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Coevolution of cooperation and layer selection strategies in multiplex networks

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Abstract

Recently, the emergent dynamics in multiplex networks, composed of layers of multiple networks, has been discussed extensively in network sciences. However, little is still known about whether and how the evolution of strategy for selecting which layer to participate in can contribute to the emergence of cooperative behaviors in multiplex network of social interactions. For this purpose, we constructed a coevolutionary model of cooperation and layer selection strategies in which each individual selects one layer from multiple layers of social networks and play Prisoner's Dilemma with neighbors in the selected layer. We found that proportion of cooperative strategies increased with increasing the number of layers regardless of the degree of dilemma, and this increase occurred due to the cyclic coevolution processes of game strategies and layer selection strategies.

Introduction

The recent progress in network sciences revealed that structures of interactions among individuals can affect the emergence and evolution of cooperative behaviors significantly (Nowak and May (1992), Ohtsuki et al. (2006)). This is because local interactions allow cooperative clusters to grow in the population of defectors in general (Nowak and May (1992)). While most of previous studies assumed that all individuals interact in a network of a single social relationship or context, there exist a bunch of networks of social interactions in a real world, and it is expected that they are affecting with each other directly or indirectly in various ways.

Such a situation of interactions among networks is known as a kind of multiplex networks, multilayer networks, interdependent networks or networks of networks, which have been discussed extensively in network sciences (Kivelä et al. (2014)), recently. A pioneering study showed that properties of cascading failures on interdependent networks differ significantly from those of single-network systems, in that the existence of inter-connecting links between networks changes the threshold and the order of transition for cascading failures (Buldyrev et al. (2010)).

There are various situations in which cooperative behaviors can evolve in multiplex networks of social interactions. Wang et al. discussed interactions between the evolution processes of cooperative behaviors in two interdependent networks (Wang et al. (2013)). In addition to the total payoff obtained from the Prisoner's Dilemma game (PDG) with its neighbors in a two-dimensional regular network, each individual obtains the additional payoff, that is the payoff obtained by another individual at the corresponding position in the other network, reflecting indirect and interdependent effects of a network on the other. They showed that the intermediate degree of interdependence contributed to the evolution of cooperation. They also showed that the degree of interdependence can self-organize to the optimal value (Wang et al. (2014)) through the individual-level adaptation of it.

In this study, we focus on the fact that an individual can belong to multiple kinds of social networks, which is a ubiquitous situation in a real life situation. We can expect that the behavior of an individual in one network can affect its future behavior in another network. Gomez-Gardenes et al. assumed that each individual belongs to multiple random networks and has a strategy of PDG (cooperate or defect) for each network that is called a layer. The population evolves according to the fitness determined by the accumulated payoff of the games with neighbors in all the layers (Gómez-Gardenes et al. (2012)). They found that the evolution of cooperation was facilitated by the multiplex structure only when the temptation to defect was large. However, it seems not natural to assume that all individuals always participate in games in all the networks, because there exist physical, social and temporal constraints in a real life situation. Instead, it is more natural to assume that each individual actively select which network to participate in depending on the state of interactions, and such a layer selection strategy can coevolve with game strategies.

Our purpose is to clarify whether and how the evolution of layer selection strategy can contribute to the emergence of cooperation in a multiplex network of social interactions. We assume multiple layers composed of random networks. Each individual belongs to all the layers but selects one layer and play games with neighbors in the selected layer. Both the layer selection strategy and the strategy for PDG for each

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layer coevolve according to the fitness based on the payoff from the games. We show that the larger the number of layers, the larger the proportion of cooperators increases, implying that multiplex networks can contribute to the evolution of cooperative behaviors. We also show it was caused by the dynamic coevolution process of strategies through which a burst of the proportion of individuals occurred in different layers repeatedly.

Model

Multiplex Network

Fig. 1 shows a schematic image of the model and Algorithm 1 also shows a pseudo-code of our model. There are M layers that abstract different channels or contexts of social interactions among individuals. For example, each layer corresponds to a social relationship in a real group or a friendship in a social networking service (SNS) in the Internet. Each layer is composed of a network of interactions among individuals in the corresponding relationship. An individual i is represented as a node n_i^l ($i = 0, 1, ..., N-1$) in the layer l ($l = 0, 1, ..., M-1$) , and thus it is represented as a set of nodes $\{n_i^0, \cdots, n_i^{M-1}\}$. The existence of a link between the individual i and j in the layer l means that i and j are neighboring individuals who can interact with each other in the layer l. In this study, the topology of each layer is defined as an Erdös-Rényi (ER) random graph with the average degree k. It is known that cooperative behavior is not easy to evolve in ER random graphs. We adopt this structure in order to see if increasing the number of layers can contribute to evolution of cooperation in spite of such a hard situation. Each time step is composed of two phases: playing games and updating strategies, as explained below. Hereafter, we describe that an individual i is in the layer l if it selects the layer l ($sl_i = l$), in that it participates in interactions in the social network represented by the layer l.

Playing games

We assume that each individual can participate in interactions in only one layer at every time step, reflecting the physical, temporal and cognitive constraints. Thus, each individual i has a layer selection strategy $sl_i \in \{0, 1, \cdots, M-1\}$. It determines the layer in which the individual i plays PDG with its neighbors. Hereafter, we describe that an individual i is in the layer l if it selects the layer l ($sl_i = l$), in that it participates in interactions in the social network represented by the layer l. Each individual i also has a strategy for PDG sp_i^l (cooperate (C) or defect (D)) for each layer l. It plays a PDG using the strategy $sp_i^{sl_i}$ with each neighboring individual j, in its selected layer sl_i, who is in the same layer ($sl_j = sl_i$) and plays $sp_j^{sl_j}$. The payoff matrix of the PDG is defined in Table 1. In addition, if there is a neighbor (j), in its selected layer sl_i, who is in a different layer ($sl_j \neq sl_i$), the individual i gets an additional payoff d regardless of the game strategy

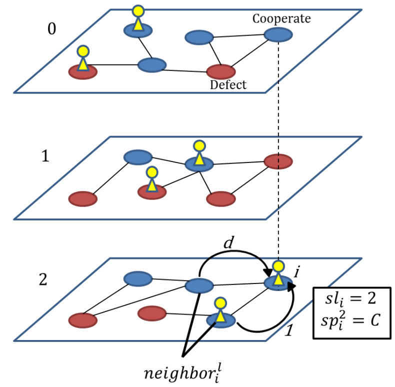

Figure 1: A schematic image of our model.

Table 1: A payoff matrix of PDG. b represents the temptation to defect.

sp_j / sp_i	cooperate (C)	defect (D)
cooperate(C)	1	0
defect(D)	b	0

of that neighbor j. d represents the payoff that can be earned by the individual alone instead of playing a game. We defined the parameter d in order to assume that not playing game is better than mutual defection but worse than mutual cooperation. This condition is satisfied if $0 < d < 1$. The total payoff P_i is regarded as the fitness of the individual i. For example, in Fig. 1, the individual i chooses the layer 2 and there are 2 neighbors in its selected layer. It obtains the payoff 1 by cooperating with a cooperator in the same layer, and earns a payoff d alone. As a result, it obtains the fitness $1 + d$.

Updating strategies

Each individual i updates its PDG strategy in the selected layer $sp_i^{sl_i}$ and its layer selection strategy sl_i according to the fitness after playing games. We assume that individuals can obtain the information about the fitness and PDG strategies of neighboring individuals in the selected layer before updating strategies. The value of PDG strategy sp_i^l in the next time step nsp_i^l is determined by the following procedure.

i One individual j is randomly selected from its neighboring individuals in the layer sl_i ($neighbor_i^{sl_i}$) regardless of sl_j.

ii If the fitness of the individual j (P_j) is higher than its own fitness P_i, $nsp_i^{sl_i}$ will be $sp_j^{sl_i}$ ($nsp_i^{sl_i} \leftarrow sp_j^{sl_i}$) with the following probability used in (Gómez-Gardenes et al.

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(2012)):

$$\prod_{i \leftarrow j} = \begin{cases} \frac{P_j - P_i}{b \max(|neighbor_i^{sl_i}|, |neighbor_j^{sl_j}|)} & \text{if } P_j > P_i \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

This equation means that each individual imitates the strategy of j with a positive probability if the fitness of the neighbor j is higher than its own fitness. The actual value of imitation probability depends on the difference between these fitness: the individual i always imitate the strategy of j when its fitness is the minimum and the fitness of j is the maximum. This imitation probability linearly decreases as the difference between these fitness values decreases. Otherwise, if the fitness of the individuals i is smaller than that of the neighbor j, it does not change the strategy ($nsp_i^{sl_i} \leftarrow sp_i^{sl_i}$). This means that it can imitate a PDG strategy of a better neighbor in its selected layer.

iii nsp_i^l is replaced with C or D randomly with a mutation probability μ.

The layer selection strategy sl_i in the next time step nsl_i is determined by the following procedures.

i One individual j is randomly selected from its neighboring individuals in its selected layer sl_i ($neighbor_i^{sl_i}$) regardless of sl_j.

ii If the fitness of the individual j (P_j) is higher than its own fitness P_i, nsl_i will be sl_j ($sl_i \leftarrow sl_j$). Otherwise, it does not change the strategy ($nsl_i \leftarrow sl_i$). This means that it can imitate a layer selection strategy of a better neighbor in its selected layer, which allows an individual to move to a different layer.

iii nsl_i is replaced with a random value from $\{0, 1, ..., M-1\}$ with the mutation probability μ.

Finally, all the strategies are updated simultaneously ($sp_i^{sl_i} \leftarrow nsp_i^{sl_i}$ and $sl_i \leftarrow nsl_i$, for all i).

In some situations, it might be plausible to assume that changing a group or network to which an individual belongs (i.e., its layer selection strategy) is easier than changing the strategy related to its personality (i.e., its game strategy). The processes described above reflects such a situation in which changes in the layer selection strategies can happen more frequently than changes in the game strategies.

Algorithm 1 A pseudo-code of our model. $payoff(a, b)$ represents the payoff value obtained by an individual who plays a strategy a with an opponent playing a strategy b. $neighbor_i^l$ represents the set of the neighboring individuals of the individual i in the layer j. $rnd(s)$ represents a function that returns a randomly selected element from the set s. rnddist() also represents a function that returns a random value from the uniform distribution [0, 1].

```
(initialize strategies)
for i = 0 → N − 1 do
    for j = 0 → M − 1 do
        sp_i^j ← rnd({C, D})
    end for
    sl_i ← rnd({0, 1...M − 1})
end for
for t = 0 → G − 1 do
    for i = 0 → N − 1 do
        (playing games)
        P_i ← 0
        for each neighbor_i^{sl_i} do
            if sl_i == sl_j then
                P_i ← P_i + payoff(sp_i^{sl_i}, sp_j^{sl_j})
            else
                P_i ← P_i + d
            end if
        end for
    end for
    (updating strategies)
    for i = 0 → N − 1 do
        (updating a PDG strategy)
        j ← rnd(neighbor_i^{sl_i})
        if P_i < P_j then
            if rnddist() < |(P_j − P_i) / (b max(|neighbor_i^{sl_i}|, |neighbor_j^{sl_j}|))| then
                nsp_i^{sl_i} ← sp_j^{sl_j}
            else
                nsp_i^{sl_i} ← sp_i^{sl_j}
            end if
        end if
        (updating a layer selection strategy)
        j ← rnd(neighbor_i^{sl_i})
        if P_i < P_j then
            nsl_i ← sl_j
        else
            nsl_i ← sl_i
        end if
    end for
```

Algorithm 1 continued

(mutation)
for $i = 0 \rightarrow N - 1$ **do**
  if rnddist() $< \mu$ **then**
    $nsp_i^{sl_i} \leftarrow rnd(\{C, D\})$
  end if
  if rnddist() $< \mu$ **then**
    $nsl_i \leftarrow rnd(\{0, 1...M - 1\})$
  end if
end for
for $i = 0 \rightarrow N - 1$ **do**
  $sp_i^{sl_i} \leftarrow nsp_i^{sl_i}$
  $sl_i \leftarrow nsl_i$
end for
end for

Experiments and Discussion

We conduced experiments on the constructed model for the purpose of revealing the co-evolution dynamics between the layer selection strategy and the cooperative behavior in multiplex networks. We used the following values as the experimental parameters: $N = 100$, $M = \{1, 3, ..., 19\}$, $k = 10.0$, $b = \{1.1, ..., 2.1\}$, $G = 10000$, $\mu = 0.02$ and $d = 0.4$. sp_i^l and sl_i were initialized with random values from their domains in the initial population. We used $d = 0.4$, which is an intermediate value in its domain ($0 < d < 1$), in order to see the basic effect of this parameter on the evolution process. The experiment results are the average of 5 trials for each combination of the parameter settings of M and b.

We aim to understand how the proportion of cooperative behaviors can be changed by the increase in the number of layers M. First, we define the selected strategy of an individual i as the one used by it in its selected layer (i.e., $sp_i^{sl_i}$), and focus on quantitative effects of M on the proportion of cooperation among the selected strategies of all the individuals R_c. We plot the average of R_c over all generations with different combinations of M and b, as a heat chart, in Fig. 2. The horizontal axis shows the number of layers M and the vertical axis shows the temptation to defect b. We see that R_c increased with increasing M and decreasing b. This means that the multiplex network facilitated the evolution of cooperation in any conditions of the Prisoner's Dilemma.

More specifically, R_c greatly decreased with increasing b when M was small. However, when M was large enough, R_c kept relatively large (or only slightly decreased) with increasing b. Thus, we can say that the negative effect of b on cooperative strategies could be reduced significantly by increasing the number of layers M.

Next, we plot the entropy of the probability distribution of sl_i, as a measure for the degree of dispersion of individuals over the networks in Fig. 3. We see that the entropy increased drastically with increasing M. This means that the wider distribution of individuals over more layers might contribute to the evolution of cooperation. We also see that the entropy slightly decreased with decreasing b, which implies that higher but not too high values of the entropy contributed to the evolution of cooperation.

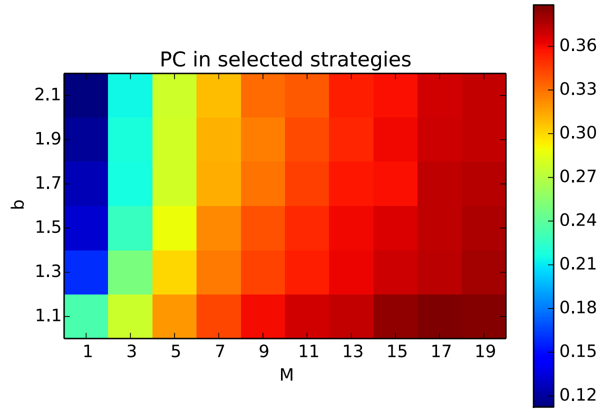

Figure 2: The heat chart of proportion of cooperative behaviors among the selected strategies $sp_i^{sl_i}$ (R_c).

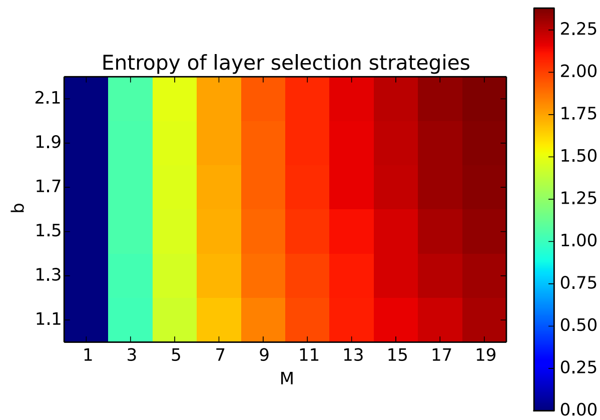

Figure 3: The entropy of the probability distribution of layer selection strategies sl_i.

Then, we focus on the transitions of the proportion of individuals in each layer and the proportion of cooperation in the selected strategies among them. We plot the transition of these indices from 2000th to 3000th step in typical trials when $b = 1.7$ and $M = 1$ (Fig. 4), 3 (Fig. 5) and 9 (Fig. 6). We focus on this period in order to observe the typical transitions after the transient process from the initial population. There are M panels, each corresponding to a layer. The horizontal axis represents step, the blue line represents the proportion of cooperation among selected strategies ($sp_i^{sl_i}$) of individuals in the corresponding layer. The red line represents the proportion of individuals that selected the corre-

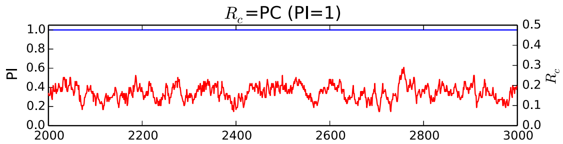

Figure 4: The transition of the proportion of cooperative behaviors among the selected strategies $sp_i^{sl_i}$ (R_c) when $M = 1$.

sponding layer. In addition, there is an additional panel in the bottom, which shows the average proportion of cooperation in the selected strategies.

When $M = 1$ (Fig. 4), all individual exist in a single layer ($PI = 1$). The proportion of cooperators slightly fluctuated at small values around 0.15. This can be said to be the baseline behavior of a standard model for the evolution of cooperation in a single and random network.

On the other hand, when $M = 3$ (Fig. 5), the average proportion of cooperators was higher than that when $M = 1$, which fluctuated at around 0.25. We also see that the proportion of individuals in each layer largely fluctuated and often reached very high values. This means that the individuals were distributed all over the layers, but they often got together in a layer.

Furthermore, when $M = 9$ (Fig. 6), the average proportion of cooperators became around 0.3, which was higher than that when $M = 3$. The occurrence of a burst-like rise and fall of the number of individuals in a layer was more pronounced and it often reached its peak around 0.8, meaning that most of individuals have selected the same layer. On the other hand, the proportion of individuals in the other layers tended to be much smaller than 0.2. We also see the gradual increase and the rapid decrease in the proportion of cooperators before and after the burst of the proportion of individuals, respectively.

The reason for this evolutionary dynamics that facilitated the cooperation can be summarized as follows. In this model, there are no games between individuals in different layers. Thus, the smaller the proportion of individuals in a layer is the higher the locality of interactions is, because it decreases the number of links used for playing games in effect. It has been pointed out that the higher locality for the smaller number of links can facilitate the evolution of cooperation (Ohtsuki et al. (2006)). Thus, cooperators can invade into a layer with the smaller number of individuals gradually. Such cooperative relationships in the layer make individuals in other less-cooperative layers (after a burst of the number of individuals) select the focal layer, which brings about a rapid increase in the proportion of individuals in the layer. However, this allows defectors to invade into the focal layer, and thus the proportion of cooperators decreases rapidly. In such a population of defectors, individuals select other cooperative layers because it is better not to play game with

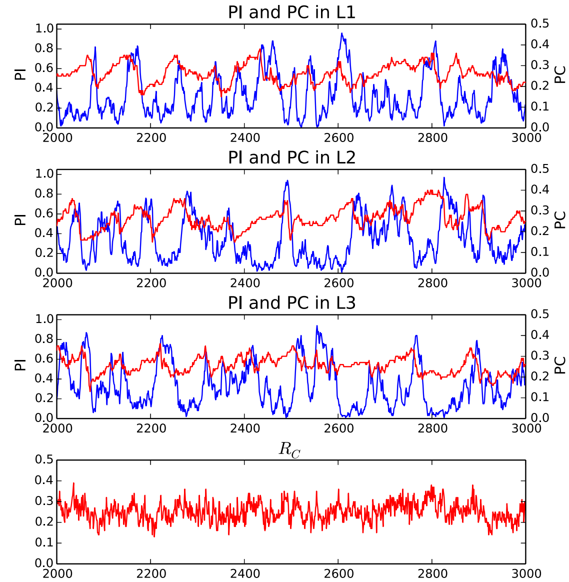

Figure 5: The transition of the proportion of cooperative behaviors (PC) and the proportion of individuals (PI) in each layer and proportion of cooperative behaviors among the selected strategies $sp_i^{sl_i}$ (R_c) when $M = 3$.

neighbors than to play games with many defectors. This further causes another burst of the proportion of individuals in another cooperative layer.

In Fig. 7 we plot the trajectory of these two indices in the 1st layer in Fig. 6 from 2000th to 4000th step. The horizontal axis represents the proportion of individuals in the layer and the vertical axis represents the proportion of cooperative strategies among the selected strategies in the layer. We see that the cyclic coevolution process of these indices occurred repeatedly, as explained above.

In addition, we conduced further experiments with different values of the parameter d. The general trend of evolution process did not change but the burst of the number of individuals occured more frequently with increasing d. This is expected to be due to the fact that individuals are easier to move to another layer by avoiding mutual defections as d increases.

Overall, repeated occurrences of this dynamic coevolution process of game strategies and layer selection strategies are expected to maintain the high proportion of cooperators in the whole population.

Conclusion

We constructed a coevolutionary model of cooperation and layer selection strategies in which each individual selects one layer from multiple layers of social networks and play Prisoner's Dilemma with neighbors in the selected layer, for the purpose to clarify whether and how the evolution of layer

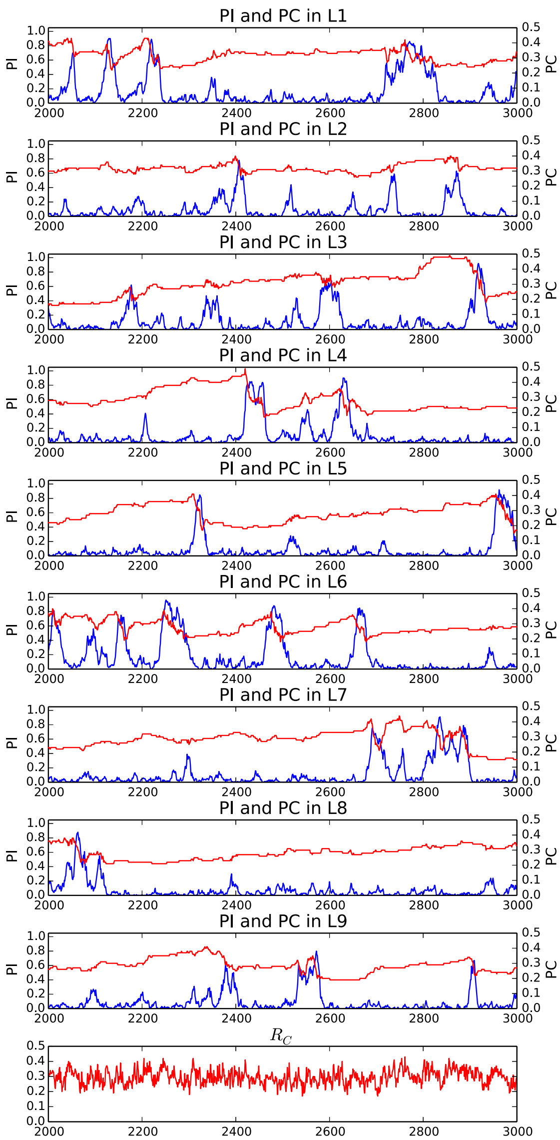

Figure 6: The transition of the proportion of cooperative behaviors (PC) and the proportion of individuals (PI) in each layer and proportion of cooperative behaviors among the selected strategies $sp_i^{sl_i}$ (R_c) when $M = 9$.

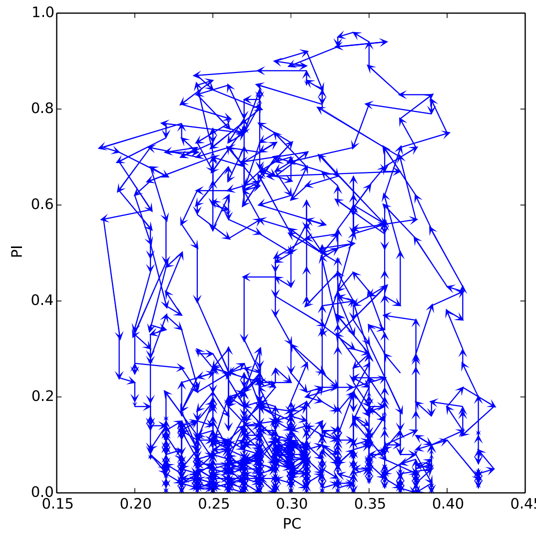

Figure 7: The trajectory of the proportion of cooperative behaviors (PC) and the proportion of individuals (PI) in the 1st layer in Fig. 6.

selection strategy can contribute to the emergence of cooperative behaviors in multiplex network of social interactions. From the results of experiments, we found that the proportion of cooperative strategies increased with increasing the number of layer regardless of the degree of dilemma, and this increase occurred due to the cyclic coevolution processes of game strategies and layer selection strategies. The emergence of such a cyclic process has been pointed out by the study of coevolution between cooperative behavior and network structures interaction in which the network rewiring strategies can coevolve with the game strategies (Suzuki et al. (2008)). Thus, this implies that such a dynamic process could be common phenomenon in a real world. It should be noticed that we further clarified that such a dynamic process can strongly facilitate the evolution of cooperation, in this paper.

Future work includes verifying whether the similar process can occur when the topology of networks is changed (e.g., scale-free or small world).

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