7 Mapping Trade Outcomes: The Shape of the Graph, Beneficial and Harmful Equilibria, and the Role of the Market

In chapter 3 we saw that the shape of the regions formed by the multitude of candidate equilibria for the two trading countries plays a crucial role in our analysis. The hill shape of this region's upper and lower frontiers enables us to determine when a country will gain and when it will lose by success in capturing an industry from its trading partner. The relative position of the peaks of the upper frontiers for the two countries shows the circumstances when the two countries will both gain from the migration of industries from one of the countries to the other, as opposed to those circumstances when their interests are in conflict. The shape of these regions of equilibria, and that of their upper frontiers in particular, show also when a country can be threatened with losses rather than gains from trade and suggests how common or rare this disturbing phenomenon can be. Thus it is important to provide some explanation of how that shape comes about. This chapter begins with such an explanation and then describes the linear program that we use to calculate the boundary from our trade model when we are given the values of the parameters.

7.1 Hill Shape of the Equilibrium Region: An Intuitive Derivation of the Upper Frontier

We will begin with the general shape of only the upper frontiers of the equilibrium regions for the two countries. These are the frontiers from which we obtain most of our conclusions. We again need to refer briefly to a third frontier, the world upper-income frontier, which indicates the upper boundary of the absolute incomes of the two countries combined. This aggregate income frontier shows the highest attainable world income for each point on the horizontal axis, that is, for each value of Z_1, which is the relative income of country 1. The combined

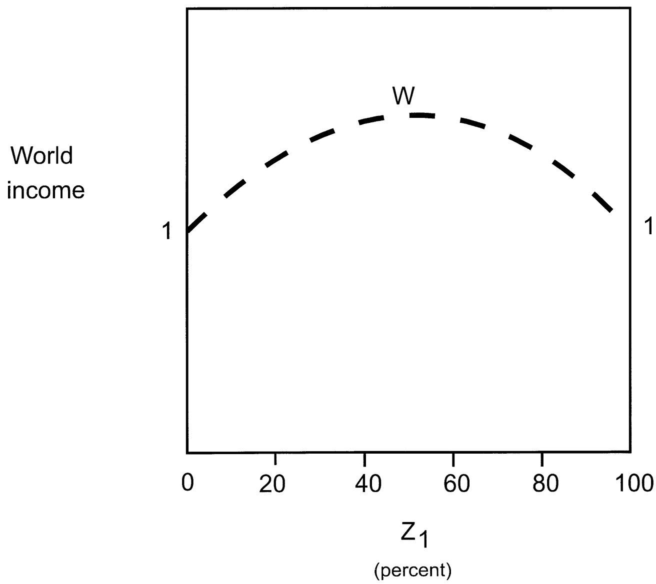

Figure 7.1
World upper income boundary

absolute income of the two countries can be calculated from our equilibrium model for each perfectly specialized assignment just as easily and in the same manner as for the aggregated income for a single country, and is shown as the dashed curve in figure 7.1.

This frontier is dome shaped. We see this in all our more than one hundred calculated models. There are two reasons for this. First the world's upper income frontier can be expected to be lower as we get near either end of the curve because at such at those extremes the entire labor force of one of the two countries is working on the very few goods that remain to that country. If we assume diminishing marginal rate of substitution among the world's products, the resulting large output quantities of those few items add much less value to the total world output than if that country's labor force were divided among a much larger share of the world's industries, as occurs in the middle of the graph. That is, near the edges of the curve there must be such an abundance of these few products that their total market value will be relatively low—much lower than the sum of the added values that could be obtained by moving much of that country's overspecialized labor force to a multitude of other industries. The second is to be found in the Ricardian explanation that output gains from trade result from specialization and the consequent opportunities to exercise comparative advantage. We recall that at both the right-hand and left-hand ends of our basic graph (see, e.g., figure 7.2 or figure 7.3), there will be no trade because at these extremes all traded goods are produced exclusively by a single country. That is, the right-hand end of the graph represents a tradeless state of autarky for country 1, in which it produces all n goods for itself. Similarly the left-hand end of the graph is a state of autarky for country 2. However, as we move toward the center of the diagram, the industries are divided ever more evenly between the two countries, until at a point near the middle (i.e., near $Z_1 = 0.5$) the two countries are each the exclusive producer of about the same number of commodities. Now, as is recognized by anyone who has studied international trade, this means that if the production functions of the two countries differ, then the potential Ricardian gains from trade will be increased as the industries are divided up between the countries. So, if there are productivity gains from trade, at all values of Z_1 intermediate between zero and unity, the height of the world's combined income frontier will be higher than it is at either of its end points. If there is some most efficient assignment of goods to producer countries, then there will be some point, call it W, in the interior of the diagram at which the world income frontier reaches its maximum. All of this means that the dashed world income frontier in figure 7.1 can be assumed to be dome shaped.

From this world income frontier we can now readily find the upper-income frontiers of the individual countries. Since country 1's income at any equilibrium point is total world income, $W(Z_1)$, multiplied by Z_1, country 1's share, that is, $Z_1 W(Z_1)$, its upper-income frontier will be dome shaped (the curve containing point M_1 in figure 7.2).[1] Its height, $0W(0)$, must be zero at the left-hand end of the graph where Z_1, country 1's share of world income, is zero. Similarly, at its right-hand end, where country 1's share is 100 percent, the height of country 1's upper frontier must be the same as that of the world income frontier. Thus, if the world frontier has the general shape depicted, the upper-income frontier of country 1 must also have the shape shown in the graph. It too will be a hill, with its peak, point M_1, to the right of that of W because at W world income is level, but country 1's *share* is increasing. The upper income frontier of country 2, the income of $(1 - Z_1)W(Z_1)$, will have the same general shape, roughly the mirror image of that of country 1. Thus the upper income frontier of country 2 will also be

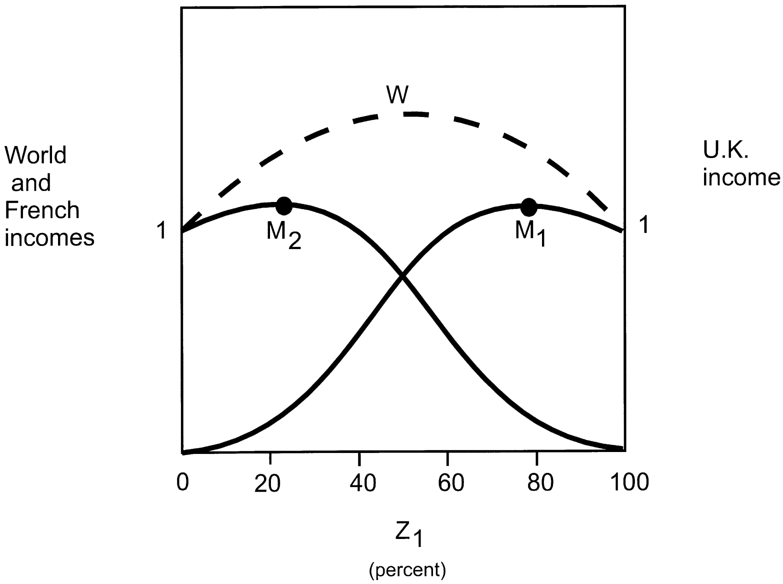

Figure 7.2
Three upper income boundaries

roughly hill shaped, with its peak, point M_2, to the left of W and, hence, to the left of the peak, M_1, of country 1′s frontier. All of this is depicted in figure 7.2, where we see that as we move to the right (so that country 1′s share of world income rises steadily), the country 1 frontier can be taken to move steadily ever closer to the world frontier, approaching the latter asymptotically. This completes the intuitive derivation of the shape of the upper-income frontiers of the two countries, on which the bulk of our policy discussion depends.[2]

7.2 Region of Candidate Equilibria: How the Boundaries Are Found

As has just been said, much of our analysis depends crucially on the shape of the region of candidate equilibria, a configuration that is characterized by its boundaries. It is consequently important for us to describe how those boundaries can be calculated, to assure the reader that they are determined in a defensible manner and that they do indeed characterize the equilibrium region accurately.

First, how do we know that these boundaries exist? We can safely assume, given the finite resources available to any economy, that the level of national income offered by any equilibrium is also finite. Moreover the formula for the number of perfectly specialized equilibria described in the previous chapter shows that while that number grows very large as the number of traded commodities increases, with any given set of commodities that number is still clearly finite. Thus for any given share of world income, Z, obtained by one of the two countries (i.e., for any given position on the horizontal axis in our graph), if there are any equilibrium points directly above that position, there must be one such point that is the highest in that set of points, and its height too must be finite. In other words, the set of candidate equilibrium points must be bounded from above at every point, Z, on the horizontal axis. Since national income must also be nonnegative, these points must also be bounded from below. The conclusion follows: Both an upper and a lower boundary for the set of equilibrium points must exist. The next problem is to find each of these frontiers.

Before describing how this is done, we want to point out again that we are carrying out our analysis at present considering only the perfectly specialized equilibria. There do exist nonspecialized equilibria too. Indeed, the real world provides us virtually with only nonspecialized equilibria—assignments of goods to countries in which most goods are produced by more than one country, though most of these, plausibly, are unstable in a world of scale economies. Later we will come back to the nonspecialized equilibria and their relation to the region of perfectly specialized equilibria.

But for now, returning to the central issue, it should be plausible that the calculation of the boundaries of the equilibrium region is a matter of mathematical programing. To find the upper frontier, we must find the equilibrium point that maximizes the national income (or that maximizes the total utility if the total utility function is known) of the county in question, for any given value of Z, subject to the constraints that ensure that the equilibrium points considered satisfy the sets of equations that constitute the basic requirements of equilibrium. As we saw in the previous chapter, these three requirements are full employment in each country, quantity of each commodity supplied equals quantity demanded, and zero profit in each industry. We can formulate a mathematical program whose constraints are these three sets of equations,[3] and whose maximum is either the total utility or the national income of the country in question. The solution to this program constitutes the point on our upper frontier corresponding to any given point Z on the horizontal axis, and repetition of this calculation for each and every value of Z yields the entire upper frontier. Similarly, for a given value of Z, we can calculate the minimum value of national income (subject to the same three constraints) and then repeat the calculation for different values of Z. This procedure traces out the lower frontier of the region of perfectly specialized equilibria. That, in principle, is how the upper and lower frontiers of the equilibrium region are determined.

7.3 Two Complications of the Boundary Determination

This description of the boundary-setting process may seem too easy. In reality there are at least two complications, as well as two simplifications, that we must describe.

First, we must list the variables whose values the programming calculation is to determine. The variables of the model include the prices of the goods produced, the quantity of each good consumed in each country, the quantity of labor assigned to its production, the wage rate in each country and the share of world output of each commodity that is produced in each country. This last variable plays a key role in our analysis because it is this variable that describes the assignments of production. We call this variable $x_{i,j}$, which denotes the share of world output of good I that, in the assignment in question, is produced in country J. In a perfectly specialized assignment $x_{i,j}$ is equal to 1 if country J is the producer of good I; and $x_{i,j}$ equals zero if country J is not its producer. Clearly, if we know the values of all of these variables, then we know exactly to which country the production of each good is assigned. In other words, each possible assignment is described by the set of zero and one values of the $x_{i,j}$ variables.

Next, we will see that this is all one needs to know to locate a perfectly specialized equilibrium on our graph of outcomes. This is because, as we saw in the previous chapter, for any given assignment the equilibrium conditions can be used to determine the value of every remaining variable. The zero-profit conditions determine how much labor will be employed in an industry, the full employment condition in each country then determines its wage rate, and so on. Thus our mathematical programming problem of boundary drawing does not have to be burdened with calculation of the values of other variables. It need merely determine the value of each $x_{i,j}$ variable, and calculation of the others can be postponed. The key role of the $x_{i,j}$ variables is the first of the complications that affects the mathematical programming calculation of the upper- and lower-income frontiers—the boundaries of the region of perfectly specialized equilibria.

The second complication arises from the fact that in a perfectly specialized equilibrium the $x_{i,j}$ variables must be integers, either zero or unity, and hence discontinuous. This poses a problem that can be solved by either of two approaches. One can turn to the methods of integer programming, using this form of mathematical programming to determine the maxima and minima. This complicates the calculation somewhat, and it yields boundaries that are not very smooth because of the gaps that the finite number of perfectly specialized equilibrium points must inevitably leave in the graph. The alternative approach is simply, during this programming calculation, to ignore the requirement that the $x_{i,j}$ variables have only integer values and to assume implicitly that the variables are continuous. It can be proved that when the number of traded commodities is relatively large, the two methods of calculation yield boundaries that are very close to one another.

7.4 Two Simplifications of the Boundary-Drawing Process

Without attempting a detailed explanation, it should be noted that the preceding description of the method of determining the upper and lower boundaries also makes the process seem more complex than it turns out to be. This is because, first, the required calculation entails a mathematical program with only a single constraint, and second, the program is linear—with all of the simplifications that result.

The elimination of all but one of the equilibrium requirements is permitted by our focus upon just the one set of variables, the $x_{i,j}$. By postponing the calculation of the other variables such as wages and prices, we are permitted to ignore temporarily the equilibrium equations that will be used to determine the values of those variables. With these equilibrium requirements out of the way, we need only determine the values of the $x_{i,j}$ variables that maximize the national income of the country in question, given the value of Z (the relative national income of that country). For this we need only one constraint: the equation that determines the country's national income as a function of the $x_{i,j}$ and Z.

That equation is straightforward. With unit price elasticity of demand for each good in each country, as our model assumes, the value of Z determines (relative) world expenditure on each commodity I. The amount of income our country receives from production of good I is then this fixed expenditure multiplied by $x_{i,j}$, its share of the world's output of I. That country's total income then is the sum of these amounts, that is, the sum for all goods I, of world expenditure on I multiplied by $x_{i,j}$.

Thus we end up with only one constraint. This constraint is clearly linear in the variables whose values are to be determined, the $x_{i,j}$. Our calculation for the upper frontier, for any given value of Z, then amounts to the one-constraint linear program in which the maximand is the national income of country J and whose constraint is the linear equation that has just been described. The entire upper frontier is then calculated by repeating this calculation for the range of Z values.

The lower frontier is patently calculated similarly, only this time by minimization of country J's national income subject to the same constraint.

7.5 On Tightness of the Boundaries

How accurately do the boundaries calculated in this manner describe the set of equilibria? The answer is that when the number of traded commodities is small, the boundaries so calculated unavoidably leave much to be desired. If the world trades in only two goods, widgets and gidgets, there are only four specialized assignments: the one in which the U.K. produces both, the one in which France produces both, the one in which France produces all the widgets and the U.K. all the gidgets, and the one in which widget and gidget production are switched. Any set of boundaries for all levels of Z must then encompass a great deal of empty space when there are only two traded goods. But, as the number of traded commodities increases, the boundaries rapidly begin to do a much better job of representing the equilibrium points, and, as our graphs in earlier chapters clearly show, by the time the number of goods exceeds ten or so, their correspondence seems quite acceptable. Indeed, we have a result considerably stronger than that. For it is possible to prove:

THEOREM 1 (Filling-in theorem) If one selects any arbitrary point on the boundary and the number of traded goods considered in the model increases, there will be a number of goods beyond which an equilibrium point will lie "right next to" the selected boundary point.

That is, if we preselect any standard of closeness (the distance between the two points should be less than some number r), then, for a number of goods sufficiently large, there will be an equilibrium point whose distance to our selected boundary point is less than r. In sum, with the number of traded goods very large the boundaries calculated by the methods just described must fit the set of equilibrium points very snugly.[4]

7.6 Observations on Imperfectly Specialized Equilibria and the Two Boundaries

We deal next with an issue that is always just below the surface in our analysis of the shape of the equilibrium region and its boundaries. In working on this matter, we focus persistently on the perfectly specialized equilibria in order to facilitate out determination of the properties of the equilibrium region and to calculate its boundaries. However, the real world is characterized by equilibria that are not perfectly specialized, though the number of countries in which the more sophisticated and more high-tech commodities are produced is often fairly small. The natural question is whether the region that we have shown to contain all the perfectly specialized equilibria, and that tends to "fill up" with perfectly specialized equilibrium points, also contains the nonspecialized equilibria. Here we will merely report the answer, without attempting to describe the proof. It can be shown that:

THEOREM 2 The upper boundary is a ceiling over all equilibria.

If we were to calculate an upper boundary of all equilibria as we have done for only the perfectly specialized ones, we would obtain the same curve. This reflects the fact that all the nonspecialized equilibria lie under the upper boundary of the specialized equilibria, and that they also press right up against the upper boundary as the number of equilibria increases.

The place where the region of nonspecialized equilibria does depart from the region of specialized equilibria is in its lower boundary. As common sense indicates, it is possible in the presence of economies of scale to get worse outcomes by dividing industries (as happens in nonspecialized equilibria), and thus losing economies of scale, than from only specialized outcomes. Consequently there can be nonspecialized equilibria below the lower boundary of the region of specialized equilibria. How many there are, and how far below that lower boundary they occur, depends on the strength of the economies of scale.

In one specially interesting case, when the economies of scale have been exhausted at the level of production actually attained in the industries so that the production curves have become linear, the nonspecialized equilibria will all lie within the region of specialized equilibria.

The general situation is illustrated in figure 7.3, where the light grey points represent unspecialized equilibria, all calculated by computer from a particular concrete model. We see that there are, indeed, such points below the lower-income frontier.

Yet this is not the end of the story. While the region we have analyzed does not generally contain all pertinent equilibrium points, in cases where the number of traded commodities is fairly large, that region does contain all (or virtually all) of the equilibria that matter. That is, it tends to contain all of the stable equilibria, thus omitting only those that are apt to be highly transitory. There are two reasons for this.

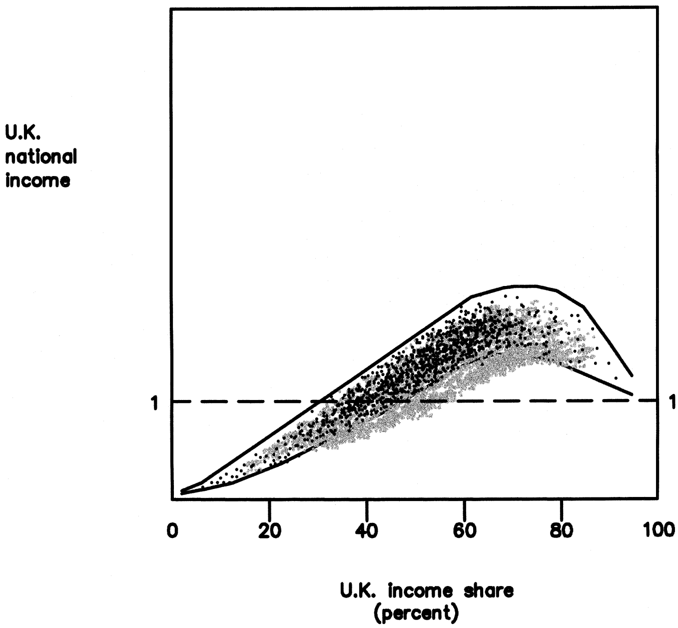

Figure 7.3
Inclusion of unspecialized equilibria

First, if economies of scale are substantial, there are strong forces working against the stability of any unspecialized equilibrium. Consider an equilibrium in which good *I* is produced simultaneously in two countries, A and B. Then equilibrium requires that the marginal cost of the product in the two countries be equal, for otherwise one of the countries could underprice the other and expand its market share of product *I*, thereby modifying the initial assignment and causing the economy to leave the initial point. But, even if the marginal cost of good *I* is the same in both countries, any fortuitous increase in sales by country A must reduce its marginal cost below that of B, thereby making possible a further rise in A's sales of good *I*, further reducing its marginal costs, and so on, until the production of I becomes perfectly specialized, with all of it being produced by A. It is even possible to show that if there is more than one shared industry, stability is not attainable if scale economies are substantial. If there is only one industry shared by several countries, however, there is a flaw in the instability argument. As a country acquires more of an industry, its wage will tend to be driven upward, and that constitutes an offset to the cost reduction that scale economies provide in the wake of the output expansion. One may well judge that the countervailing power of the wage rise is virtually certain to be weak in a model with many traded commodities, since expansion in the output of only one of the country's large number of products is unlikely to have much of an effect on the country's wage rate, though this expansion can cut the cost of that one industry significantly as a result of its scale economies. It is plausible that the weak force of the wage increase will not be overwhelmed by the cost reduction contributed by scale economies if those scale economies come to an end beyond some level of output. Then after some point the production functions become nearly linear.

In reality the erosion of scale economies beyond some level of output seems quite common, and this explains the patent stability and frequency of shared-industry assignments. Fortunately, when production functions are linear or near linear after a certain point, the shared equilibrium points will all lie inside (in the linear case) or almost inside (in the almost-linear case) the region of equilibria.

We can conclude that while the space between the upper- and lower-income frontiers does not contain all of the nonspecialized equilibria, that space can quite justifiably be characterized as a close approximation to the region of stable equilibria.

7.7 Poor Welfare Performance and Inefficiency of Some Equilibria

We saw in chapter 3 that many of the locally stable equilibria that arise in a scale economies model can be damaging to the interests of one of the trading countries. We learned that there exist many equilibria near either vertical axis of the graph at which both countries are worse off and possibly very much worse off than they would be at equilibria closer to the center of the diagram. In other words, the analysis indicates that under scale economies the invisible hand can blunder; it can sustain an equilibrium point that is locally stable and yet does not enjoy the beneficial welfare properties that the economics literature associates with the market mechanism, at least in a regime of perfect competition. We will presently consider explicitly why the market mechanism can lose some of its benign powers in a world of scale economies. First, however, we offer a brief and simple proof that equilibria of the scale-economies model need not even satisfy the requirements of productive efficiency (defined in the usual way to mean that productive efficiency yields the largest possible output of any given commodity that is achievable without any offsetting reduction in the output of any other commodity).

The discussion is framed in terms of Ricardo's two countries, England and Portugal, but substituting for his cloth and wine two contemporary products, computers and Walkman radios, in whose production we may expect scale economies. In such a two-good, two-country model it must be remembered that there are always exactly two perfectly specialized assignments. Portugal can produce all the Walkmans and England all the computers, or the reverse can be true.

In figure 7.4 curves PP' and EE' are the production frontiers for Portugal and England. Here the convexity (downward) of the two frontiers represents the presence of scale economies because toward the center of the frontier, where the countries' production is unspecialized and the output of each good is relatively small, the output vector is held down (the output point is closer to the origin than it would be in the linear case).[5] The two specialized solutions are $S' = (E', P)$ and $S = (P', E)$. In the first of these England produces all the world's Walkmans and Portugal produces all of the computers, while in the second of these the production assignment is reversed. Both of these solutions are obviously locally stable equilibria at prices that clear the markets, because if either country tries to produce a small quantity of the other's

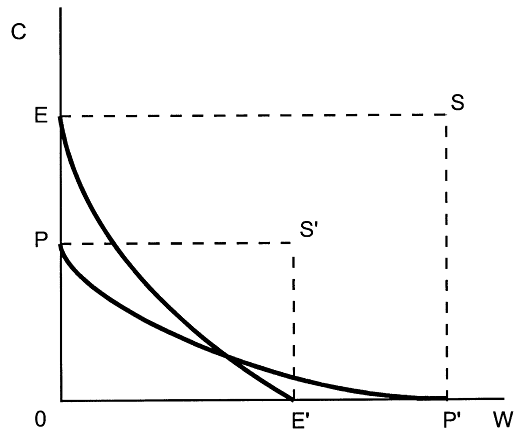

Figure 7.4
Inefficiency of some specialized equilibria

product it will fail because of its high costs. Yet, as shown in figure 7.4, where the two frontiers intersect, the one specialized point *S* clearly dominates the other, *S′*, so the latter must be inefficient. This must always be so where the two countries' production frontiers have an odd number of intersections because then one country must be able to produce a maximum of more Walkmans than the other, and the other country must then be able to produce more computers than the first. Hence the specialized equilibrium in which the first country produces all the Walkmans and the second country produces all the computers must then dominate the equilibrium in which the assignment of commodity production to the two countries is reversed. This example clearly shows that:

THEOREM 3 (Existence of inefficient equilibria) In a world of scale economies, stable equilibria can be inefficient.

Numerical examples are clearly not difficult to construct.

7.8 Why the Market Cannot Generally Prevent Equilibria with Poor Welfare Properties

One could well ask why the market mechanism does not automatically eliminate those equilibria of the model that are decidedly inferior in their welfare performance. That is, suppose equilibrium *B* offers both trading countries higher incomes or even larger quantities of all commodities than another equilibrium *A*. Why don't entrepreneurs in both countries recognize the shareable profit opportunities that are available if the world economy is initially at *A*, and why do they not act so as to move the world to *B*? Alternatively, why does not arbitrage enable the market to do the welfare-maximizing job we expect of it in a world of universally diminishing returns? The realistic answer is that in practice business managers neither are aware of the identity of the distant opportunities that are really promising, nor do they know how to get to that better equilibrium. Getting to *B* requires entrepreneurs in each country to select among the vast array of goods that they may never have produced before. Neither previous experience nor any other market signal will make these possibilities, much less the best or even the viable choices, obvious in advance. Moreover the investments required for the move from *A* to *B* are not only very risky—they must also be very large, since entry into the requisite industries on a small scale is doomed to failure. In such circumstances one could surely expect no automatic forces to move the economy from *A* to *B*. The risks of the necessary investments are too great, and the available information too sparse to ensure that such welfare-enhancing moves will generally be carried out by market forces. Anyone with any experience in business or government is surely aware that no one knows which, if any, equilibrium points at a distance from the current assignment guarantee mutual gains to the trading nations involved or even higher profits or a higher national income to either country alone. With such great gaps in the pertinent information, great risks, and high outlays required for the move, how can one expect arbitrage or any other instrument of the market mechanism to move the economy from an undesirable equilibrium point? Even a country stuck in an equilibrium that is worse for it than autarky is surely kept there by ignorance. This is simply because autarky is a state of affairs that it has never experienced and that it has no way of evaluating. Thus, though a country might benefit by simply closing its borders to all imports of goods whose production is characterized by scale economies, the prospect must be a frightening leap into the unknown.

Another reason, perhaps more fundamental, prevents the market mechanism from carrying out such improving moves from one equilibrium to another. Market signals are entirely local. They tell the business managers only about the partial derivatives of profit that small moves from an initial position will yield. When there is a profusion of local maxima, a move that brings one higher on the current profit hill does not offer any information about other faraway hills whose peaks may be much higher than that of the current hillock. A move toward the summit of the current hill can easily bring one farther away from a far higher peak of a distant hill. In sum, in a world of scale economies market forces can be relied on to drive the economy toward a local maximum of profits and even of welfare. But these forces have no power to free the world economy from that local optimum and move it toward one that is unambiguously better. That is why, under scale economies, decidedly inferior equilibria can nevertheless be locally stable. The invisible hand can indeed, by the happenstance of history, find itself stuck at an equilibrium that is locally optimal but globally far inferior to others, even inferior to the autarky equilibrium for at least one of the trading countries.

7.9 Concluding Comment

While, in practice, the calculation of the boundaries of the region of perfectly specialized equilibria involves complications in the requisite computer programs, the logic of the process is not difficult. We have seen how boundaries can be computed by a simple linear programming exercise. We have seen that the region between the two frontiers characterizes the perfectly specialized equilibria, since it contains all of them, and since the equilibrium points tend to fill up the region as the number of traded commodities in the model grows. Finally, we have seen that while unspecialized equilibria can lie below this region (but never above it), the nonspecialized points that are not inside or close to the region tend to be unstable. In other words, in a world of scale economies all stable equilibria tend to lie inside the region. In the next chapter we will see that there is a corresponding result for a world of constant returns to scale—a result that offers significant additional insights of its own.

We have also seen from the shape of the region of equilibria that some of the locally stable equilibria will keep the absolute incomes of one of the two countries, and in many cases those of both countries, below their maximal attainable levels. In addition it has been shown that the equilibria can be inefficient. Yet, as we have explained, in our scale economies world the market mechanism loses its ability to move the economy automatically to a superior equilibrium. This is a consequence of the likelihood that the superior equilibria will be located in a relatively distant portion of the region, meaning that entrepreneurs may not be aware of the existence of the opportunities they offer and have may have little idea of the means that can move the economy there. The values of the observable economic parameters provide only local information and offer no guidance on the direction in which the true global optimum lies, or even on the location of equilibria markedly superior to the economy's current position.

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