hybrid mechanism demonstrated the effectiveness of local elitism for spatial coevolution over the more strict elitism that ignores the improvements of the offspring. However we were surprised by the reduction of strength of the effect when used in a non-coevolutionary spatial GA.

A-priori it is obvious that elites will always help on unimodal problems such as pure one-max, as the GA cannot be trapped at any local optima. Keeping the best values found so far can only benefit the GA with no downside. After testing the GA on a problem with numerous, albeit small, local optima (one-max matching) and still seeing a significant increase in performance it is clear that elitism will still be helpful on all but the most deceptive of problems. Our version of local elitism allows evolution to occur once nodes across populations becomes 'locked in' through the elitism and continually 'ratchets up' their combined fitness in a synergistic manner. While this could theoretically allow the local neighborhood within the spatial coevolutionary GA to be trapped at a "local optima", we hypothesize that the global selection component of the mechanism allows these "locked in" pairs to be separated by the reproductive grid structure and thus allowing for gene-flow from nearby neighbors to allow the system to escape. Finally we observed that it required two elites in both of the coevolutionary populations for the extreme beneficial effects to be realized. Why one elite member per population was insufficient to see any measurable benefit cannot yet be explained.

## Acknowledgements

We wish to acknowledge the support and funding of the Natural Sciences and Engineering Research Council of Canada for this research.

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