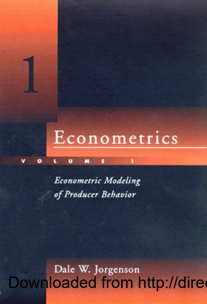

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Econometrics

VOLUME 1

Econometric Modeling of Producer Behavior

Dale W. Jorgenson

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Volume 1:
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Preface

Dale W. Jorgenson

This volume contains my econometric studies of producer behavior. The volume includes a self-contained presentation of duality in the theory of production, statistical methods for estimation and inference in systems of nonlinear simultaneous equations, and econometric models based on flexible functional forms. The innovations embodied in these models—duality, simultaneity, and flexibility—have become standard in modeling producer behavior.

The centerpiece of the volume is a suite of econometric models generated from the dual formulation of the theory of producer behavior. The companion volume, *Aggregate Consumer Behavior*, provides a parallel treatment of my econometric studies of consumer behavior. These flexible representations of technology and preferences serve as building blocks for the general equilibrium models presented in my earlier volumes, *Econometric General Equilibrium Modeling and Energy, the Environment, and Economic Growth.*

In chapter 1 I survey econometric methods for modeling producer behavior. The goal of empirical research is to determine the nature of substitution among inputs, the character of differences in technology, and the role of economies of scale. Econometric methodology based on duality in the theory of production has generated an extensive body of empirical work. I summarize studies of substitution, technical change, and economies of scale that draw on this methodology.

The traditional approach to econometric modeling begins with additive and homogeneous production functions. Demand and supply functions are derived from the conditions for producer equilibrium. However, the constraints imposed by additivity and homogeneity frustrate the objective of characterizing technology empirically. For example, the production function originated by Charles Cobb and Paul Douglas (1928) requires that elasticities of substitution among all inputs must be equal to unity.

The constant elasticity of substitution (CES) production function introduced by Kenneth Arrow, Hollis Chenery, Bagicha Minhas, and Robert Solow (1961) achieves flexibility by treating the elasticity of substitution as an unknown parameter. However, the CES production function retains additivity and homogeneity and imposes stringent limitations on patterns of substitution. Daniel McFadden (1963) and Hirofumi Uzawa (1962) have shown, essentially, that elasticities of substitution among all inputs must be the same.

The innovations in econometric methodology for modeling producer behavior, summarized in this volume, stem from the dual formulation of production theory originated by Harold Hotelling (1932). Lawrence Lau and I give a self-contained presentation of duality in production theory in chapters 5 and 6. Technology is characterized by a price or cost function that is dual to the production function. Demand and supply functions are generated without imposing arbitrary restrictions on the underlying technology.

The responses of demands and supplies to changes in prices, technology, and economies of scale characterize the behavior of producers. For example, measures of substitution are specified in terms of the impacts of price changes on demands and supplies. Similarly, measures of technical change are specified in terms of the impacts of changes in technology. A judicious choice of these measures results in a flexible approach to econometric modeling.

In chapter 1 I outline the generation of transcendental logarithmic or translog price and cost functions. I define the share elasticity as the impact on the share of an input in the value of output of a proportional change in the price of an input. If a share elasticity is positive, the corresponding value share increases with the input price. If a share elasticity is negative, the value share decreases with the price. Finally, if a share elasticity is zero, the value share is independent of the price, as in the Cobb-Douglas production function.

Similarly, the bias of technical change is the impact of a change in technology on the input value share. If the bias of some technical change is positive, the corresponding value share increases with a change in the level of technology and we say that the technical change is input-using. If the bias is negative, the corresponding value share decreases with a change in the level of technology and the technical change is input-saving. Finally, if the bias is zero, the value share is independent of technology; in this case we say that the technical change is neutral.

An important feature of models of production based on the translog price function is that the rate of technical change is endogenous, but does not affect future production possibilities. The biases of technical change can be used to derive the implications of changes in input prices for the rate of technical change. If the bias is positive, the rate of technical change decreases with the input price. If the bias is negative, the rate of technical change increases with the input price. Finally, if the bias is zero, so that technical change is neutral, the rate of technical change is independent of the price.

The description of technology is completed by the deceleration of technical change. This is defined as the negative of the rate of change of the rate of technical change. If the deceleration is positive, negative, or zero, the rate of technical change is decreasing, increasing, or independent of the level of technology.

In the system of demand and supply functions generated from the translog price function the share elasticities and biases of technical change are unknown parameters. The dependent variables are the value shares of all inputs and the rate of technical change. All the dependent variables are functions of the same independent variables, namely, prices and the level of technology. These functions are nonlinear in the variables; the functions may also be nonlinear in the parameters. Finally, the parameters may be subject to nonlinear constraints arising from the theory of production. Additional constraints arise from restrictions on technology such as additivity and homogeneity.

Myopic decision rules for econometric models of producer behavior can be derived by treating the price of capital input as the rental price of capital services. Production decisions depend only on current prices, including the price of investment goods. More details about myopic decision rules are given in my paper, “Technology and Decision Rules in the Theory of Investment Behavior,” in the companion volume, *Tax Policy and the Cost of Capital.* These decision rules greatly facilitate the implementation of the econometric models.

The constraints on the system of demand and supply functions implied by the theory of production are:

1. *Homogeneity*. The value shares and the rate of technical change are homogeneous of degree zero in the input prices.
2. *Product exhaustion*. The sum of the value shares is equal to unity.
3. *Symmetry*. The matrix of share elasticities, biases of technical change, and the deceleration of technical change must be symmetric.

4. *Nonnegativity*. The value shares must be nonnegative.
5. *Monotonicity*. The matrix of share elasticities must be nonpositive definite.

Statistical methods for estimating the unknown parameters of systems of demand and supply functions depend on the character of the data set. For cross-section observations on individual producing units, prices can be treated as exogenous variables. Unknown parameters can be estimated by the nonlinear multivariate regression techniques introduced by Robert Jennrich (1969) and Edmond Malinvaud (1970, 1980). These techniques deal with nonlinearities in the parameters, nonlinearities in the variables, or both.

For time series observations on industry groups, the prices that determine demands and supplies must be treated as endogenous variables. Unknown parameters can be estimated by techniques for nonlinear simultaneous equations. In chapter 7 Jean-Jacques Laffont and I present the method of nonlinear three-stage least squares for estimation of parameters in systems of demand and supply functions. These methods deal with simultaneity, as well as nonlinearities in the parameters and the variables.

The theory of production can be tested statistically by deriving constraints on the parameters of a system of demand and supply functions implied by the theory. Additional constraints, for example, the constraints implied by additivity and homogeneity, can also be tested. Jennrich and Malinvaud have introduced methods for statistical inference in nonlinear multivariate regression models. Ronald Gallant and I present methods for statistical inference in systems of nonlinear simultaneous equations in chapter 11.

I conclude chapter 1 by considering flexible representations of technology for econometric general equilibrium modeling. I also describe the use of panel data techniques for modeling technical change and economies of scale simultaneously. Finally, I outline methods for constructing dynamic models of production that incorporate internal costs of adjustment. The optimal production plan at each point of time depends on the initial level of “quasi-fixed” inputs, such as capital inputs, as well as expectations about future prices of outputs and inputs.

In chapter 2 I survey empirical studies of depreciation, an important special topic in econometric modeling of producer behavior. The measurement of depreciation requires modeling substitution among different vintages of capital inputs, corresponding to units of capital accumulated at different points of time. In principle each vintage could be treated as a separate input in an econometric model of production. However, the number of parameters would increase with the square of the number of inputs in a flexible functional form like the translog, rendering this approach infeasible.

The key simplifying assumption in the vintage model of capital is that different vintages are perfect substitutes in production. The services of these different vintages are proportional to initial investments with constants of proportionality given by relative efficiencies in production. The prices of different vintages of capital inputs are proportional to the relative efficiencies. Empirical research on the vintage model of capital reduces to modeling the relative efficiencies. Although the assumption of perfect substitutes is restrictive, the vintage approach has become the method of choice for modeling substitution among capital inputs.

I have presented a vintage model of capital in “The Economic Theory of Replacement and Depreciation,” chapter 5 of the companion volume, *Tax Policy and the Cost of Capital.* This model, originally formulated by Hotelling (1925), is characterized by price-quantity duality. Capital goods decline in efficiency with age, requiring replacement investments to maintain productive capacity. The price of a capital good also falls with age, reflecting both the current decline and the present value of future declines in efficiency. Depreciation is the decrease in the value of a capital good with age.

Laurits Christensen and I have presented a vintage accounting system for prices and quantities of capital goods in our paper, “Measuring the Performance of the Private Sector of the U.S. Economy, 1929–1969,” chapter 5 in the companion volume, *Postwar U.S. Economic Growth.* This accounting system provides an internally consistent framework for measuring depreciation and capital stock. We have extended this framework to encompass income, product, and wealth data for the econometric general equilibrium models described below.

In the model of capital goods prices introduced by Robert Hall (1971) the relative efficiencies of a capital good are expressed as functions of age and calendar time. The unknown parameters of the model can be estimated from observations on the prices of capital goods of different vintages. This model can be generalized to capital goods with different varieties that are perfect substitutes in production.

Relative efficiencies are represented as functions of the technical characteristics of each variety.

Providing an illustration of modeling the relative efficiencies of different vintages of capital, Charles Hulten and Frank Wykoff (1981b) have constructed econometric models for the prices of eight categories of capital goods. Making use of the asset classification scheme of the Bureau of Economic Analysis (1987) capital stock study, Kun-Young Yun and I (1991b) have derived economic depreciation rates for thirty-five asset categories. These estimates of depreciation have been incorporated into price and quantity indices of capital services for thirty-five industries in "Productivity and Economic Growth," chapter 1 of the companion volume, *International Comparisons of Economic Growth.* The research of Hulten and Wykoff has been successfully exploited by the Bureau of Economic Analysis in measuring depreciation in the U.S. national accounts, as described by Barbara Fraumeni (1997).

Ellen Dulberger (1989) has employed speed of processing and main memory as technical characteristics of different varieties of computer processors. The Bureau of Economic Analysis (1986) has introduced price indices for computers based on this model of relative efficiencies into the U.S. National Income and Product Accounts. Kevin Stiroh and I (1999) have derived price and quantity indices for the capital services of computers in our paper, "Information Technology and Economic Growth."

In chapter 3 Lawrence Lau and I present an economic theory of agricultural household behavior. This theory relates household consumption and production decisions to the prices of outputs, variable inputs, and consumption goods. Additional determinants of these decisions include stocks of quasi-fixed inputs, household wealth, and the composition of the household. Extensive empirical studies of agricultural households based on this approach were published in *Resource Use in Agriculture, Applications of the Profit Function to Selected Countries,* a special issue of *Food Research Institute Studies,* edited by Pan Yotopoulos and Lau (1978).

Our theory of agricultural household behavior expresses household welfare as a function of the utility functions of individual household members. An important simplifying assumption is that the utility functions for all individuals are identical, except for proportional transformations of units of measurement. These transformations are equivalence scales that depend on the characteristics of the individual such as age and sex. Daniel Slesnick and I have used this approach to modeling household behavior in our paper, "Aggregate Consumer Behavior and Household Equivalence Scales," chapter 5 in the companion volume, *Measuring Social Welfare*.

The objective of the agricultural household is to maximize household welfare, subject to the technology of the enterprise, the constraints on the time available to household members, and the total expenditure of the household. The household takes the profits of the agricultural enterprise and non-agricultural income as given in making consumption decisions. It maximizes welfare with respect to leisure, consumption of goods produced within the agricultural enterprise, and purchased consumption goods.

Given competitive markets for agricultural inputs, including hired labor, production decisions depend only on technology and are independent of preferences. Lau and I represent the technology of the agricultural enterprise in terms of outputs, variable inputs such as labor, materials, energy, and quasi-fixed inputs such as land and reproducible capital. We derive a profit function that is dual to the agricultural production function. This gives the maximized value of profit of the enterprise as a function of the prices of the outputs and the variable inputs and the quantities of the quasi-fixed inputs.

In chapter 4 Christensen, Lau and I, present an econometric model of production that embodies three innovations. The model utilizes translog functional forms, statistical methods for nonlinear systems of simultaneous equations, and duality in production theory. In this model the economy supplies outputs of consumption and investment goods and demands inputs of capital and labor services. Price and quantity data for the inputs and outputs of the U.S. private domestic economy are taken from the system of U.S. national accounts that Christensen and I have constructed for the period 1929–1969.

An increase in the output of investment goods requires foregoing a part of the output of consumption goods, so that adjusting the rate of investment is costly. However, costs of adjustment are fully reflected in the market price of investment goods. The cost of capital input is a function of this price, so that costs of adjustment are external to the production process. In models of production with internal costs of adjustment, like those presented in section 1.7 of chapter 1, the cost of capital input must be inferred from the shadow value of the adjustment costs. Further details are given in my paper, "Technology and

Decision Rules in the Theory of Investment Behavior," in the companion volume, *Tax Policy and the Cost of Capital*.

Our first objective is to develop tests of the theory of production that do not employ additivity and homogeneity as part of the maintained hypothesis. For this purpose we generate an econometric model of aggregate producer behavior from the translog production possibility frontier. The dependent variables are ratios of the values of investment goods and labor services to the value of capital services. The independent variables are logarithms of the quantities of outputs of investment and consumption goods, quantities of inputs of capital and labor services, and the level of technology.

Under constant returns to scale our model of aggregate producer behavior implies the existence of a price possibility frontier, defined by the set of prices consistent with zero profits. The price possibility frontier and the system of demand and supply functions are dual to the production possibility frontier and the necessary conditions for producer equilibrium. An econometric model generated from the translog price possibility frontier has the same dependent variables. The independent variables are logarithms of the prices of investment and consumption goods, logarithms of the prices of capital and labor services, and the level of technology.

The translog production and price possibility frontiers correspond to two distinct representations of technology. We have estimated the unknown parameters of both models by the method of nonlinear three-stage least squares presented in chapter 7. We have tested hypotheses implied by the theory of production for both models, using the test statistics presented in chapter 10. Results for both models are consistent with the validity of an extensive set of restrictions implied by the theory.

Our second objective is to test the additivity and homogeneity restrictions that underly the constant elasticity of the substitution production function. We employ the same data and econometric methodology as in our tests of the theory of production. The constraints implied by additivity and homogeneity conflict sharply with the empirical evidence. We further simplify the technology by requiring that the elasticity of substitution between capital and labor inputs is equal to unity, as in the Cobb-Douglas production function. Conditional on additivity and homogeneity, this is also strongly rejected by our tests.

Our overall conclusion is that flexible representations of technology are appropriate for dynamic general equilibrium modeling at the aggregate level. A representation incorporating additivity and homogeneity is much less satisfactory. Yun and I have employed the translog price function in our dynamic general equilibrium model of the impact of U.S. tax policy. We have presented the model in our paper, "The Efficiency of Capital Allocation," chapter 10 in the companion volume, *Tax Policy and the Cost of Capital.* We have used this model in analyzing the impact of U.S. tax reforms in our paper, "Tax Policy and Capital Allocation," chapter 11 in the same volume.

Our dynamic general equilibrium model of tax policy also incorporates a flexible representation of preferences. This is based on the translog indirect utility function presented in my paper with Christensen and Lau in the companion volume, *Aggregate Consumer Behavior*. Equilibrium in the tax model is characterized by an intertemporal price system that clears markets for consumption and investment goods and for capital and labor services. The price of investment goods reflects the present value of capital services and links the present to the future. Capital has been accumulated through previous investments, linking the present to the past.

In chapters 8, 9, and 10 Klaus Conrad and I have applied the econometric methodology presented in chapter 1 to aggregate data for the Federal Republic of Germany. These data are presented in our 1975 book, *Measuring Performance in the Private Economy of the Federal Republic of Germany, 1950–1973.* The economy supplies investment and consumption goods and demands capital and labor services. An additional feature of this model is that the rate of technical change is endogenous and depends on the same independent variables as the demands and supplies. We utilize translog price and production functions to generate systems of nonlinear simultaneous equations that describe aggregate producer behavior.

In chapter 8 we derive constraints on the parameters of econometric models of producer behavior implied by the theory of production. Since the price and production functions provide two distinct representations of technology, we present tests of these constraints for both. The theory of production is consistent with the results of both sets of tests, corroborating and extending the findings of chapter 4. We test inequality restrictions implied by monotonicity and convexity of the price and production functions, as well as the equality restrictions tested in chapter 4.

In chapter 10 Conrad and I have tested and rejected restrictions on technology associated with the additivity and homogeneity implied by the constant elasticity of substitution production function. These findings also corroborate and extend those of chapter 4. In chapter 9 we represent technical change by commodity augmentation factors that are analogous to the equivalence scales of chapter 3. We find that technical change is factor-augmenting, so that inputs of capital and labor services can be transformed into efficiency units, while investment and consumption goods outputs can be represented in natural units.

In chapter 12 Fraumeni and I present econometric models for each of thirty-five industrial sectors of the U.S. economy. These models are based on a translog price function for each sector. The price of output is a function of the prices of the primary factors of production—capital and labor services—prices of inputs of energy and materials, and time as an index of technology. An important feature of these models is that the rate of technical change is endogenous, but does not affect future production possibilities.

The econometric model of producer behavior for each of the thirty-five industries consists of a system of nonlinear simultaneous equations. The equations give the value shares of capital, labor, energy, and materials (KLEM) inputs and the rate of technical change as functions of relative prices and time. Price and quantity data for the inputs and outputs of each industry are taken from the system of national accounts presented in my 1980 paper, “Accounting for Capital,” while the rate of technical change is an index number constructed from these data. This paper extends the vintage accounting system I had developed with Christensen to include both sectoral and aggregate production accounts.

The parameters of the system of input demand equations for each industry are estimated by the method of nonlinear three-stage least squares presented in chapter 7. These parameters included the share elasticities that describe substitution and the biases that describe technical change. We have estimated these parameters from time-series data for each industry. The industry-level data for the U.S. are described in my 1980 paper with Fraumeni, “The Role of Capital in U.S. Economic Growth, 1948–1976.” In 1987 we published updated sectoral and aggregate production accounts in our book with Frank Gollop, *Productivity and U.S. Economic Growth.* The results are summarized in chapter 1 of the companion volume, *Postwar U.S. Economic Growth.*

As before, we describe substitution patterns by share elasticities, giving the impact of a proportional price change on the share of an input in the value of an input. As an illustration, the share elasticity of capital with respect to the price of labor is zero if the elasticity of substitution between the two inputs is equal to unity, since the share of capital is constant. We describe patterns of technical change by biases, giving the impact of a change in technology on the input value share. For example, we say that technical change is capital-using if the capital share increases with time, holding input prices constant.

The empirical findings on patterns of substitution and technical change reveal striking similarities among industries. In general, share elasticities are nonnegative, so that shares increase with proportional input price changes and elasticities of substitution are greater than unity. The elasticities of the shares of capital with respect to the price of labor are nonnegative for thirty-three of the thirty-five industries. Elasticities of substitution between capital and labor are greater than unity for these industries.

Similarly, elasticities of the shares of capital with respect to the price of energy are nonnegative for thirty-four industries and elasticities with respect to the price of materials are nonnegative for all thirty-five industries. The share elasticities of labor with respect to the price of materials are nonnegative for all thirty-five industries. However, the share elasticities of labor with respect to the price of energy are nonnegative for only nineteen of the thirty-five industries. Finally, the share elasticities of energy with respect to the price of materials are nonnegative for thirty of the thirty-five industries.

A classification of industries by patterns of the biases of technical change is given in table 1.1 of chapter 1. The most common pattern is capital-using, labor-using, energy-using, and materials-saving technical change. The economic interpretation is that changes in technology conserve material inputs or increase value added through inputs of capital, labor, and energy. This occurs for nineteen of the thirty-five industries. Technical change is capital-using for twenty-five of the thirty-five industries, labor-using for thirty-one, energy-using for twenty-nine, and materials-using for only two.

We have emphasized that rates of technical change are endogenous in our econometric models of producer behavior. These rates depend on prices of inputs and the level of technology. If the bias of technical change is capital-using, then an increase in the price of a capital input reduces the rate of technical change. Since this is typical of the patterns we have described, an increase in the price of capital inputs reduces the rate of technical change. Similarly, increases in the prices of labor and energy inputs typically depress the rate of technical change, while an increase in the price of materials inputs raises the rate of technical change.

Over extended periods of time, energy prices have fallen relative to the prices of other inputs, elevating rates of technical change at the industry level. However, prices of labor inputs have risen relative to other input prices, depressing these rates of technical change. The substantial increases in energy prices after 1973 have had the effect of reducing sectoral rates of technical change, decreasing the aggregate rate of technical change, and diminishing the rate of growth of the U.S. economy.

In chapter 1 of the companion volume, *Energy, the Environment, and Economic Growth,* Peter Wilcoxen and I present a dynamic general equilibrium model of the U.S. economy with a flexible representation of technology for each of thirty-five industries. For this purpose we have employed econometric models of producer behavior for these industries based on translog price functions. We have used our dynamic general equilibrium model to analyze the economic impact of alternative energy, environmental, and tax policies. Mun Ho and I use this model to analyze the impact of trade policies in chapters 8, 9 and 10 of the same volume.

Our dynamic general equilibrium model of the U.S. economy also incorporates a flexible representation of preferences. This is presented in my paper with Lau and Thomas Stoker, “Transcendental Logarithmic Model of Aggregate Consumer Behavior,” chapter 8 of the companion volume, *Aggregate Consumer Behavior*. The model is based on exact aggregation over systems of household demand functions derived from the translog indirect utility function. Equilibrium is characterized by an intertemporal price system that clears markets for the outputs of all thirty-five industries as well as for capital and labor services. The price of investment goods is forward-looking and depends on future capital service prices, while the stock of capital is backward-looking and depends on past investments.

In chapter 14 I consider the relationship between energy prices and rates of technical change in greater detail. I present an econometric model of producer behavior for thirty-five U.S. industries, based on data for the period 1958–1979. I divide energy inputs between electricity and nonelectrical energy, so that the model for each sector includes the shares of five inputs–capital and labor services, electricity and nonelectrical energy, and materials. The shares are functions of the prices of these inputs, as well as time is an index of technology. Finally, the model includes an endogenous rate of technical change, also a function of the five input prices and time.

The patterns of substitution for the models presented in chapter 14 are similar to those for the models of chapter 12. Technical change is electricity-using for twenty-three of the thirty-five industries and non-electrical energy-using for twenty-eight of the thirty-five industries. An increase in the price of electricity reduces the rate of technical change for twenty-three industries and reduces this rate for the remaining twelve industries. An increase in the price of non-electrical energy reduces the rate of technical change for twenty-eight industries and reduces the rate for the remaining seven.

Historically, the price of electricity has fallen relative to the price of nonelectrical energy over extended periods of time. Prices of both types of energy have fallen relative to prices of capital and labor services and materials inputs. Electrification associated with the positive bias of technical change for electricity has raised rates of technical change in a wide range of industries. However, greater use of nonelectrical energy has increased rates of technical change in an even broader range. Jumps in the prices of both forms of energy after 1973 have had a depressing effect on rates of technical change at the industry level, slowing the aggregate rate of technical change and the growth rate of the U.S. economy.

In chapter 13 Masahiro Kuroda, Kanji Yoshioka, and I present econometric models of producer behavior for thirty industries of the Japanese economy. These models are based on the translog price function introduced in chapter 12. We implement this model for price and quantity data for inputs and outputs of Japanese industries, as well as rates of technical change for these industries. The data are also employed in my paper with Kuroda and Mieko Nishimizu, “Japan-U.S. Industry-Level Productivity Comparisons, 1960–1979,” chapter 7 in the companion volume, *International Comparisons of Economic Growth.*

In table 13.3 we compare patterns of substitution between U.S. and Japanese industries. These are broadly similar. In figure 13.2 we compare patterns of technical change for the two countries. The bias of technical change is labor-using for all thirty industries in Japan, while the bias is material-saving for twenty-eight of the thirty industries. The bias is energy-using for twenty-six of these industries and capital-saving for twenty-two. Capital-saving bias predominates in Japan, while capital-using bias predominates in the U.S. For both countries an increase in the price of energy results in a reduction in the rate of technical change and a slowdown in economic growth.

In chapter 15 Kuroda, Hikaru Sakuramoto, Yoshioka, and I present bilateral econometric models of producer behavior for twenty-eight Japanese and U.S. industries. We treat data on production patterns for Japan and the U.S. as separate sets of observations. However, we assume that econometric models of producer behavior for the two countries have common parameters. The point of departure for the econometric model is a bilateral translog production function. Output depends on a dummy variable—one for the U.S. and zero for Japan—that allows for differences in technology between the two countries, as well as inputs and time as an index of the level of technology.

We combine data for the U.S. and Japan to estimate the parameters that describe substitution and technical change. These data are employed for bilateral productivity comparisons in my paper with Kuroda and Nishimizu. For the same input prices and level of technology production is more capital-intensive and intermediate input-intensive in Japanese industries and more labor-intensive in U.S. industries. Rates of technical change are higher for Japanese industries and lower for U.S. industries. Not surprisingly, the technology gap between Japan and the U.S. is gradually closing, as Kuroda, Nishimizu, and I have shown.

One of the key innovations in the econometric models of production presented in this volume is the application of duality in production theory. The starting point of the theory of production is the set of production possibilities, containing all the production plans available to the producing unit. The production function gives the maximum net output of any commodity as a function of the net outputs of all other commodities. The theory of marginal productivity completes the theory of production; this may be identified with the gradient of the production function.

In chapter 5 Lau and I present equivalent specifications of the set of production possibilities, the production function, and the marginal productivities. We can take a characterization of any one of the three as a starting point of the theory of production and derive the properties of the other two. Marginal productivities provide the vehicle for generating econometric models of production in chapters 4, 8, 9, and 10 at the aggregate level and in chapter 15 at the sectoral level. The theory of production provides the links between these econometric models and representations of technology in terms of the production function and the set of production possibilities.

The second objective of chapter 5 is to characterize the set of feasible production plans from the point of view of economic behavior. For this purpose Lau and I present a theory of supply that parallels the theory of marginal productivity. We develop equivalent specifications of the profit function, the net supplies, and the set of price and profit possibilities. Any one of the three can be taken as the starting point for the theory of production. The net supplies provide the vehicle for generating econometric models of production in chapters 4, 8, 9, and 10 and chapters 12, 13, and 14. The theory of production provides links between these econometric models and the alternative representations of technology we consider in chapter 5.

The final objective of chapter 5 is to link the theory of supply with the theory of marginal productivity. Lau and I demonstrate the equivalence of technological ands behavioral viewpoints of the theory of production. For this purpose we employ equivalent specifications of the production and profit functions. The theory also implies equivalent specifications of marginal productivities and net supplies and of the sets of production and price possibilities. The econometric models of production considered in this volume can be linked to any of these six alternative specifications of the theory of production.

Sets of production and price possibilities are not employed in econometric modeling. However, Kenneth Hoffman and I have utilized these specifications of technology in our paper, “Economic and Technological Models for Evaluation of Energy Policy” included in the companion volume, *Econometric General Equilibrium Modeling*. We present a linear activity analysis model of the U.S. energy sector. This model is based on information from detailed engineering studies of technologies that are not currently available, but could be implemented under alternative technology policies. The activity analysis model of the energy sector is linked to econometric models for the nonenergy sectors of the U.S. economy to provide a complete representation of technology.

The linear activity analysis model does not require that the marginal products corresponding to a given production plan and the net supplies corresponding to a given price system are unique. However, uniqueness of the marginal products and the net supplies is essential for econometric modeling. In chapter 6 Lau and I consider the implications of differentiability of the production and profit functions or, equivalently, uniqueness of the marginal products and net supplies. By strengthening convexity assumptions for the production and profit functions we are able to develop the theory of production in terms of properties of differentiable convex functions and their gradients.

A second key innovation in the econometric models of production we present in this volume is the application of methods for systems of nonlinear simultaneous equations. In chapter 7 Laffont and I present the method of nonlinear three-stage least squares for estimation of the parameters of these models. Gallant and I provide the corresponding methods for statistical inference in chapter 11. These methods were greatly extended by Lars Hansen (1982) and became the basis for the Generalized Method of Moments that is now the standard approach to estimation and inference in macroeconometric modeling.

In chapter 7 Laffont and I consider two lines of attack on efficient estimation of systems of nonlinear simultaneous equations. The nonlinear three-stage least squares estimator is obtained by minimizing a weighted sum of squared residuals, where the weights depend on a set of instrumental variables. The first step in constructing this estimator is to linearize the system of nonlinear simultaneous equations. Arnold Zellner and Henri Theil's (1962) method of three-stage least squares is applied to the linearized model. This process is reiterated until the weighted sum of squared residuals is a minimum.

An alternative approach to efficient estimation of systems of nonlinear simultaneous equations is an extension of the method of efficient instrumental variables for linear systems developed in my papers with James Brundy (1971, 1973). Laffont and I show that efficient instrumental variables and minimum distance estimators achieve the same efficiency. Both are less efficient than the full information maximum likelihood estimator for systems of nonlinear simultaneous equations; however, the maximum likelihood estimator is computationally burdensome, and requires estimating all the equations of the model at the same time.

In chapter 11 Gallant and I present statistics for testing hypotheses about the parameters of systems of nonlinear simultaneous equations. We consider statistics based on likelihood ratio and Wald approaches, applied to the nonlinear three-stage least-squares estimator. The likelihood ratio approach involves a comparison of values of the minimized criterion function with and without the constraints implied by the hypothesis to be tested. Both approaches can be used to generate confidence intervals and regions for the unknown parameters. These methods are available in many econometric software packages.

The econometric models of producer behavior presented in this volume have been incorporated into the dynamic general equilibrium models. These general equilibrium models also include econometric models of consumer behavior presented in the companion volume, *Aggregate Consumer Behavior*. The advantage of the econometric approach to general equilibrium modeling is that responses of production and consumption decisions to changes in energy prices, environmental controls, trade restrictions, and tax policies can be derived from historical experience. This experience is an indispensable guide to economic policy making.

Implementation of the econometric approach to general equilibrium modeling has necessitated innovations in economic theory and econometric method. Duality has been especially critical in generating econometric models that provide flexible representations of technology and preferences. These models have required the development of new statistical methods for systems of nonlinear simultaneous equations. Duality, simultaneity, and flexibility in econometric modeling have led to a burgeoning empirical literature, characterizing technology and preferences in a wide range of empirical settings.

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1

Econometric Methods for Modeling Producer Behavior

Dale W. Jorgenson

1.1 Introduction

The purpose of this chapter is to provide an exposition of econometric methods for modeling producer behavior. The objective of econometric modeling is to determine the nature of substitution among inputs, the character of differences in technology, and the role of economies of scale. The principal contribution of recent advances in methodology has been to exploit the potential of economic theory in achieving this objective.

Important innovations in specifying econometric models have arisen from the dual formulation of the theory of production. The chief advantage of this formulation is in generating demands and supplies as explicit functions of relative prices. By using duality in production theory, these functions can be specified without imposing arbitrary restrictions on patterns of production.

The econometric modeling of producer behavior requires parametric forms for demand and supply functions. Patterns of production can be represented in terms of unknown parameters that specify the responses of demands and supplies to changes in prices, technology, and scale. New measures of substitution, technical change, and economies of scale have provided greater flexibility in the empirical determination of production patterns.

Econometric models of producer behavior take the form of systems of demand and supply functions. All the dependent variables in these functions depend on the same set of independent variables. However, the variables and the parameters may enter the functions in a nonlinear manner. Efficient estimation of these parameters has necessitated the development of statistical methods for systems of nonlinear simultaneous equations.

The new methodology for modeling producer behavior has generated a rapidly expanding body of empirical work. We illustrate the application of this methodology by summarizing empirical studies of substitution, technical change, and economies of scale. In this introductory section we first review recent methodological developments and then provide a brief overview of the chapter.

1.1.1 Production Theory

The economic theory of production—as presented in such classic treatises as Hicks's *Value and Capital* (1946) and Samuelson's *Foundations of Economic Analysis* (1983)—is based on the maximization of profit, subject to a production function. The objective of this theory is to characterize demand and supply functions, using only the restrictions on producer behavior that arise from optimization. The principal analytical tool employed for this purpose is the implicit function theorem.[1]

Unfortunately, the characterization of demands and supplies as implicit functions of relative prices is inconvenient for econometric applications. In specifying an econometric model of producer behavior the demands and supplies must be expressed as explicit functions. These functions can be parameterized by treating measures of substitution, technical change, and economies of scale as unknown parameters to be estimated on the basis of empirical data.

The traditional approach to modeling producer behavior begins with the assumption that the production function is additive and homogeneous. Under these restrictions demand and supply functions can be derived explicitly from the production function and the necessary conditions for producer equilibrium. However, this approach has the disadvantage of imposing constraints on patterns of production—thereby frustrating the objective of determining these patterns empirically.

The traditional approach was originated by Cobb and Douglas (1928) and was employed in empirical research by Douglas and his associates for almost two decades.[2] The limitations of this approach were made strikingly apparent by Arrow, Chenery, Minhas, and Solow (1961, henceforward ACMS), who pointed out that the Cobb-Douglas production function imposes *a priori* restrictions on patterns of substitution among inputs. In particular, elasticities of substitution among all inputs must be equal to unity.

The constant elasticity of substitution (CES) production function introduced by ACMS adds flexibility to the traditional approach by treating the elasticity of substitution as an unknown parameter.[3] However, the CES production function retains the assumptions of additivity and homogeneity and imposes very stringent limitations on patterns of substitution. McFadden (1963) and Uzawa (1962) have shown, essentially, that elasticities of substitution among all inputs must be the same.

The dual formulation of production theory has made it possible to overcome the limitations of the traditional approach to econometric modeling. This formulation was introduced by Hotelling (1932) and later revived and extended by Samuelson (1953, 1960)[4] and Shephard (1953, 1970).[5] The key features of the dual formulation are, first, to characterize the production function by means of a dual representation such as a price or cost function and, second, to generate explicit demand and supply functions as derivatives of the price or cost function.[6]

The dual formulation of production theory embodies the same implications of optimizing behavior as the theory presented by Hicks (1946) and Samuelson (1983). However, the dual formulation has a crucial advantage in the development of econometric methodology: Demands and supplies can be generated as explicit functions of relative prices without imposing the arbitrary constraints on production patterns required in the traditional methodology. In addition, the implications of production theory can be incorporated more readily into an econometric model.

1.1.2 Parametric Form

Patterns of producer behavior can be described most usefully in terms of the behavior of the derivatives of demand and supply functions.[7] For example, measures of substitution can be specified in terms of the response of demand patterns to changes in input prices. Similarly, measures of technical change can be specified in terms of the response of these patterns to changes in technology. The classic formulation of production theory at this level of specificity can be found in Hicks's *Theory of Wages* (1932).

Hicks (1932) introduced the elasticity of substitution as a measure of substitutability. The elasticity of substitution is the proportional change in the ratio of two inputs with respect to a proportional change in their relative price. Two inputs have a high degree of substitutability if this measure exceeds unity and a low degree of substitutability if the measure is less than unity. The unitary elasticity of substitution employed in the Cobb-Douglas production function is a borderline case between high and low degrees of substitutability.

Similarly, Hicks introduced the bias of technical change as a measure of the impact of changes in technology on patterns of demand for inputs. The bias of technical change is the response of the share of an input in the value of output to a change in the level of technology. If the bias is positive, changes in technology increase demand for the input and are said to use the input; if the bias is negative, changes in technology decrease demand for the input and are said to save the input. If technical change neither uses nor saves an input, the change is neutral in the sense of Hicks.

By treating measures of substitution and technical change as fixed parameters the system of demand and supply functions can be generated by integration. Provided that the resulting functions are themselves integrable, the underlying price or cost function can be obtained by a second integration. As we have already pointed out, Hicks's elasticity of substitution is unsatisfactory for this purpose, since it leads to arbitrary restrictions on patterns of producer behavior.

The introduction of a new measure of substitution, the share elasticity, by Christensen, Jorgenson, and Lau (1971, 1973) and Samuelson (1973) has made it possible to overcome the limitations of parametric forms based on constant elasticities of substitution.[8] Share elasticities, like biases of technical change, can be defined in terms of shares of inputs in the value of output. The share elasticity of a given input is the response of the share of that input to a proportional change in the price of an input.

By taking share elasticities and biases of technical change as fixed parameters, demand functions for inputs with constant share elasticities and constant biases of technical change can be obtained by integration. The shares of each input in the value of output can be taken to be linear functions of the logarithms of input prices and of the level of technology. The share elasticities and biases of technical change can be estimated as unknown parameters of these functions.

The constant share elasticity (CSE) form of input demand functions can be integrated a second time to obtain the underlying price or cost function. For example, the logarithm of the price of output can be expressed as a quadratic function of the logarithms of the input prices and the level of technology. The price of output can be expressed as a transcendental or, more specifically, an exponential function of the logarithms of the input prices.[9] Accordingly, Christensen, Jorgenson, and Lau refer to this parametric form as the translog price function.[10]

1.1.3 Statistical Method

Econometric models of producer behavior take the form of systems of demand and supply functions. All the dependent variables in these functions depend on the same set of independent variables—for example, relative prices and the level of technology. The variables may enter these functions in a nonlinear manner, as in the translog demand functions proposed by Christensen, Jorgenson, and Lau. The functions may also be nonlinear in the parameters. Finally, the parameters may be subject to nonlinear constraints arising from the theory of production.

The selection of a statistical method for estimation of systems of demand and supply functions depends on the character of the data set. For cross-section data on individual producing units, the prices that determine demands and supplies can be treated as exogenous variables. The unknown parameters can be estimated by means of nonlinear multivariate regression techniques. Methods of estimation appropriate for this purpose were introduced by Jennrich (1969) and Malinvaud (1970, 1980).[11]

For time-series data on aggregates such as industry groups, the prices that determine demands and supplies can be treated as endogenous variables. The unknown parameters of an econometric model of producer behavior can be estimated by techniques appropriate for systems of nonlinear simultaneous equations. One possible approach is to apply the method of full information maximum likelihood. However, this approach has proved to be impractical, since it requires the likelihood function for the full econometric model, not only for the model of producer behavior.

Jorgenson and Laffont (1974) have developed limited information methods for estimating the systems of nonlinear simultaneous equations that arise in modeling producer behavior. Amemiya (1974) proposed to estimate a single nonlinear structural equation by the method of nonlinear two-stage least squares. The first step in this procedure is to linearize the equation and to apply the method of two-stage least squares to the linearized equation. Using the resulting estimates of the coefficients of the structural equation, a second linearization can be obtained and the process can be repeated.

Jorgenson and Laffont extended Amemiya's approach to a system of nonlinear simultaneous equations by introducing the method of nonlinear three-stage least squares. This method requires an estimate of the covariance matrix of the disturbances of the system of equations as well as an estimate of the coefficients of the equations. The procedure is initiated by linearizing the system and applying the method of three-stage least squares to the linearized system. This process can be repeated, using a second linearization.[12]

It is essential to emphasize the role of constraints on the parameters of econometric models implied by the theory of production. These constraints may take the form of linear or nonlinear restrictions on the parameters of a single equation or may involve restrictions on parameters that occur in several equations. An added complexity arises from the fact that the restrictions may take the form of equalities or inequalities. Estimation under inequality restrictions requires nonlinear programming techniques.[13]

The constraints that arise from the theory of production can be used to provide tests of the validity of the theory. Similarly, constraints that arise from simplification of the patterns of production can be tested statistically. Methods for statistical inference in multivariate nonlinear regression models were introduced by Jennrich (1969) and Malinvaud (1970, 1980). Methods for inference in systems of nonlinear simultaneous equations were developed by Gallant and Jorgenson (1979) and Gallant and Holly (1980).[14]

1.1.4 Overview

This chapter begins with the simplest form of the econometric methodology for modeling producer behavior. This methodology is based on production under constant returns to scale. The dual representation of the production function is a price function, giving the price of output as a function of the prices of inputs and the level of technology. An econometric model of producer behavior is generated by differentiating the price function with respect to the prices and the level of technology.

We present the dual formulation of the theory of producer behavior under constant returns to scale in section 1.2. We parameterize this model by taking measures of substitution and technical change to be constant parameters. We then derive the constraints on these parameters implied by the theory of production. In section 1.3 we present statistical methods for estimating this model of producer behavior under linear and nonlinear restrictions. Finally, we illustrate the application of this model by studies of data on individual industries in section 1.4.

In section 1.5 we consider the extension of econometric modeling of producer behavior to nonconstant returns to scale. In regulated industries the price of output is set by regulatory authority. Given the demand for output as a function of the regulated price, the level of output can be taken as exogenous to the producing unit. Necessary conditions for producer equilibrium can be derived from cost minimization. The minimum value of total cost can be expressed as a function of the level of output and the prices of all inputs. This cost function provides a dual representation of the production function.

The dual formulation of the theory of producer behavior under nonconstant returns to scale parallels the theory under constant returns. However, the level of output replaces the level of technology as an exogenous determinant of production patterns. An econometric model can be parameterized by taking measures of substitution and economies of scale to be constant parameters. In section 1.6 we illustrate this approach by means of studies of data on individual firms in regulated industries.

In section 1.7 we conclude the chapter by outlining frontiers for future research. Current empirical research has focused on the development of more elaborate and more detailed data sets. We consider, in particular, the modeling of consistent time series of interindustry transactions tables and the application of the results to general equilibrium analysis of the impact of economic policy. We also discuss the analysis of panel data sets, that is, time series of cross sections of observations on individual producing units.

Current methodological research has focused on dynamic modeling of production. At least two promising approaches to this problem have been proposed. Both employ optimal control models of producer behavior. The first is based on static expectations with all future prices taken to be equal to current prices. The second approach is based on stochastic optimization under rational expectations, utilizing information about expectations of future prices contained in current production patterns.

1.2 Price Functions

The purpose of this section is to present the simplest form of the econometric methodology for modeling producer behavior. We base this methodology on a production function with constant returns to scale. Producer equilibrium implies the existence of a price function, giving the price of output as a function of the prices of inputs and the level of technology. The price function is dual to the production function and provides an alternative and equivalent description of technology.

An econometric model of producer behavior takes the form of a system of simultaneous equations, determining the distributive shares of the inputs and the rate of technical change. Measures of substitution and technical change give the responses of the distributive shares and the rate of technical change to changes in prices and the level of technology. To generate an econometric model of producer behavior we treat these measures as unknown parameters to be estimated.

The economic theory of production implies restrictions on the parameters of an econometric model of producer behavior. These restrictions take the form of linear and nonlinear constraints on the parameters. Statistical methods employed in modeling producer behavior involve the estimation of systems of nonlinear simultaneous equations with parameters subject to constraints. These constraints give rise to tests of the theory of production and tests of restrictions on patterns of substitution and technical change.

1.2.1 Duality

In order to present the theory of production we first require some notation. We denote the quantity of output by y and the quantities of J inputs by x_j $(j = 1, 2, \dots, J)$. Similarly, we denote the price of output by q and the prices of the J inputs by p_j $(j = 1, 2, \dots, J)$. We find it convenient to employ vector notation for the input quantities and prices:

$x = (x_1, x_2, \dots, x_J)$—vector of input quantities,
$p = (p_1, p_2, \dots, p_J)$—vector of input prices.

We assume that the technology can be represented by a *production function*, say F, where

$$y = F(x, t)\ , \tag{1.2.1}$$

and t is an index of the level of technology. In the analysis of time series data for a single producing unit the level of technology can be represented by time. In the analysis of cross-section data for different producing units the level of technology can be represented by one-zero dummy variables corresponding to the different units.[15]

We can define the *shares* of inputs in the value of output by

$$v_j = \frac{p_j\ x_j}{q\ y}\ , \quad (j = 1, 2, \dots, J)\ .$$

Under competitive markets for output and all inputs the necessary conditions for producer equilibrium are given by equalities between the share of each input in the value of output and the elasticity of output with respect to that input

$$v = \frac{\partial \ln y}{\partial \ln x}\,(x, t)\ , \tag{1.2.2}$$

where

$v = (v_1, v_2, \dots, v_J)$—vector of value shares.

$\ln x = (\ln x_1, \ln x_2, \dots, \ln x_J)$—vector of logarithms of input quantities.

Under constant returns to scale the elasticities and the value shares for all inputs sum to unity

$$i'v = i'\,\frac{\partial \ln y}{\partial \ln x} = 1\ ,$$

where i is a vector of ones. The value of output is equal to the sum of the values of the inputs.

Finally, we can define the *rate of technical change,* say v_t, as the rate of growth of the quantity of output holding all inputs constant

$$v_t = \frac{\partial \ln y}{\partial t}\,(x, t)\ . \tag{1.2.3}$$

It is important to note that this definition does not impose any restriction on patterns of substitution among inputs.

Given the identity between the value of output and the value of all inputs and given equalities between the value share of each input and the elasticity of output with respect to that input, we can express the price of output as a function, say Q, of the prices of all inputs and the level of technology

$$q = Q(p, t) . \tag{1.2.4}$$

We refer to this as the *price function* for the producing unit.

The price function Q is dual to the production function F and provides an alternative and equivalent description of the technology of the producing unit.[16] We can formalize this description in terms of the following properties of the price function:

1. *Positivity*. The price function is positive for positive input prices.
2. *Homogeneity*. The price function is homogeneous of degree one in the input prices.
3. *Monotonicity*. The price function is increasing in the input prices.
4. *Concavity*. The price function is concave in the input prices.

Given differentiability of the price function, we can express the value shares of all inputs as elasticities of the price function with respect to the input prices

$$v = \frac{\partial \ln q}{\partial \ln p} (p, t) , \tag{1.2.5}$$

where $\ln p = (\ln p_1, \ln p_2, \ldots, \ln p_J)$—vector of logarithms of input prices.

Further, we can express the negative of the rate of technical change as the rate of growth of the price of output, holding the prices of all inputs constant

$$- v_t = \frac{\partial \ln q}{\partial t} (p, t) . \tag{1.2.6}$$

Since the price function Q is homogeneous of degree one in the input prices, the value shares and the rate of technical change are homogeneous of degree zero and the value shares sum to unity

$$i'v = i' \frac{\partial \ln q}{\partial \ln p} = 1 .$$

Since the price function is increasing in the input prices the value shares must be nonnegative,

$v \geqq 0$.

Since the value shares sum to unity, we can write

$v \geq 0$,

where $v \geq 0$ implies $v \geqq 0$ and $v \neq 0$.

1.2.2 Substitution and Technical Change

We have represented the value shares of all inputs and the rate of technical change as functions of the input prices and the level of technology. We can introduce measures of substitution and technical change to characterize these functions in detail. For this purpose we differentiate the logarithm of the price function twice with respect to the logarithms of input prices to obtain measures of substitution

$$U_{pp} = \frac{\partial^2 \ln q}{\partial \ \ln p^2} \, (p,t) = \frac{\partial \ v}{\partial \ln p} \, (p,t) \ . \tag{1.2.7}$$

We refer to the measures of substitution (1.2.7) as *share elasticities*, since they give the response of the value shares of all inputs to proportional changes in the input prices. If a share elasticity is positive, the corresponding value share increases with the input price. If a share elasticity is negative, the value share decreases with the input price. Finally, if a share elasticity is zero, the value share is independent of the price.[17]

Second, we can differentiate the logarithm of the price function twice with respect to the logarithms of input prices and the level of technology to obtain measures of technical change

$$u_{pt} = \frac{\partial^2 \ln q}{\partial \ln p \partial t} \, (p,t) = \frac{\partial v}{\partial t} = - \, \frac{\partial v_t}{\partial \ln p} \, (p,t) \ . \tag{1.2.8}$$

We refer to these measures as *biases of technical change*. If a bias of technical change is positive, the corresponding value share increases with a change in the level of technology and we say that technical change is *input-using*. If a bias of technical change is negative, the value share decreases with a change in technology and technical change is *input-saving*. Finally, if a bias is zero, the value share is independent of technology; in this case we say that technical change is *neutral*.[18]

Alternatively, the vector of biases of technical change u_{pt} can be employed to derive the implications of changes in input prices for the rate of technical change. If a bias of technical change is positive, the rate of technical change decreases with the input price. If a bias is negative, the rate of technical change increases with the input price. Finally, if a bias is zero so that technical change is neutral, the rate of technical change is independent of the price.

To complete the description of technical change we can differentiate the logarithm of the price function twice with respect to the level of technology

$$u_{tt} = \frac{\partial^2 \ln q}{\partial t^2}(p,t) = -\frac{\partial v_t}{\partial t}(p,t) \,. \tag{1.2.9}$$

We refer to this measure as the *deceleration* of technical change, since it is the negative of rate of change of the rate of technical change. If the deceleration is positive, negative, or zero, the rate of technical change is decreasing, increasing, or independent of the level of technology.

The matrix of second-order logarithmic derivatives of the logarithm of the price function Q must be symmetric. This matrix includes the matrix of share elasticities U_{pp}, the vector of biases of technical change u_{pt}, and the deceleration of technical change u_{tt}. Concavity of the price function in the input prices implies that matrix of second-order derivatives, say H, is nonpositive definite, so that the matrix $U_{pp} + vv' - V$ is nonpositive definite, where

$$\frac{1}{q}\, N \cdot H \cdot N = U_{pp} + vv' - V \;;$$

the price of output q is positive and the matrices N and V are diagonal

$$N = \begin{bmatrix} p_1 & 0 & \cdots & 0 \\ 0 & p_2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & p_J \end{bmatrix}, \qquad V = \begin{bmatrix} v_1 & 0 & \cdots & 0 \\ 0 & v_2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & v_J \end{bmatrix}.$$

We can define substitution and complementarity of inputs in terms of the matrix of share elasticities U_{pp} and the vector of value shares v. We say that two inputs are *substitutes* if the corresponding element of the matrix $U_{pp} + vv' - V$ is negative. Similarly, we say that two inputs are *complements* if the corresponding element of this matrix is positive. If the element of this matrix corresponding to the two inputs is zero, we say that the inputs are *independent*. The definition of substitution and complementarity is symmetric in the two inputs, reflecting the symmetry of the matrix $U_{pp} + vv' - V$. If there are only two inputs, nonpositive definiteness of this matrix implies that the inputs cannot be complements.[19]

We next consider restrictions on patterns of substitution and technical change implied by separability of the price function Q. The most important applications of separability are associated with aggregation over inputs. Under separability, the price of output can be represented as a function of the prices of a smaller number of inputs by introducing price indexes for input aggregates. By treating the price of each aggregate as a function of the prices of the inputs making up the aggregate, we can generate a second stage of the model.

We say that the price function Q is *separable* in the K input prices $\{p_1, p_2, \ldots, p_K\}$ if and only if the price function can be represented in the form

$$q = Q[P(p_1, p_2, \ldots, p_K), p_{K+1}, \ldots, p_J, t] , \tag{1.2.10}$$

where the function P is independent of the $J - K$ input prices $\{p_{K+1}, p_{K+2}, \ldots, p_J\}$ and the level of technology t.[20] We say that the price function is *homothetically separable* if the function P in (2.10) is homogeneous of degree one.[21] Separability of the price function implies homothetic separability.[22]

The price function Q is homothetically separable in the K input prices $\{p_1, p_2, \ldots, p_K\}$ if and only if the production function F is homothetically separable in the K input quantities $\{x_1, x_2, \ldots, x_K\}$

$$y = F[G(x_1, x_2, \ldots, x_K), x_{K+1}, \ldots, x_J, t] , \tag{1.2.11}$$

where the function G is homogeneous of degree one and independent of $J - K$ quantities $\{x_{K+1}, x_{K+2}, \ldots, x_J\}$ and the level of technology t.[23]

We can interpret the function P in the definition of a separability of the price function as a price index; similarly, we can interpret the function G as a quantity index. The price index is dual to the quantity index and has properties analogous to those of the price function:

1. *Positivity*. The price index is positive for positive input prices.
2. *Homogeneity*. The price index is homogeneous of degree one in the input prices.
3. *Monotonicity*. The price index is increasing in the input prices.
4. *Concavity*. The price index is concave in the input prices.

The total cost of the K inputs included in the price index P, say c, is the sum of expenditures on all K inputs

$$c = \sum_{k=1}^{K} p_k x_k \ .$$

We can define the quantity index G for this aggregate as the ratio of total cost to the price index P

$$G = \frac{c}{P} \ . \tag{1.2.12}$$

The product of the price and quantity indexes for the aggregate is equal to the cost of the K inputs.[24]

We can analyze the implications of homothetic separability by introducing price and quantity indexes of aggregate input and defining the value share of aggregate input in terms of these indexes. An aggregate input can be treated in precisely the same way as any other input, so that price and quantity indexes can be used to reduce the dimensionality of the space of input prices and quantities. The price index generates a second stage of the model, by treating the price of each aggregate as a function of the prices of the inputs making up the aggregate.[25]

1.2.3 Parameterization

In the theory of producer behavior the dependent variables are value shares of all inputs and the rate of technical change. The independent variables are prices of inputs and the level of technology. The purpose of an econometric model of producer behavior is to characterize the value shares and the rate of technical change as functions of the input prices and the level of technology.

To generate an econometric model of producer behavior a natural approach is to treat the measures of substitution and technical change as unknown parameters to be estimated. For this purpose we introduce the parameters

$$B_{pp} = U_{pp} \ , \qquad \beta_{pt} = u_{pt} \ , \qquad \beta_{tt} = u_{tt} \ , \tag{1.2.13}$$

where B_{pp} is a matrix of constant share elasticities, β_{pt} is a vector of constant biases of technical change, and β_{tt} is a constant deceleration of technical change.[26]

We can regard the matrix of share elasticities, the vector of biases of technical change, and the deceleration of technical change as a system of second-order partial differential equations. We can integrate this system to obtain a system of first-order partial differential equations

$$\begin{aligned} v \ &= \alpha_p + B_{pp} \ln p + \beta_{pt} \cdot t \ , \\ -v_t &= \alpha_t + \beta'_{pt} \ln p + \beta_{tt} \cdot t \ , \end{aligned} \tag{1.2.14}$$

where the parameters—α_p, α_t—are constants of integration.

To provide an interpretation of the parameters—α_p, α_t—we first normalize the input prices. We can set the prices equal to unity where the level of technology t is equal to zero. This represents a choice of origin for measuring the level of technology and a choice of scale for measuring the quantities and prices of inputs. The vector of parameters α_p is the vector of value shares and the parameter α_t is the negative of the rate of technical change where the level of technology t is zero.

Similarly, we can integrate the system of first-order partial differential equations (1.2.14) to obtain the price function

$$\ln p = \alpha'_0 + a'_p \ln p + \alpha_t \cdot t + \frac{1}{2} \ln p' \, B_{pp} \ln p + \ln p' \, \beta_{pt} \cdot t + \frac{1}{2} \beta_{tt} \cdot t^2 \ , \tag{1.2.15}$$

where the parameter α_0 is a constant of integration. Normalizing the price of output so that it is equal to unity where t is zero, we can set this parameter equal to zero. This represents a choice of scale for measuring the quantity and price of output.

For the price function (1.2.15) the price of output is a transcendental or, more specifically, an exponential function of the logarithms of the input prices. We refer to this form as the *transcendental logarithmic* price function or, more simply, the translog price function, indicating the role of the variables. We can also characterize this price function as the *constant share elasticity* or CSE price function, indicating the role of the fixed parameters. In this representation the scalars—α_t, β_t—the vectors—α_p, β_{pt}—and the matrix B_{pp} are constant parameters that reflect the underlying technology. Differences in levels of technology among time periods for a given producing unit or among producing units at a given point of time are represented by differences in the level of technology t.

For the translog price function the negative of the average rates of technical change at any two levels of technology, say t and $t-1$, can be expressed as the difference between successive logarithms of the price of output, less a weighted average of the differences between successive logarithms of the input prices with weights given by the average value shares

$$-\bar{v}_t = \ln q(t) - \ln q(t-1) - \bar{v}' \, [\ln p(t) - \ln p(t-1)] \, , \tag{1.2.16}$$

In the expression (1.2.16) $\bar{v}_t$ is the average rate of technical change,

$$\bar{v}_t = \frac{1}{2} \, [v_t(t) + v_t(t-1)] \, ,$$

and the vector of average value shares $\bar{v}$ is given by

$$\bar{v} = \frac{1}{2} \, [v(t) + v(t-1)] \, .$$

We refer to the expression (1.2.16), introduced by Christensen and Jorgenson (1970), as the *translog rate of technical change*.

We have derived the translog price function as an exact representation of a model of producer behavior with constant share elasticities and constant biases and deceleration of technical change.[27] An alternative approach to the translog price function, based on a Taylor's series approximation to an arbitrary price function, was originated by Christensen, Jorgenson, and Lau (1971, 1973). Diewert (1976, 1980) has shown that the translog rate of technical change (2.16) is exact for the translog price function and the converse.

Diewert (1971, 1973, 1974b) introduced the Taylor's series approach for parameterizing models of producer behavior based on the dual formulation of the theory of production. He utilized this approach to generate the "generalized Leontief" parametric form, based on square root rather than logarithmic transformations of prices. Earlier, Heady and Dillon (1961) had employed Taylor's series approximations to generate parametric forms for the production function, using both square root and logarithmic transformations of the quantities of inputs.

The limitation of Taylor's series approximations has been emphasized by Gallant (1981) and Elbadawi, Gallant, and Souza (1983). Taylor's series provide only a local approximation to an arbitrary price or production function. The behavior of the error of approximation must be specified in formulating an econometric model of producer behavior. To remedy these deficiencies Gallant (1981) has introduced global approximations based on Fourier series.[28]

1.2.4 Integrability

The next step in generating our econometric model of producer behavior is to incorporate the implications of the econometric theory of production. These implications take the form of restrictions on the system of equations (1.2.14), consisting of value shares of all inputs v and the rate of technical change v_t. These restrictions are required to obtain a price function Q with the properties we have listed above. Under these restrictions we say that the system of equations is *integrable*. A complete set of conditions for integrability is the following:

1.2.4.1 Homogeneity

The value shares and the rate of technical change are homogeneous of degree zero in the input prices.

We first represent the value shares and the rate of technical change as a system of equations (1.2.14). Homogeneity of the price function implies that the parameters —B_{pp}, β_{pt}—in this system must satisfy the restrictions

$$B_{pp}i = 0 \ ,$$
$$\beta'_{pt}i = 0 \ , \qquad (1.2.17)$$

where i is a vector of ones. For J inputs there are $J+1$ restrictions implied by homogeneity.

1.2.4.2 Product Exhaustion

The sum of the value shares is equal to unity.

Product exhaustion implies that the value of the J inputs is equal to the value of the product. Product exhaustion implies that the parameters—α_p, B_{pp}, β_{pt}—must satisfy the restrictions

$$\alpha'_p i = 1 \ ,$$
$$B'_{pp}i = 0 \ ,$$
$$\beta'_{pt}i = 0 \ . \qquad (1.2.18)$$

For J inputs there are $J+2$ restrictions implied by product exhaustion.

1.2.4.3 Symmetry

The matrix of share elasticities, biases of technical change, and the deceleration of technical change must be symmetric.

A necessary and sufficient condition for symmetry is that the matrix of parameters must satisfy the restrictions

$$\begin{bmatrix} B_{pp} & \beta_{pt} \\ \beta'_{pt} & \beta_{tt} \end{bmatrix} = \begin{bmatrix} B_{pp} & \beta_{pt} \\ \beta'_{pt} & \beta_{tt} \end{bmatrix}' . \tag{1.2.19}$$

For J inputs the total number of symmetry restrictions is $1/2J(J+1)$.

1.2.4.4 Nonnegativity

The value shares must be nonnegative.

Nonnegativity is implied by monotonicity of the price function

$$\frac{\partial \ln q}{\partial \ln p} \geqq 0 .$$

For the translog price function the conditions for monotonicity take the form

$$\frac{\partial \ln q}{\partial \ln p} = \alpha_p + B_{pp} \ln p + \beta_{pt} \cdot t \geqq 0 . \tag{1.2.20}$$

Since the translog price function is quadratic in the logarithms of the input prices, we can always choose prices so that the monotonicity of the price function is violated. Accordingly, we cannot impose restrictions on the parameters that would imply nonnegativity of the value shares for all prices and levels of technology. Instead, we consider restrictions that imply monotonicity of the value shares wherever they are nonnegative.

1.2.4.5 Monotonicity

The matrix of share elasticities must be nonpositive definite.

Concavity of the price function implies that the matrix $B_{pp} + vv' - V$ is nonpositive definite. Without violating the product exhaustion and nonnegativity restrictions we can set the matrix $vv' - V$ equal to zero. For example, we can choose one of the value shares equal to unity and all the others equal to zero. A necessary condition for the matrix $B_{pp} + vv' - V$ to be nonpositive definite is that the matrix of constant share elasticities B_{pp} must be nonpositive definite. This condition is also sufficient, since the matrix $vv' - V$ is nonpositive definite and the sum of two nonpositive definite matrices is nonpositive definite.[29]

We can impose concavity on the translog price functions by representing the matrix of constant share elasticities B_{pp} in terms of its Cholesky factorization

$$B_{pp} = TDT' ,$$

where T is a unit lower triangular matrix and D is a diagonal matrix. For J inputs we can write the matrix B_{pp} in terms of its Cholesky factorization as follows:

$$B_{pp} = \begin{bmatrix} \delta_1 & \lambda_{21}\delta_1 & \cdots & \lambda_{J1}\delta_1 \\ \lambda_{21}\delta_1 & \lambda_{21}\lambda_{21}\delta_1 + \delta_2 & \cdots & \lambda_{J1}\lambda_{21}\delta_1 + \lambda_{J2}\delta_2 \\ \vdots & \vdots & & \vdots \\ \lambda_{J1}\delta_1 & \lambda_{J1}\lambda_{21}\delta_1 + \lambda_{J2}\delta_2 & \cdots & \lambda_{J1}\lambda_{J1}\delta_1 + \lambda_{J2}\lambda_{J2}\delta_2 + \cdots + \delta_J \end{bmatrix},$$

where

$$T = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \lambda_{21} & 1 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ \lambda_{J1} & \lambda_{J2} & \cdots & 1 \end{bmatrix}, \qquad D = \begin{bmatrix} \delta_1 & 0 & \cdots & 0 \\ 0 & \delta_2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \delta_J \end{bmatrix}.$$

The matrix of constant share elasticities B_{pp} must satisfy restrictions implied by symmetry and product exhaustion. These restrictions imply that the parameters of the Cholesky factorization must satisfy the following conditions

$$1 + \lambda_{21} + \lambda_{31} + \cdots + \lambda_{Jl} = 0 ,$$

$$1 + \lambda_{32} + \lambda_{42} + \cdots + \lambda_{J2} = 0 ,$$

$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$

$$\delta_J = 0 .$$

Under these conditions there is a one-to-one transformation between the elements of the matrix of share elasticities B_{pp} and the parameters of the Cholesky factorization—T, D. The matrix of share elasticities is nonpositive definite if and only if the diagonal elements $\{\delta_1, \delta_2, \ldots, \delta_{J-1}\}$ of the matrix D are nonpositive.[30]

1.3 Statistical Methods

Our model of producer behavior is generated from a translog price function for each producing unit. To formulate an econometric model of production and technical change we add a stochastic component to the equations for the value shares and the rate of technical change. We associate this component with unobservable random disturbances at the level of the producing unit. The producer maximizes profits for given input prices, but the value shares of inputs are subject to a random disturbance.

The random disturbances in an econometric model of producer behavior may result from errors in implementation of production plans, random elements in the technology not reflected in the model of producer behavior, or errors of measurement in the value shares. We assume that each of the equations for the value shares and the rate of technical change has two additive components. The first is a nonrandom function of the input prices and the level of technology; the second is an unobservable random disturbance that is functionally independent of these variables.[31]

1.3.1 Stochastic Specification

To represent an econometric model of production and technical change we require some additional notation. We consider observations on the relative distribution of the value of output among all inputs and the rate of technical change. We index the observations by levels of technology $(t = 1, 2, \ldots, T)$. We employ a level of technology indexed by time t as an illustration throughout the following discussion. The vector of value shares in the t-th time period is denoted v^t $(t = 1, 2, \ldots, T)$. Similarly, the rate of technical change in the t-th time period is denoted v_t^t. The vector of input prices in the t-th time period is denoted p_t $(t = 1, 2, \ldots, T)$. Similarly, the vector of logarithms of input prices is denoted $\ln p_t$ $(t = 1, 2, \ldots, T)$.

We obtain an econometric model of production and technical change corresponding to the translog price function by adding random disturbances to the equations for the value shares and the rate of technical change

$$\begin{aligned} v^t &= \alpha_p + B_{pp} \ln p_t + \beta_{pt} \cdot t + \varepsilon^t \ , \\ v_t^t &= \alpha_t + \beta'_{pt} \ln p_t + \beta_{tt} \cdot t + \varepsilon_t^t \ , \qquad (t = 1, 2, \ldots, T) \ , \end{aligned} \tag{1.3.1}$$

where ε^t is the vector of unobservable random disturbances for the value shares of the t-th time period and ε_t^t is the corresponding disturbance for the rate of technical change. Since the value shares for all inputs sum to unity in each time period, the random disturbances corresponding to the J value shares sum to zero in each time period

$$i'\varepsilon^t = 0 \ , \quad (t = 1, 2, \ldots, T) \ , \tag{1.3.2}$$

so that these disturbances are not distributed independently.

We assume that the unobservable random disturbances for all $J+1$ equations have expected value equal to zero for all observations

$$E\begin{bmatrix} \varepsilon^t \\ \varepsilon_t^t \end{bmatrix} = 0 \ , \quad (t = 1, 2, \ldots, T) \ . \tag{1.3.3}$$

We also assume that the disturbances have a covariance matrix that is the same for all observations; since the random disturbances corresponding to the J value shares sum to zero, this matrix is nonnegative definite with rank at most equal to J. We assume that the covariance matrix of the random disturbances corresponding to the value shares and the rate of technical change, say Σ, has rank J, where

$$V\begin{bmatrix} \varepsilon^t \\ \varepsilon_t^t \end{bmatrix} = \Sigma \ , \quad (t = 1, 2, \ldots, T) \ .$$

Finally, we assume that the random disturbances corresponding to distinct observations in the same or distinct equations are uncorrelated. Under this assumption the covariance matrix of random disturbances for all observations has the Kronecker product form

$$V\begin{bmatrix} \varepsilon_1^1 \\ \varepsilon_1^2 \\ \vdots \\ \varepsilon_1^T \\ \varepsilon_2^1 \\ \vdots \\ \varepsilon_t^T \end{bmatrix} = \Sigma \otimes I \ . \tag{1.3.4}$$

1.3.2 Autocorrelation

The rate of technical change v_t^t is not directly observable; we assume that the equation for the translog price index of the rate of technical change can be written

$$-\bar{v}_t^t = \alpha_t + \beta'_{pt}\,\overline{\ln p_t} + \beta_{tt} \cdot \bar{t} + \bar{\varepsilon}_t^t\,, \quad (t = 1, 2, \ldots, T)\,, \tag{1.3.5}$$

where $\bar{\varepsilon}_t^t$ is the average disturbance in the two periods

$$\bar{\varepsilon}_t^t = \frac{1}{2}\,[\varepsilon_t^t + \varepsilon_t^{t-1}]\,, \quad (t = 1, 2, \ldots, T)\,.$$

Similarly, $\overline{\ln p_t}$ is a vector of averages of the logarithms of the input prices and $\bar{t}$ is the average of time as an index of technology in the two periods.

Using our new notation, the equations for the value shares of all inputs can be written

$$\bar{v}^t = \alpha_p + B_{pp}\,\overline{\ln p_t} + \beta_{pt} \cdot \bar{t} + \bar{\varepsilon}^t\,, \quad (t = 1, 2, \ldots, T)\,, \tag{1.3.6}$$

where $\bar{\varepsilon}^t$ is a vector of averages of the disturbances in the two periods. As before, the average value shares sum to unity, so that the average disturbances for the equations corresponding to value shares sum to zero

$$i'\bar{\varepsilon}^t = 0\,, \quad (t = 1, 2, \ldots, T)\,. \tag{1.3.7}$$

The covariance matrix of the average disturbances corresponding to the equation for the rate of technical change for all observations is proportional to a Laurent matrix

$$V\begin{bmatrix} \bar{\varepsilon}_t^2 \\ \bar{\varepsilon}_t^3 \\ \vdots \\ \bar{\varepsilon}_t^T \end{bmatrix} \sim \Omega\,, \tag{1.3.8}$$

where

$$\Omega = \begin{bmatrix} 1/2 & 1/4 & 0 & \cdots & 0 \\ 1/4 & 1/2 & 1/4 & \cdots & 0 \\ 0 & 1/4 & 1/2 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1/2 \end{bmatrix} .$$

The covariance matrix of the average disturbance corresponding to the equation for each value share is proportional to the same Laurent matrix. The covariance matrix of the average disturbances for all observations has the Kronecker product form

$$V \begin{bmatrix} \bar{\varepsilon}_1^2 \\ \bar{\varepsilon}_1^3 \\ \vdots \\ \bar{\varepsilon}_1^T \\ \bar{\varepsilon}_2^2 \\ \vdots \\ \bar{\varepsilon}_t^T \end{bmatrix} = \Sigma \otimes \Omega . \tag{1.3.9}$$

Since the matrix Ω in (1.3.9) is known, the equations for the average rate of technical change and the average value shares can be transformed to eliminate autocorrelation. The matrix Ω is positive definite, so that there is a matrix P such that

$$P\Omega P' = I ,$$
$$P'P = \Omega^{-1} .$$

To construct the matrix P we first invert the matrix Ω to obtain the inverse matrix Ω^{-1}, a positive definite matrix. We then calculate the Cholesky factorization of the inverse matrix Ω^{-1},

$$\Omega^{-1} = TDT' ,$$

where T is a unit lower triangular matrix and D is a diagonal matrix with positive elements along the main diagonal. Finally, we can write the matrix P in the form

$$P = D^{1/2}T' ,$$

where $D^{1/2}$ is a diagonal matrix with elements along the main diagonal equal to the square roots of the corresponding elements of D.

We can transform equations for the average rates of technical change by the matrix $P = D^{1/2}T'$ to obtain equations with uncorrelated random disturbances

$$D^{1/2}T'\begin{bmatrix} \bar{v}_t^2 \\ \bar{v}_t^3 \\ \vdots \\ \bar{v}_t^T \end{bmatrix} = D^{1/2}T'\begin{bmatrix} 1 & \overline{\ln p_{12}} & \cdots & 2-1/2 \\ 1 & \overline{\ln p_{13}} & \cdots & 3-1/2 \\ \vdots & \vdots & & \vdots \\ 1 & \overline{\ln p_{1T}} & \cdots & T-1/2 \end{bmatrix}\begin{bmatrix} \alpha_t \\ \beta_{1t} \\ \vdots \\ \beta_{tt} \end{bmatrix} + D^{1/2}T'\begin{bmatrix} \bar{\varepsilon}_t^2 \\ \bar{\varepsilon}_t^3 \\ \vdots \\ \bar{\varepsilon}_t^T \end{bmatrix}, \tag{1.3.10}$$

since

$$P\Omega P' = D^{1/2}T' \, \Omega \, (D^{1/2} \, T')' = I \, .$$

The transformation $P = D^{1/2}T'$ is applied to data on the average rates of technical change $\bar{v}_t$ and data on the average values of the variables that appear on the right-hand side of the corresponding equation.

We can apply the transformation $P = D^{1/2}T'$ to the equations for average value shares to obtain equations with uncorrelated disturbances. As before, the transformation is also applied to data on the average values of variables that appear on the right-hand side of the corresponding equations. The covariance matrix of the transformed disturbances from the equations for the average value shares and the equation for the average rates of technical change has the Kronecker product form

$$(I \otimes D^{1/2}T')(\Sigma \otimes \Omega)(I \otimes D^{1/2}T')' = \Sigma \otimes I \, . \tag{1.3.11}$$

To estimate the unknown parameters of the translog price function we combine the first $J-1$ equations for the average value shares with the equation for the average rate of technical change to obtain a complete econometric model of production and technical change. We can estimate the parameters of the equation for the remaining average value share, using the product exhaustion restrictions on these parameters. The complete model involves $1/2\,J(J+3)$ unknown parameters. A total of $1/2(J^2+4J+5)$ additional parameters can be estimated as functions of these parameters, using the homogeneity, product exhaustion, and symmetry restrictions.[32]

1.3.3 Identification and Estimation

We next discuss the estimation of the econometric model of production and technical change given in (1.3.5) and (1.3.6). The assumption that the input prices and the level of technology are exogenous variables implies that the model becomes a nonlinear multivariate regression model with additive errors, so that nonlinear regression techniques can be employed. This specification is appropriate for cross-section data on individual producing units. For aggregate time series data the existence of supply functions for all inputs makes it essential to treat the prices as endogenous. Under this assumption the model becomes a system of nonlinear simultaneous equations.

To estimate the complete model of production and technical change by the method of full information maximum likelihood it would be necessary to specify the full econometric model, not merely the model of producer behavior. Accordingly, to estimate the model of production in (1.3.5) and (1.3.6) we consider limited information techniques. For nonlinear multivariate regression models we can employ the method of maximum likelihood proposed by Malinvaud (1980).[33] For systems of nonlinear simultaneous equations we outline the estimation of the model by the nonlinear three-stage least squares (NL3SLS) method originated by Jorgenson and Laffont (1974). Wherever the right-hand side variables can be treated as exogenous, this method reduces to limited information maximum likelihood for nonlinear multivariate regression models.

Application of NL3SLS to our model of production and technical change would be straightforward, except for the fact that the covariance matrix of the disturbances is singular. We obtain NL3SLS estimators of the complete system by dropping one equation and estimating the resulting system of J equations by NL3SLS. The parameter estimates are invariant to the choice of the equation omitted in the model.

The NL3SLS estimator can be employed to estimate all parameters of the model of production and technical change, provided that these parameters are identified. The necessary order condition for identification is that

$$\frac{1}{2}(J+3) < (J-1)\min(V, T-1)\,, \tag{1.3.12}$$

where V is the number of instruments. A necessary and sufficient rank condition is given below; this amounts to the nonlinear analogue of the absence of multicollinearity.

Our objective is to estimate the unknown parameters—α_p, B_{pp}, β_{pt}—subject to the restrictions implied by homogeneity, product exhaustion, symmetry, and monotonicity. By dropping the equation for one of the value shares, we can eliminate the restrictions implied by summability. These restrictions can be used in estimating the parameters that occur in the equation that has been dropped. We impose the restrictions implied by homogeneity and symmetry as equalities. The restrictions implied by monotonicity take the form of inequalities.

We can write the model of production and technical change in (1.3.5) and (1.3.6) in the form

$$\begin{aligned} v_1 &= f_1(\gamma) + \varepsilon_1 , \\ v_2 &= f_2(\gamma) + \varepsilon_2 , \\ &\cdots\cdots\cdots\cdots \\ v_J &= f_J(\gamma) + \varepsilon_J , \end{aligned} \tag{1.3.13}$$

where v_j $(j = 1, 2, \ldots, J-1)$ is the vector of observations on the distributive share of the j-th input for all time periods, transformed to eliminate autocorrelation, v_J is the corresponding vector of observations on the rates of technical change; the vector γ includes the parameters—α_p, α_t, B_{pp}, β_{pt}, β_{tt}; f_j $(j = 1, 2, \ldots, J)$ is a vector of nonlinear functions of these parameters; finally, ε_j $(j = 1, 2, \ldots, J)$ is the vector of disturbances in the j-th equation, transformed to eliminate autocorrelation.

We can stack the equations in (1.3.13), obtaining

$$v = f(\gamma) + \varepsilon , \tag{1.3.14}$$

where

$$v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_J \end{bmatrix}, \qquad f = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_J \end{bmatrix}, \qquad \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_J \end{bmatrix}.$$

By the assumptions in section 1.3.1 above the random vector ε has mean zero and covariance matrix $\Sigma_\varepsilon \otimes I$ where Σ_ε is obtained from the covariance matrix Σ in (1.3.11) by striking the row and column corresponding to the omitted equation.

The nonlinear three-stage least squares (NL3SLS) estimator for the model of production and technical change is obtained by minimizing the weighted sum of squared residuals

$$S(\gamma) = [v - f(\gamma)]'[\hat{\Sigma}_{\varepsilon}^{-1} \otimes Z(Z'Z)^{-1}Z'][v - f(\gamma)] , \tag{1.3.15}$$

with respect to the vector of unknown parameters γ, where Z is the matrix of $T - 1$ observations on the V instrumental variables. Provided that the parameters are identified, we can apply the Gauss-Newton method to minimize (1.3.15). First, we linearize the model (1.3.14), obtaining

$$v = f(\gamma_0) + \frac{\partial f}{\partial \gamma}(\gamma_0)\Delta\gamma + u , \tag{1.3.16}$$

where γ_0 is the initial value of the vector of unknown parameters γ and

$$\Delta\gamma = \gamma_1 - \gamma_0 ,$$

where γ_1 is the revised value of this vector. The fitted residuals u depend on the initial and revised values.

To revise the initial values we apply Zellner and Theil's (1962) three-stage least-squares method to the linearized model, obtaining

$$\begin{aligned}\Delta\gamma = {} & \left\{\frac{\partial f}{\partial \gamma}(\gamma_0)'(\hat{\Sigma}_{\varepsilon}^{-1} \otimes Z(Z'Z)^{-1}Z') \frac{\partial f}{\partial \gamma}(\gamma_0)\right\}^{-1} \\ & \times \frac{\partial f}{\partial \gamma}(\gamma_0)'\{\hat{\Sigma}_{\varepsilon}^{-1} \otimes Z(Z'Z)^{-1}Z'\}b[v - f(\gamma_0)] .\end{aligned} \tag{1.3.17}$$

If $S(\gamma_0) > S(\gamma_1)$, a further iteration is performed by replacing γ_0 by γ_1 in (1.3.16) and (1.3.17), resulting in a further revised value, say γ_2, and so on. If this condition is not satisfied, we divide the revision $\Delta\gamma$ by two and evaluate the criteria $S(\gamma)$ again; we continue reducing the revision $\Delta\gamma$ until the criterion improves or the convergence criterion $\max_j \Delta\gamma_j/\gamma_j$ is less than some prespecified limit. If the criterion improves, we continue with further iterations. If not, we stop the iterative process and employ the current value of the vector of unknown parameters as our NL3SLS estimator.[34]

The final step in estimation of the model of production and technical change is to minimize the criterion function (1.3.15) subject to the restrictions implied by monotonicity of the distributive shares. We have eliminated the restrictions that take the form of equalities. Monotonicity of the distributive shares implies inequality restrictions on the parameters of the Cholesky factorization of the matrix of constant share elasticities B_{pp}. The diagonal elements of the matrix D in this factorization must be nonpositive.

We can represent the inequality constraints on the matrix of share elasticities B_{pp} in the form

$$\phi_j(\gamma) \geqq 0 \,, \qquad (j = 1, 2, \ldots, J-1), \tag{1.3.18}$$

where $J-1$ is the number of restrictions. We obtain the inequality constrained nonlinear three-stage least-squares estimator for the model by minimizing the criterion function subject to the constraints (1.3.18). This estimator corresponds to the saddlepoint of the Lagrangian function

$$L = S(\gamma) + \lambda'\phi \,, \tag{1.3.19}$$

where λ is a vector of $J-1$ Lagrange multipliers and ϕ is a vector of $J-1$ constraints.

The Kuhn-Tucker (1951) conditions for a saddlepoint of the Lagrangian (1.3.19) are the first-order conditions

$$\frac{\partial L}{\partial \gamma} = \frac{\partial S(\gamma)}{\partial \gamma} + \lambda' \frac{\partial \phi}{\partial \gamma} = 0 \,, \tag{1.3.20}$$

and the complementary slackness condition

$$\lambda'\phi = 0 \,, \qquad \lambda \geq 0 \,. \tag{1.3.21}$$

To find a saddlepoint of the Lagrangian (1.3.19) we begin by linearizing the model of production and technical change (1.3.14) as in (1.3.16). Second, we linearize the constraints as

$$\phi(\gamma) = \frac{\partial \phi}{\partial \gamma} \, \Delta\gamma + \phi(\gamma_0) \,, \tag{1.3.22}$$

where γ_0 is a vector of initial values of the unknown parameters. We apply Liew's (1976) inequality constrained three-stage least-squares method to the linearized model, obtaining

$$\Delta\gamma^* = \Delta\gamma + \left\{ \frac{\partial f}{\partial \gamma} (\gamma_0)' (\hat{\Sigma}_\varepsilon^{-1} \otimes Z(Z'Z)^{-1}Z') \frac{\partial f}{\partial \gamma} (\gamma_0) \right\}^{-1} \frac{\partial \phi}{\partial \gamma} (\gamma_0)'\lambda^*, \tag{1.3.23}$$

where $\Delta\delta$ is the change in the values of the parameters (1.3.17) and λ^* is the solution of the linear complementarity problem

$$\frac{\partial \phi}{\partial \gamma}(\gamma_0)\left\{\frac{\partial f}{\partial \gamma}(\gamma_0)'(\hat{\Sigma}_\varepsilon^{-1} \otimes Z(Z'Z)^{-1}Z')\,\frac{\partial f}{\partial \gamma}(\gamma_0)\right\}^{-1} \frac{\partial \phi}{\partial \gamma}(\gamma_0)\,\lambda$$
$$+\frac{\partial \phi}{\partial \gamma}(\gamma_0)\Delta\gamma - \phi(\gamma_0) \geqq 0\,,$$

where

$$\left[\frac{\partial \phi}{\partial \gamma}(\gamma_0)\left\{\frac{\partial f}{\partial \gamma}(\gamma_0)'(\hat{\Sigma}_\varepsilon^{-1} \otimes Z(Z'Z)^{-1}Z')\,\frac{\partial f}{\partial \gamma}(\gamma_0)\right\}^{-1} \frac{\partial \phi}{\partial \gamma}(\gamma_0)\right.$$
$$\left. +\frac{\partial \phi}{\partial \gamma}(\gamma_0)\Delta\gamma - \phi(\gamma_0)\right]'\lambda = 0\,, \qquad \lambda \geq 0\,.$$

Given an initial value of the unknown parameters γ_0 that satisfies the $J-1$ constraints (1.3.18), if $S(\gamma_1) < S(\gamma_0)$ and δ_1 satisfies the constraints, the iterative process continues by linearizing the model (1.3.14) as in (1.3.16) and the constraints (1.3.18) as in (1.3.22) at the revised value of the vector of unknown parameters $\gamma_1 = \gamma_0 + \Delta\gamma$. If not, we shrink $\Delta\gamma$ as before, continuing until an improvement is found subject to the constraints or $\max_j \Delta\gamma_j/\gamma_j$ is less than a convergence criterion.

The nonlinear three-stage least-squares estimator obtained by minimizing the criterion function (1.3.15) is a consistent estimator of the vector of unknown parameters γ. A consistent estimator of the covariance matrix Σ_ε with typical element σ_{jk} is given by

$$\hat{\sigma}_{jk} = \frac{1}{T}\,[v_j - f_j(\hat{\gamma})]'\,[v_k - f_k(\hat{\gamma})]\,, \qquad (j, k = 1, 2, \ldots, J)\,. \tag{1.3.24}$$

Under suitable regularity conditions the estimator $\hat{\gamma}$ is asymptotically normal with covariance matrix

$$V(\hat{\gamma}) = \left\{\frac{\partial f}{\partial \gamma}(\gamma)'(\Sigma_\varepsilon^{-1} \otimes Z(Z'Z)^{-1}Z')\,\frac{\partial f}{\partial \gamma}(\gamma)\right\}^{-1}. \tag{1.3.25}$$

We obtain a consistent estimator of this matrix by inserting the consistent estimators $\hat{\gamma}$ and $\hat{\Sigma}_\varepsilon$ in place of the parameters γ and Σ_ε. The nonlinear three-stage least-squares estimator is efficient in the class of instrumental variables estimators using Z as the matrix of instrumental variables.[35]

The rank condition necessary and sufficient for identifiability of the vector of unknown parameters γ is the nonsingularity of the following matrix in the neighborhood of the true parameter vector

$$\frac{\partial f}{\partial \gamma}(\gamma)'(\Sigma_\varepsilon^{-1} \otimes Z(Z'Z)^{-1}Z')\frac{\partial f}{\partial \gamma}(\gamma) . \tag{1.3.26}$$

The order condition (1.3.12) given above is necessary for the nonsingularity of this matrix.

Finally, we can consider the problem of testing equality restrictions on the vector of unknown parameters γ. For example, suppose that the maintained hypothesis is that there are $r = 1/2\, J(J+3)$ elements in this vector after solving out the homogeneity, product exhaustion, and symmetry restrictions. Additional equality restrictions can be expressed in the form

$$\gamma = g(\delta) , \tag{1.3.27}$$

where δ is a vector of unknown parameters with S elements, $s < r$. We can test the hypothesis

$$H : \gamma = g(\delta) ,$$

against the alternative

$$A : \gamma \neq g(\delta) .$$

Test statistics appropriate for this purpose have been analyzed by Gallant and Jorgenson (1979) and Gallant and Holly (1980).[36]

A statistic for testing equality restrictions in the form (1.3.27) can be constructed by analogy with the likelihood ratio principle. First, we can evaluate the criterion function (1.3.15) at the minimizing value $\hat{\gamma}$, obtaining

$$S(\hat{\gamma}) = [v - f(\hat{\gamma})]'[\hat{\Sigma}_\varepsilon^{-1} \otimes Z(Z'Z)^{-1}Z'][v - f(\hat{\gamma})].$$

Second, we can replace the vector of unknown parameters γ by the function $g(\delta)$ in (1.3.27)

$$S(\delta) = \{v - f[g(\delta)]\}'[\hat{\Sigma}_\varepsilon^{-1} \otimes Z(Z'Z)^{-1}Z']\{v - f[g(\delta)]\} ;$$

minimizing the criterion function with respect to δ, we obtain the minimizing value $\hat{\delta}$, the constrained estimator of γ, $g(\hat{\delta})$, and the constrained value of the criterion itself $S(\hat{\delta})$.

The appropriate test statistic, say $T(\hat{\gamma}, \hat{\delta})$, is equal to the difference between the constrained and unconstrained values of the criterion function

$$T(\hat{\gamma}, \hat{\delta}) = S(\hat{\delta}) - S(\hat{\gamma}). \tag{1.3.28}$$

Gallant and Jorgenson (1979) show that this statistic is distributed asymptotically as chi-squared with $r - s$ degrees of freedom. Wherever the right-hand side variables can be treated as exogenous, this statistic reduces to the likelihood ratio statistic for nonlinear multivariate regression models proposed by Malinvaud (1980). The resulting statistic is distributed asymptotically as chi-squared.[37]

1.4 Applications of Price Functions

We first illustrate the econometric modeling of substitution among inputs in section 1.4.1 by presenting an econometric model for nine industrial sectors of the U.S. economy implemented by Berndt and Jorgenson (1973). The Berndt–Jorgenson model is based on a price function for each sector, giving the price of output as a function of the prices of capital and labor inputs and the prices of inputs of energy and materials. Technical change is assumed to be neutral, so that all biases of technical change are set equal to zero.

In section 1.4.2 we illustrate the economectric modeling of both substitution and technical change. We present an econometric model of producer behavior that has been implemented for thirty-five industrial sectors of the U.S. economy by Jorgenson and Fraumeni (1983). In this model the rate of technical change and the distributive shares of productive inputs are determined simultaneously as functions of relative prices. Although the rate of technical change is endogenous, this model must be carefully distinguished from models of induced technical change.

Aggregation over inputs has proved to be an extremely important technique for simplifying the description of technology for empirical implementation. The corresponding restrictions can be used to generate a two-stage model of producer behavior. Each stage can be parameterized separately; alternatively, the validity of alternative simplifications can be assessed by testing the restrictions. In section 1.4.3 we conclude with illustrations of aggregation over inputs in studies by Berndt and Jorgenson (1973) and Berndt and Wood (1975).

1.4.1 Substitution

In the Berndt–Jorgenson (1973) model, production is divided among nine sectors of the U.S. economy:

1. Agriculture, nonfuel mining, and construction
2. Manufacturing, excluding petroleum refining
3. Transportation
4. Communications, trade, and services
5. Coal mining
6. Crude petroleum and natural gas
7. Petroleum refining
8. Electric utilities
9. Gas utilities.

The nine producing sectors of the U.S. economy included in the Berndt–Jorgenson model can be divided among five sectors that produce energy commodities—coal, crude petroleum and natural gas, refined petroleum, electricity, and natural gas as a product of gas utilities—and four sectors that produce nonenergy commodities—agriculture, manufacturing, transportation, and communications. For each sector output is defined as the total domestic supply of the corresponding commodity group, so that the input into the sector includes competitive imports of the commodity, inputs of energy, and inputs of nonenergy commodities.

The Berndt–Jorgenson model of producer behavior includes a system of equations for each of the nine producing sectors giving the shares of capital, labor, energy and materials inputs in the value of output as functions of the prices of the four inputs. To formulate an econometric model stochastic components are added to this system of equations. The rate of technical change is taken to be exogeneous, so that the adjustment for autocorrelation described in section 1.3.2 is not required. However, all prices are treated as endogenous variables; estimates of the unknown parameters of the econometric model are based on the nonlinear three-stage least-squares estimator presented in section 1.3.3.

The endogenous variables in the Berndt–Jorgenson model of producer behavior include value shares of capital, labor, energy, and materials input for each sector. Three equations can be estimated for each sector, corresponding to three of the value shares, as in (1.2.14). The unknown parameters include three elements of the vector $\{\alpha_p\}$ and six share elasticities in the matrix $\{B_{pp}\}$, which is constrained to be symmetric, so that there is a total of nine unkown parameters. Berndt and Jorgenson estimate these parameters from time series data for the period 1947–1971 for each industry; the estimates are presented by Hudson and Jorgenson (1974).

As a further illustration of modeling of substitution among inputs, we consider an econometric model of the total manufacturing sector of the U.S. economy implemented by Berndt and Wood (1975). This sector combines the manufacturing and petroleum refining sectors of the Berndt-Jorgenson model. Berndt and Wood generate this model by expressing the price of aggregate input as a function of the prices of capital, labor, energy, and materials inputs into total manufacturing. They find that capital and energy inputs are complements, while all other pairs of inputs are substitutes.

By comparison with the results of Berndt and Wood, Hudson and Jorgenson (1978) have classified patterns of substitution and complementarity among inputs for the four nonenergy sectors of the Berndt-Jorgenson model. For agriculture, nonfuel mining and construction, capital and energy are complements and all other pairs of inputs are substitutes. For manufacturing, excluding petroleum refining, energy is complementary with capital and materials, while other pairs of inputs are substitutes. For transportation, energy is complementary with capital and labor while other pairs of inputs are substitutes. Finally, for communications, trade and services, energy and materials are complements and all other pairs of inputs are substitutes.

Berndt and Wood have considered further simplification of the Berndt-Jorgenson model of producer behavior by imposing separability restrictions on patterns of substitution among capital, labor, energy, and materials inputs.[38] This would reduce the number of input prices at the first stage of the model through the introduction of additional input aggregates. For this purpose additional stages in the allocation of the value of sectoral output among inputs would be required. Berndt and Wood consider all possible pairs of capital. Labor, energy, and materials inputs, but find that only the input aggregate consisting of capital and energy is consistent with the empirical evidence.[39]

Berndt and Morrison (1979) have disaggregated the Berndt-Wood data on labor input between blue collar and white collar labor and have studied the substitution among the two types of labor and capital, energy, and materials inputs for U.S. total manufacturing, using a translog price function. Anderson (1981) has reanalyzed the Berndt-Wood data set, testing alternative specifications of the model of substitution among inputs. Gallant (1981) has fitted an alternative model of substitution among inputs to these data, based on the Fourier functional form for the price function. Elbadawi, Gallant, and Souza (1983) have employed this approach in estimating price elasticities of demand for inputs, using the Berndt-Wood data as a basis for Monte Carlo simulations of the performance of alternative functional forms.

Cameron and Schwartz (1979), Denny, May, and Pinto (1978), Fuss (1977a), and McRae (1981) have constructed econometric models of substitution among capital, labor, energy, and materials inputs based on translog functional forms for total manufacturing in Canada. Technical change is assumed to be neutral, as in the study of U.S. total manufacturing by Berndt and Wood (1975), but nonconstant returns to scale are permitted. McRae and Webster (1982) have compared models of substitution among inputs in Canadian manufacturing, estimated from data for different time periods.

Friede (1979) has analyzed substitution among capital, labor, energy, and materials inputs for total manufacturing in the Federal Republic of Germany. He assumes that technical change is neutral and utilizes a translog price function. He has disaggregated the results to the level of fourteen industrial groups, covering the whole of the West German economy. He has separated materials inputs into two groups—manufacturing and transportation services as one group and other nonenergy inputs as a second group. Ozatalay, Grumbaugh, and Long (1979) have modeled substitution among capital, labor, energy and materials inputs, on the basis of a translog price function. They use time-series data for total manufacturing for the period 1963–1974 in seven countries—Canada, Japan, the Netherlands, Norway, Sweden, the U.S., and West Germany.

Longva and Olsen (1983) have analyzed substitution among capital, labor, energy, and materials inputs for total manufacturing in Norway. They assume that technical change is neutral and utilize a generalized Leontief price function. They have disaggregated the results to the level of nineteen industry groups. These groups do not include the whole of the Norwegian economy; eight additional industries are included in a complete multisectoral model of production for Norway. Dargay (1983) has constructed econometric models of substitution among capital, labor, energy, and materials inputs based on translog functional forms for total manufacturing in Sweden. She assumes that technical change is neutral, but permits nonconstant returns to scale. She has disaggregated the results to the level of twelve industry groups within Swedish manufacturing.

Although the breakdown of inputs among capital, labor, energy, and materials has come to predominate in econometric models of production at the industry level Humphrey and Wolkowitz (1976) have grouped energy and materials inputs into a single aggregate input in a study of substitution among inputs in several U.S. manufacturing industries that utilizes translog price functions. Friedlaender and Spady (1980) have disaggregated transportation services between trucking and rail service and have grouped other inputs into capital, labor and materials inputs. Their study is based on cross-section data for ninety-six three-digit industries in the United States for 1972 and employs a translog functional form with fixed inputs.

Parks (1971) has employed a breakdown of intermediate inputs among agricultural materials, imported materials and commercial services, and transportation services in a study of Swedish manufacturing based on the generalized Leontief functional form. Denny and May (1978) have disaggregated labor input between white collar and blue collar labor, capital input between equipment and structures, and have grouped all other inputs into a single aggregate input for Canadian total manufacturing, using a translog functional form. Frenger (1978) has analyzed substitution among capital, labor, and materials inputs for three industries in Norway, breaking down intermediate inputs in a different way for each industry, and utilizing a generalized Leontief functional form.

Griffin (1977a,b,c, 1978) has estimated econometric models of substitution among inputs for individual industries based on translog functional forms. For this purpose he has employed data generated by process models of the U.S. electric power generation, petroleum refining, and petrochemical industries constructed by Thompson *et al.* (1977). Griffin (1979) and Kopp and Smith (1980a,b, 1981a,b) have analyzed the effects of alternative aggregations of intermediate inputs on measures of substitution among inputs in the steel industry. For this purpose they have utilized data generated from a process analysis model of the U.S. steel industry constructed by Russell and Vaughan (1976).[40]

Although we have concentrated attention on substitution among capital, labor, energy, and materials inputs, there exists a sizable literature on substitution among capital, labor, and energy inputs alone. In this literature the price function is assumed to be homothetically separable in the prices of these inputs. This requires that all possible pairs of the inputs—capital and labor, capital and energy, and labor and energy—are separable from materials inputs. As we have observed above, only capital-energy separability is consistent with the results of Berndt and Wood (1975) for U.S. total manufacturing.

Appelbaum (1979b) has analyzed substitution among capital, labor, and energy inputs in the petroleum and natural gas industry of the United States, based on the data of Berndt and Jorgenson. Field and Grebenstein (1980) have analyzed substitution among physical capital, working capital, labor, and energy for ten two-digit U.S. manufacturing industries on the basis of translog price functions, using cross-section data for individual states for 1971.

Griffin and Gregory (1976) have modeled substitution among capital, labor, and energy inputs for total manufacturing in nine major industrialized countries—Belgium, Denmark, France, Italy, the Netherlands, Norway, the U.K., the U.S., and West Germany—using a translog price function. They pool four cross sections for these countries for the years 1955, 1960, 1965, and 1969, allowing for differences in technology among countries by means of one-zero dummy variables. Their results differ substantially from those of Berndt and Jorgenson and Berndt and Wood. These differences have led to an extensive discussion among Berndt and Wood (1979, 1981), Griffin (1981a,b), and Kang and Brown (1981), attempting to reconcile the alternative approaches.

Substitution among capital, labor, and energy inputs requires a price function that is homothetically separable in the prices of these inputs. An alternative specification is that the price function is homothetically separable in the prices of capital, labor, and natural resource inputs. This specification has been utilized by Humphrey, Burras, and Moroney (1975), Moroney and Toevs (1977, 1979) and Moroney and Trapani (1981a,b) in studies of substitution among these inputs for individual manufacturing industries in the United States based on translog price functions.

A third alternative specification is that the price function is separable in the prices of capital and labor inputs. Berndt and Christensen (1973b, 1974) have used translog price functions employing this specification in studies of substitution among individual types of capital and labor inputs for U.S. total manufacturing. Berndt and Christensen (1973b) have divided capital input between structures and equipment inputs and have tested the separability of the two types of capital input from labor input. Berndt and Christensen (1974) have divided labor input between blue collar and white collar inputs and have tested the separability of the two types of labor input from capital input. Hamermesh and Grant (1979) have surveyed the literature on econometric modeling of substitution among different types of labor input.

Woodland (1975) has analyzed substitution among structures, equipment and labor inputs for Canadian manufacturing, using generalized Leontief price functions. Woodland (1978) has presented an alternative approach to testing separability and has applied it in modeling substitution among two types of capital input and two types of labor input for U.S. total manufacturing, using the translog parametric form. Field and Berndt (1981) and Berndt and Wood (1979, 1981) have surveyed econometric models of substitution among inputs. They focus on substitution among capital, labor, energy and materials inputs at the level of individual industries.

1.4.2 Technical Change

The Jorgenson-Fraumeni (1983) model is based on a production function characterized by constant returns to scale for each of thirty-five industrial sectors of the U.S. economy. Output is a function of inputs of primary factors of production—capital and labor services—inputs of energy and materials, and time as an index of the level of technology. While the rate of technical change is endogenous in this econometric model, the model must be carefully distinguished from models of induced technical change, such as those analyzed by Hicks (1932), Kennedy (1964), Samuelson (1965b), von Weizsäcker (1962), and many others. In those models the biases of technical change are endogenous and depend on relative prices. As Samuelson (1965b) has pointed out, models of induced technical change require intertemporal optimization since technical change at any point of time affects future production possibilities.[41]

In the Jorgenson-Fraumeni model of producer behavior myopic decision rules can be derived by treating the price of capital input as a rental price of capital service.[42] The rate of technical change at any point of time is a function of relative prices, but does not affect future production possibilities. This greatly simplifies the modeling of producer behavior and facilitates the implementation of the econometric model. Given myopic decision rules for producers in each industrial sector, all of the implications of the economic theory of production can be described in terms of the properties of the sectoral price functions given in section 1.2.1.[43]

The Jorgenson-Fraumeni model of producer behavior consists of a system of equations giving the shares of capital, labor, energy, and materials inputs in the value of output and the rate of technical change as functions of relative prices and time. To formulate an econometric model a stochastic component is added to these equations. Since the rate of technical change is not directly observable, we consider a form of the model with autocorrelated disturbances; the data are transformed to eliminate the autocorrelation. The prices are treated as endogenous variables and the unknown parameters are estimated by the method of nonlinear three-stage least squares presented in section 1.3.3.

The endogenous variables in the Jorgenson-Fraumeni model include value shares of sectoral inputs for four commodity groups and the sectoral rate of technical change. Four equations can be estimated for each industry, corresponding to three of the value shares and the rate of technical change. As unknown parameters there are three elements of the vector $\{\alpha_p\}$, the scalar $\{\alpha_t\}$, six share elasticities in the matrix $\{B_{pp}\}$, which is constrained to be symmetric, three biases of technical change in the vector $\{\beta_{pt}\}$, and the scalar $\{\beta_{tt}\}$, so that there is a total of fourteen unknown parameters for each industry. Jorgenson and Fraumeni estimate these parameters from time-series data for the period 1958–1974 for each industry, subject to the inequality restrictions implied by monotonicity of the sectoral input value shares.[44]

The estimated share elasticities with respect to price $\{B_{pp}\}$ describe the implications of patterns of substitution for the distribution of the value of output among capital, labor, energy, and materials inputs. Positive share elasticities imply that the corresponding value shares increase with an increase in price; negative share elasticities imply that the value shares decrease with price; zero share elasticities correspond to value shares that are independent of price. The concavity constraints on the sectoral price functions contribute substantially to the precision of the estimates, but require that the share of each input be nonincreasing in the price of the input itself.

The empirical findings on patterns of substitution reveal some striking similarities among industries.[45] The elasticities of the shares of capital with respect to the price of labor are nonnegative for thirty-three of the thirty-five industries, so that the shares of capital are nondecreasing in the price of labor for these thirty-three sectors. Similarly, elasticities of the share of capital with respect to the price of energy are nonnegative for thirty-four industries and elasticities with respect to the price of materials are nonnegative for all thirty-five industries. The share elasticities of labor with respect to the prices of energy and materials are nonnegative for nineteen and for all thirty-five industries, respectively. Finally, the share elasticities of energy with respect to the price of materials are nonnegative for thirty of the thirty-five industries.

We continue the interpretation of the empirical results with estimated biases of technical change with respect to price $\{\beta_{pt}\}$. These parameters can be interpreted as changes in the sectoral value shares (1.2.14) with respect to time, holding prices constant. This component of change in the value shares can be attributed to changes in technology rather than to substitution among inputs. For example, if the bias of technical change with respect to the price of capital input is positive, we say that technical change is capital-using; if the bias is negative, we say that technical change is capital-saving.

Considering the rate of technical change (1.2.14). The biases of technical change $\{\beta_{pt}\}$ can be interpreted in an alternative and equivalent way. These parameters are changes in the negative of the rate of technical change with respect to changes in prices. As substitution among inputs takes place in response to price changes, the rate of technical change is altered. For example, if the bias of technical change with respect to capital input is positive, an increase in the price of capital input decreases the rate of technical change; if the bias is negative, an increase in the price of capital input increases the rate of technical change.

A classification of industries by patterns of the biases of technical change is given in table 1.1. The pattern that occurs with greatest frequency is capital-using, labor-using, energy-using, and materials-saving technical change. This pattern occurs for nineteen of the thirty-five industries for which biases are fitted. Technical change is capital-using for twenty-five of the thirty-five industries, labor-using for thirty-one industries, energy-using for twenty-nine industries, and materials-using for only two industries.

The patterns of biases of technical change given in table 1.1 have important implications for the relationship between relative prices and the rate of economic growth. An increase in the price of materials increases the rate of technical change in thirty-three of the thirty-five industries. By contrast, increases in the prices of capital, labor, and energy reduced the rates of technical change in twenty-five, thirty-one, and twenty-nine industries, respectively. The substantial in-

Table 1.1
Classification of industries by biases of technical change

Pattern of biases	Industries
Capital using Labor using Energy using Material saving	Agriculture, metal mining, crude petroleum and natural gas, nonmetallic mining, textiles, apparel, lumber, furniture, printing, leather, fabricated metals, electrical machinery, motor vehicles, instruments, miscellaneous manufacturing, transportation, trade, finance, insurance and real estate, services
Capital using Labor using Energy saving Material saving	Coal mining, tobacco manufacturers, communications, government enterprises
Capital using Labor saving Energy using Material saving	Petroleum refining
Capital using Labor saving Energy saving Material using	Construction
Capital saving Labor saving Energy using Material saving	Electric utilities
Capital saving Labor using Energy saving Material saving	Primary metals
Capital saving Labor using Energy using Material saving	Paper, chemicals, rubber, stone, clay and glass, machinery except electrical, transportation equipment and ordnance, gas utilities
Capital saving Labor saving Energy using Material using	Food

Source: Jorgenson and Fraumeni (1983b), p. 264.

creases in energy prices since 1973 have had the effect of reducing sectoral rates of technical change, slowing the aggregate rate of technical change, and diminishing the rate of growth for the U.S. economy as a whole.[46]

While the empirical results suggest a considerable degree of similarity across industries, it is necessary to emphasize that the Jorgenson-Fraumeni model of producer behavior requires important simplifying assumptions. First, conditions for producer equilibrium under perfect competition are employed for all industries. Second, constant returns to scale at the industry level are assumed. Finally, a description of technology that leads to myopic decision rules is employed. These assumptions must be justified primarily by their usefulness in implementing production models that are uniform for all thirty-five industrial sectors of the U.S. economy.

Binswanger (1974a,c, 1978c) has analyzed substitution and technical change for U.S. agriculture, using cross sections of data for individual states for 1949, 1954, 1959, and 1964. Binswanger was the first to estimate biases of technical change based on the translog price function. He permits technology to differ among time periods and among groups of states within the United States. He divides capital inputs between land and machinery and divides intermediate inputs between fertilizer and other purchased inputs. He considers substitution among these four inputs and labor input.

Binswanger employs time-series data on U.S. agriculture as a whole for the period 1912–1964 to estimate biases of technical change on an annual basis. Brown and Christensen (1981) have analyzed time-series data on U.S. agriculture for the period 1947–1974. They divide labor services between hired labor and self-employed labor and capital input between land and all other—machinery, structures, and inventories. Other purchased inputs are treated as a single aggregate. They model substitution and technical change with fixed inputs, using a translog functional form.

Berndt and Khaled (1979) have augmented the Berndt-Wood data set for U.S. manufacturing to include data on output. They estimate biases of technical change and permit nonconstant returns to scale. They employ a Box-Cox transformation of data on input prices, generating a functional form that includes the translog, generalized Leontief, and quadratic as special cases. The Box-Cox transformation is also employed by Appelbaum (1979a) and by Caves, Christensen, and

Trethaway (1980). Denny (1974) has proposed a closely related approach to parameterization based on mean value functions.

Kopp and Diewert (1982) have employed a translog parametric form to study technical and allocative efficiency. For this purpose they have analyzed data on U.S. total manufacturing for the period 1947–1971 compiled by Berndt and Wood (1975) and augmented by Berndt and Khaled (1979). Technical change is not required to be neutral and nonconstant returns to scale are permitted. They have interpreted the resulting model of producer behavior as a representation of average practice. They have then rescaled the parameters to obtain a "frontier" representing best practice and have employed the results to obtain measures of technical and allocative efficiency for each year in the sample.[47]

Wills (1979) has modeled substitution and technical change for the U.S. steel industry, using a translog price function. Norsworthy and Harper (1981) have extended and augmented the Berndt-Wood data set for total manufacturing and have modeled substitution and technical change, using a translog price function. Woodward (1983) has reanalyzed these data and has derived estimates of rates of factor augmentation for capital, labor, energy, and materials inputs, using a translog price function.

Jorgenson (1984b) has modeled substitution and technical change for thirty-five industries of the United States for the period 1958–1979, dividing energy inputs between electricity and nonelectrical energy inputs. He employs translog price functions with capital, labor, two kinds of energy, and materials inputs and finds that technical change is electricity-using and nonelectrical energy-using for most U.S. industries. Nakamura (1984) has developed a similar model for twelve sectors covering the whole of the economy for the Federal Republic of Germany for the period 1960–1974. He has disaggregated intermediate inputs among energy, materials, and services.

We have already discussed the work of Kopp and Smith on substitution among inputs, based on data generated by process models of the U.S. steel industry. Kopp and Smith (1981c, 1982) have also analyzed the performance of different measures of technical change, also using data generated by these models. They show that measures of biased technical change based on the methodology developed by Binswanger can be explained by the proportion of investment in specific technologies.

Econometric models of substitution among inputs at the level of individual industries have incorporated intermediate inputs—broken down between energy and materials inputs—along with capital and labor inputs. However, models of substitution and technical change have also been constructed at the level of the economy as a whole. Output can be divided between consumption and investment goods, as in the original study of the translog price function by Christensen, Jorgenson, and Lau (1971, 1973), and input can be divided between capital and labor services.

Hall (1973) has considered nonjointness of production of investment and consumption goods outputs for the United States. Kohli (1981, 1983) has also studied nonjointness in production for the United States. Burgess (1974) has added imports as an input to inputs of capital and labor services. Denny and Pinto (1978) developed a model with this same breakdown of inputs for Canada. Conrad and Jorgenson (1977, 1978a) have considered nonjointness of production and alternative models of technical change for the Federal Republic of Germany.

1.4.3 Two-Stage Allocation

Aggregation over inputs has proved to be a very important means for simplifying the description of technology in modeling producer behavior. The price of output can be represented as a function of a smaller number of input prices by introducing price indexes for input aggregates. These price indexes can be used to generate a second stage of the model by treating the price of each aggregate as a function of the prices of the inputs making up the aggregate. We can parameterize each stage of the model separately.

The Berndt-Jorgenson (1973) model of producer behavior is based on two-stage allocation of the value of output of each sector. In the first stage the value of sectoral output is allocated among capital, labor, energy, and materials inputs, where materials include inputs of nonenergy commodities and competitive imports. In the second stage the value of energy expenditure is allocated among expenditures on individual types of energy and the value of materials expenditure is allocated among expenditures on competitive imports and nonenergy commodities.

The first stage of the econometric model is generated from a price function for each sector. The price of sectoral output is a function of the prices of capital and labor inputs and the prices of inputs of energy and materials. The second stage of the model is generated from price indexes for energy and materials inputs. The price of energy is a function of the prices of five types of energy inputs, while the price of materials is a function of the prices of four types of nonenergy inputs and the price of competitive imports.

The Berndt-Jorgenson model of producer behavior consists of three systems of equations. The first system gives the shares of capital, labor, energy and materials inputs in the value of output, the second system gives the shares of energy inputs in the value of energy input, and the third system gives the shares of nonenergy inputs and competitive imports in the value of materials inputs. To formulate an econometric model stochastic components are added to these systems of equations. The rate of technical change is taken to be exogenous; all prices—including the prices of energy and materials inputs for each sector—are treated as endogenous variables. Estimates of the unknown parameters of all three systems of equations are based on the nonlinear three-stage least-squares estimator.

The Berndt-Jorgenson model illustrates the use of two stage allocation to simplify the description of producer behavior. By imposing the assumption that the price of aggregate input is separable in the prices of individual energy and materials inputs, the price function that generates the first stage of the model can be expressed in terms of four input prices rather than twelve. However, simplifications of the first stage of the model requires the introduction of a second stage, consisting of price functions for energy and materials inputs. Each of these price functions can be expressed in terms of five prices of individual inputs.

Fuss (1977a) has constructed a two-stage model of Canadian total manufacturing using translog functional forms. He treats substitution among coal, liquid petroleum gas, fuel oil, natural gas, electricity, and gasoline as a second stage of the model. Friede (1979) has developed two-stage models based on translog price functions for fourteen industries of the Federal Republic of Germany. In these models the second stage consists of three separate models—one for substitution among individual types of energy and two for substitution among individual types of nonenergy inputs. Dargay (1983) has constructed a two-stage model of twelve Swedish manufacturing industries utilizing a translog functional form. She has analyzed substitution among electricity, oil, and solid fuels inputs at the second stage of the model.

Nakamura (1984) has constructed three-stage models for twelve industries of the Federal Republic of Germany, using translog price functions. The first stage encompasses substitution and technical change among capital, labor, energy, materials, and services inputs. The second stage consists of three models—a model for substitution among individual types of energy, a model for substitution among individual types of materials, and a model for substitution among individual types of services. The third stage consists of models for substitution between domestically produced input and the corresponding imported input of each type.

Pindyck (1979a,b) has constructed a two-stage model of total manufacturing for ten industrialized countries—Canada, France, Italy, Japan, the Netherlands, Norway, Sweden, the U.K., the U.S., and West Germany—using a translog price function. He employs annual data for the period 1959–1973 in estimating a model for substitution among four energy inputs—coal, oil, natural gas, and electricity. He uses annual data for the period 1963–1973 in estimating a model for substitution among capital, labor, and energy inputs. Magnus (1979) and Magnus and Woodland (1980) have constructed a two-stage model for total manufacturing in the Netherlands along the same lines. Similarly, Ehud and Melnik (1981) have developed a two-stage model for the Israeli economy.

Halvorsen (1977) and Halvorsen and Ford (1979) have constructed a two-stage model for substitution among capital, labor, and energy inputs for nineteen two-digit U.S. manufacturing industries on the basis of translog price functions. For this purpose they employ cross-section data for individual states in 1971. The second stage of the model provides a disaggregation of energy input among inputs of coal, oil, natural gas, and electricity. Halvorsen (1978) has analyzed substitution among different types of energy on the basis of cross-section data for 1958, 1962, and 1971.

1.5 Cost Functions

In section 1.2, we have considered producer behavior under constant returns to scale. The production function (1.2.1) is homogeneous of degree one, so that a proportional change in all inputs results in a change in output in the same proportion. Necessary conditions for producer equilibrium (1.2.2) are that the value share of each input is equal to the elasticity of output with respect to that input. Under constant returns to scale the value shares and the elasticities sum to unity.

In this section, we consider producer behavior under increasing returns to scale. Under increasing returns and competitive markets for output and all inputs, producer equilibrium is not defined by profit maximization, since no maximum of profit exists. However, in regulated industries the price of output is set by regulatory authority. Given demand for output as a function of the regulated price, the level of output is exogenous to the producing unit.

With output fixed from the point of view of the producer, necessary conditions for equilibrium can be derived from cost minimization. Where total cost is defined as the sum of expenditures on all inputs, the minimum value of cost can be expressed as a function of the level of output and the prices of all inputs. We refer to this function as the cost function. We have described the theory of production under constant returns to scale in terms of properties of the price function (1.2.4); similarly, we can describe the theory under increasing returns in terms of properties of the cost function.

1.5.1 Duality

Utilizing the notation of section 1.2, we can define total cost, say c, as the sum of expenditures on all inputs

$$c = \sum_{j=l}^{J} p_j \, x_j \, .$$

We next define the shares of inputs in total cost by

$$v_j = \frac{p_j \, x_j}{c} \, , \quad (j = 1, 2, \ldots, J) \, .$$

With output fixed from the point of view of the producing unit and competitive markets for all inputs, the necessary conditions for producer equilibrium are given by equalities between the shares of each input in total cost and the ratio of the elasticity of output with respect to that input and the sum of all such elasticities

$$v = \frac{\dfrac{\partial \ln y}{\partial \ln x}}{i' \dfrac{\partial \ln y}{\partial \ln x}} \, , \tag{1.5.1}$$

where i is a vector of ones and

$v = (v_1, v_2, \ldots, v_J)$—vector of cost shares.

Given the definition of total cost and the necessary conditions for producer equilibrium, we can express total cost, say c, as a function of the prices of all inputs and the level of output

$$c = C(p, y). \tag{1.5.2}$$

We refer to this as the *cost function*. The cost function C is dual to the production function F and provides an alternative and equivalent description of the technology of the producing unit.[48]

We can formalize the theory of production in terms of the following properties of the cost function:

1. *Positivity*. The cost function is positive for positive input prices and a positive level of output.
2. *Homogeneity*. The cost function is homogeneous of degree one in the input prices.
3. *Monotonicity*. The cost function is increasing in the input prices and in the level of output.
4. *Concavity*. The cost function is concave in the input prices.

Given differentiability of the cost function, we can express the cost shares of all inputs as elasticities of the cost function with respect to the input prices

$$v = \frac{\partial \ln c}{\partial \ln p}\,(p, y). \tag{1.5.3}$$

Further, we can define an index of returns to scale as the elasticity of the cost function with respect to the level of output

$$v_y = \frac{\partial \ln c}{\partial \ln y}\,(p, y)\,. \tag{1.5.4}$$

Following Frisch (1965), we can refer to this elasticity as the *cost flexibility*.

The cost flexibility v_y is the reciprocal of the *degree of returns to scale*, defined as the elasticity of output with respect to a proportional increase in all inputs

$$v_y = \frac{1}{i'\dfrac{\partial \ln y}{\partial \ln x}}\,. \tag{1.5.5}$$

If output increases more than in proportion to the increase in inputs, cost increases less than in proportion to the increase in output.

Since the cost function C is homogeneous of degree one in the input prices, the cost shares and the cost flexibility are homogeneous of degree zero and the cost shares sum to unity

$$i'v = i' \frac{\partial \ln c}{\partial \ln p} = 1 .$$

Since the cost function is increasing in the input prices, the cost shares must be nonnegative and not all zero

$$v \geq 0 .$$

The cost function is also increasing in the level of output, so that the cost flexibility is positive

$$v_y > 0 .$$

1.5.2 Substitution and Economies of Scale

We have represented the cost shares of all inputs and the cost flexibility as functions of the input prices and the level of output. We can characterize these functions in terms of measures of substitution and economies of scale. We obtain *share elasticities* by differentiating the logarithm of the cost function twice with respect to the logarithms of input prices

$$U_{pp} = \frac{\partial^2 \ln c}{\partial \ln p^2} (p, y) = \frac{\partial v}{\partial \ln p} (p, y) . \tag{1.5.6}$$

These measures of substitution give the response of the cost shares of all inputs to proportional changes in the input prices.

Second, we can differentiate the logarithm of the cost function twice with respect to the logarithms of the input prices and the level of output to obtain measures of economies of scale

$$u_{py} = \frac{\partial^2 \ln c}{\partial \ln p \, \partial \ln y} (p, y) = \frac{\partial v}{\partial \ln y} (p, y) = \frac{\partial v_y}{\partial \ln p} (p, y) . \tag{1.5.7}$$

We refer to these measures as *biases of scale*. The vector of biases of scale u_{py} can be employed to derive the implications of economies of scale for the relative distribution of total cost among inputs. If a scale bias is positive, the cost share of the corresponding input increases with a change in the level of output. If a scale bias is negative, the cost share decreases with a change in output. Finally, if a scale bias is zero, the cost share is independent of output.

Alternatively, the vector of biases of scale u_{py} can be employed to derive the implications of changes in input prices for the cost flexibility. If the scale bias is positive, the cost flexibility increases with the input price. If the scale bias is negative, the cost flexibility decreases with the input price. Finally, if the bias is zero, the cost flexibility is independent of the input price.

To complete the description of economies of scale we can differentiate the logarithm of the cost function twice with respect to the level of output

$$u_{yy} = \frac{\partial^2 \ln c}{\partial \ln y^2} (p, y) = \frac{\partial v_y}{\partial \ln y} (p, y) \, . \tag{1.5.8}$$

If this measure is positive, zero, or negative, the cost flexibility is increasing, decreasing, or independent of the level of output.

The matrix of second-order logarithmic derivatives of the logarithms of the cost function C must be symmetric. This matrix includes the matrix of share elasticities U_{pp}, the vector of biases of scale u_{py}, and the derivative of the cost flexibility with respect to the logarithm of output u_{yy}. Concavity of the cost function in the input prices implies that the matrix of second-order derivatives, say H, is nonpositive definite, so that the matrix $U_{pp} + vv' - V$ is nonpositive definite, where

$$\frac{1}{c} \, N \cdot H \cdot N = U_{pp} + vv' - V \, .$$

Total cost c is positive and the diagonal matrices N and V are defined in terms of the input prices p and the cost shares v, as in section 1.2. Two inputs are *substitutes* if the corresponding element of the matrix $U_{pp} + vv' - V$ is negative, *complements* if the element is negative, and *independent* if the element is zero.

In section 1.2.2 we have introduced price and quantity indexes of aggregate input implied by homothetic separability of the price function. We can analyze the implications of homothetic separability of the cost function by introducing price and quantity indexes of aggregate input and defining the cost share of aggregate input in terms of these indexes. An aggregate input can be treated in precisely the same way as any other input, so that price and quantity indexes can be used to reduce the dimensionality of the space of input prices and quantities.

We say that the cost function C is *homothetic* if and only if the cost function is separable in the prices of all J inputs $\{p_1, p_2, \ldots, p_J\}$, so that

$$c = C[P(p_1, p_2, \ldots, p_J), y] , \tag{1.5.9}$$

where the function P is homogeneous of degree one and independent of the level of output y. The cost function is homothetic if and only if the production function is *homothetic,* where

$$y = F[G(x_1, x_2, \ldots, x_J)] , \tag{1.5.10}$$

where the function G is homogeneous of degree one.[49]

Since the cost function is homogeneous of degree one in the input prices, it is homogeneous of degree one in the function P, which can be interpreted as the price index for a single aggregate input; the function G is the corresponding quantity index. Furthermore, the cost function can be represented as the product of the price index of aggregate input P and a function, say H, of the level of output

$$c = P(p_1, p_2, \ldots, p_J) \cdot H(y) . \tag{1.5.11}$$

Under homotheticity, the cost flexibility v_y is independent of the input prices

$$v_y = \frac{\partial \ln H}{\partial \ln y} (y) . \tag{1.5.12}$$

If the cost flexibility is also independent of the level of output, the cost function is homogeneous in the level of output and the production function is homogeneous in the quantity index of aggregate input G. The degree of homogeneity of the production function is the degree of returns to scale and is equal to the reciprocal of the cost flexibility. Under constant returns to scale the degree of returns to scale and the cost flexibility are equal to unity.

1.5.3 Parameterization and Integrability

In section 1.2.3 we have generated an econometric model of producer behavior by treating the measures of substitution and technical change as unknown parameters to be estimated. In this section we generate an econometric model of cost and production by introducing the parameters

$$B_{pp} = U_{pp}\ , \qquad \beta_{py} = u_{py}\ , \qquad \beta_{yy} = u_{yy}\ , \tag{1.5.13}$$

where B_{pp} is a matrix of constant share elasticities, β_{py} is a vector of constant biases of scale, and β_{yy} is a constant derivative of the cost flexibility with respect to the logarithm of output.

We can regard the matrix of share elasticities, the vector of biases of scale, and the derivative of the cost flexibility with respect to the logarithm of output as a system of second-order partial differential equations. We can integrate this system to obtain a system of first-order partial differential equations

$$\begin{aligned} v &= \alpha_p + B_{pp} \ln p + \beta_{py} \ln y\ , \\ v_y &= \alpha_y + \beta'_{py} \ln p + \beta_{yy} \ln y\ , \end{aligned} \tag{1.5.14}$$

where the parameters—α_p, α_y—are constants of integration. Choosing scales for measuring the quantities and prices of output and the inputs, we can consider values of input prices and level of output equal to unity. At these values the vector of parameters α_p is equal to the vector of cost shares and the parameters α_y is equal to the cost flexibility.

We can integrate the system of first-order partial differential equations (1.5.14) to obtain the cost function

$$\begin{aligned} \ln c = \alpha_0 &+ \alpha_p \ln p + \alpha_y \ln y + \frac{1}{2} \ln p' B_{pp} \ln p \\ &+ \ln p' \beta_{py} \ln y + \frac{1}{2} \beta_{yy} (\ln y)^2\ , \end{aligned} \tag{1.5.15}$$

where the parameter α_0 is a constant of integration. This parameter is equal to the logarithm of total cost where the input prices and level of output are equal to unity. We can refer to this form as the translog cost function, indicating the role of the variables, or the constant share elasticity (CSE) cost function, indicating the role of the parameters.

To incorporate the implications of the economic theory of production we consider restrictions on the system of equations (1.5.14) required to obtain a cost function with properties listed above. A complete set of conditions for integrability is the following:

1.5.3.1 Homogeneity

The cost shares and the cost flexibility are homogeneous of degree zero in the input prices.

Homogeneity of degree zero of the cost shares and the cost flexibility implies that the parameters—B_{pp} and β_{py}—must satisfy the restrictions

$$B_{pp}i = 0 \tag{1.5.16}$$
$$\beta'_{py}i = 0 \,,$$

where i is a vector of ones. For J inputs there are $J+1$ restrictions implied by homogeneity.

1.5.3.2 Cost Exhaustion

The sum of the cost shares is equal to unity.

Cost exhaustion implies that the value of the J inputs is equal to total cost. Cost exhaustion implies that the parameters—α_p, B_{pp}, β_{py}—must satisfy the restrictions

$$\alpha'_p \; i = 1 \,,$$
$$B'_{pp}i = 0 \,,$$
$$\beta'_{py}i = 0 \,. \tag{1.5.17}$$

For J inputs there are $J+2$ restrictions implied by cost exhaustion.

1.5.3.3 Symmetry

The matrix of share elasticities, biases of scale, and the derivative of the cost flexibility with respect to the logarithm output must be symmetric.

A necessary and sufficient condition for symmetry is that the matrix of parameters must satisfy the restrictions

$$\begin{bmatrix} B_{pp} & \beta_{py} \\ \beta'_{py} & \beta_{yy} \end{bmatrix} = \begin{bmatrix} B_{pp} & \beta_{py} \\ \beta'_{py} & \beta_{yy} \end{bmatrix}' \,. \tag{1.5.18}$$

For J inputs the total number of symmetry restrictions is 1/2 $J(J+1)$.

1.5.3.4 *Nonnegativity*

The cost shares and the cost flexibility must be nonnegative.

Since the translog cost function is quadratic in the logarithms of the input prices and the level of output, we cannot impose restrictions on the parameters that imply nonnegativity of the cost shares and the cost flexibility. Instead, we consider restrictions on the parameters that imply monotonicity of the cost shares wherever they are nonnegative.

1.5.3.5 *Monotonicity*

The matrix of share elasticities $B_{pp} + vv' - V$ is nonpositive definite.

The conditions on the parameters assuring concavity of the cost function wherever the cost shares are nonnegative are precisely analogous to the conditions given in section 1.2.4 for concavity of the price function wherever the value shares are nonnegative. These conditions can be expressed in terms of the Cholesky factorization of the matrix of constant share elasticities B_{pp}.

1.5.4 Stochastic Specification

To formulate an econometric model of cost and production we add a stochastic component to the equations for the cost shares and the cost function itself. To represent the econometric model we require some additional notation. Where there are K producing units we index the observations by producing unit $(k = 1, 2, \ldots, K)$. The vector of cost shares for the k-th unit is denoted v_k and total cost of the unit is c_k $(k = 1, 2, \ldots, K)$. The vector of input prices faced by the k-th unit is denoted p_k and the vector of logarithms of input prices is $\ln\, p_k$ $(k = 1, 2, \ldots, K)$. Finally, the level of output of the i-th unit is denoted y_k $(k = 1, 2, \ldots, K)$.

We obtain an econometric model of cost and production corresponding to the translog cost function by adding random disturbances to the equations for the cost shares and the cost function

$$\begin{aligned} v_k &= \alpha_p + B_{pp} \ln p_k + \beta_{py} \ln y + \varepsilon_k \,, \\ \ln c_k &= \alpha_0 + \alpha_p \ln p_k + \alpha_y \ln y_k + \frac{1}{2} \ln p_k \; B_{pp} \ln p_k \\ &\qquad + \ln p_k \; \beta_{py} \ln y_k + \frac{1}{2} (\ln y_k)^2 + \varepsilon_k^c \,, \quad (k = 1, 2, \ldots, K) \,, \end{aligned} \tag{1.5.19}$$

where ε_k is the vector of unobservable random disturbances for the cost shares of the k-th producing unity and ε_k^c is the corresponding disturbance for the cost function $(k = 1, 2, \ldots, K)$. Since the cost shares for all inputs sum to unity for each producing unit, the random disturbances corresponding to the J cost shares sum to zero for each unit

$$i' \varepsilon_k = 0 \quad (k = 1, 2, \ldots, K) , \tag{1.5.20}$$

so that these disturbances are not distributed independently.

We assume that the unobservable random disturbances for all $J+1$ equations have expected value equal to zero for all observations

$$E\begin{bmatrix} \varepsilon_k \\ \varepsilon_k^c \end{bmatrix} = 0 , \quad (k = 1, 2, \ldots, K) . \tag{1.5.21}$$

We also assume that the disturbances have a covariance matrix that is the same for all producing units and has rank J, where

$$V\begin{bmatrix} \varepsilon_k \\ \varepsilon_k^c \end{bmatrix} = \Sigma , \quad (k = 1, 2, \ldots, K).$$

Finally, we assume that random disturbances corresponding to distinct observations are uncorrelated, so that the covariance matrix of random disturbances for all observations has the Kronecker product form

$$V\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{12} \\ \vdots \\ \varepsilon_{1K} \\ \varepsilon_{21} \\ \vdots \\ \varepsilon_K^c \end{bmatrix} = \Sigma \otimes I . \tag{1.5.22}$$

We can test the validity of restrictions on economies of scale by expressing them in terms of the parameters of an econometric model of cost and production. Under homotheticity the cost flexibility is independent of the input prices. A necessary and sufficient condition for homotheticity is given by

$$\beta_{py} = 0 ; \tag{1.5.23}$$

the vector of biases of scale is equal to zero. Under homogeneity the cost flexibility is also independent of output, so that

$$\beta_{yy} = 0 \; ;$$

the derivative of the flexibility with respect to the logarithm of output is zero. Finally, under constant returns to scale, the cost flexibility is equal to unity; given the restrictions implied by homogeneity, constant returns requires

$$\alpha_y = 1 \; . \tag{1.5.24}$$

1.6 Applications of Cost Functions

To illustrate the econometric modeling of economies of scale in section 1.6.1, we present an econometric model that has been implemented for the electric power industry in the United States by Christensen and Greene (1976). This model is based on cost functions for cross sections of individual electric utilities in 1955 and 1970. Total cost of steam generation is a function of the level of output and the prices of capital, labor, and fuel inputs. Steam generation accounts for more than ninety percent of total power generation for each of the firms in the Christensen-Greene sample.

A key feature of the electric power industry in the United States is that individual firms are subject to price regulation. The regulatory authority sets the price for electric power. Electric utilities are required to supply the electric power that is demanded at the regulated price. This model must be carefully distinguished from the model of a regulated firm proposed by Averch and Johnson (1962).[50] In the Averch-Johnson model firms are subject to an upper limit on the rate of return rather than price regulation. Firms minimize costs under rate of return regulation only if the regulatory constraint is not binding.

The literature on econometric modeling of scale economies in U.S. transportation and communications industries parallels the literature on the U.S. electric power industry. Transportation and communications firms, like electric utilities, are subject to price regulation and are required to supply all the services that are demanded at the regulated price. However, the modeling of transportation and communications services is complicated by joint production of several outputs. We review econometric models with multiple outputs in section 1.6.2.

1.6.1 Economies of Scale

The Christensen-Greene model of the electric power industry consists of a system of equations giving the shares of all inputs in total cost and total cost itself as functions of relative prices and the level of output. To formulate an econometric model Christensen and Greene add a stochastic component to these equations. They treat the prices and levels of output as exogenous variables and estimate the unknown parameters by the method of maximum likelihood for nonlinear multivariate regression models.

The endogenous variables in the Christensen-Greene model are the cost shares of capital, labor, and fuel inputs and total cost. Christensen and Greene estimate three equations for each cross section, corresponding to two of the cost shares and the cost function. As unknown parameters they estimate two elements of the vector α_p, the two scalars—α_0 and α_y—three elements of the matrix of share elasticities B_{pp}, two biases of scale in the vector β_{py}, and the scalar β_{yy}. They estimate a total of ten unknown parameters for each of two cross sections of electric utilities for the years 1955 and 1970.[51] Estimates of the remaining parameters of the model are calculated by using the cost exhaustion, homogeneity, and symmetry restrictions. They report that the monotonicity and concavity restrictions are met at every observation in both cross-section data sets.

The hypothesis of constant returns to scale can be tested by first considering the hypothesis that the cost function is homothetic; under this hypothesis the cost flexibility is independent of the input prices. Given homotheticity the additional hypothesis that the cost function is homogeneous can be tested; under this hypothesis the cost flexibility is independent of output as well as prices. These hypotheses can be nested, so that the test of homogeneity is conditional on the test of homotheticity. Likelihood ratio statistics for these hypotheses are distributed, asymptotically, as chi-squared.

We present the results of Christensen and Greene for 1955 and 1970 in table 1.2. Test statistics for the hypotheses of homotheticity and homogeneity for both cross-section data sets and critical values for chi-squared are presented in table 1.2. Homotheticity can be rejected, so that both homotheticity and homogeneity are inconsistent with the evidence; homogeneity, given homotheticity, is also rejected. If all other parameters involving the level of output were set equal to zero, the parameter α_y would be the reciprocal of the degree of returns to

Table 1.2
Cost function for U.S. electric power industry (parameter estimates, 1955 and 1970; *t*-ratios in parentheses).[a]

Parameter	1955	1970
α_0	8.412	7.14
	(31.52)	(32.45)
α_y	0.386	0.587
	(6.22)	(20.87)
α_K	0.094	0.208
	(0.94)	(2.95)
α_L	0.348	–0.151
	(4.21)	(–1.85)
α_E	0.558	0.943
	(8.57)	(14.64)
β_{yy}	0.059	0.049
	(5.76)	(12.94)
β_{Ky}	–0.008	0.003
	(–1.79)	(–1.23)
β_{Ly}	–0.016	–0.018
	(–10.10)	(–8.25)
β_{Ey}	0.024	0.021
	(5.14)	(6.64)
β_{KK}	0.175	0.118
	(5.51)	(6.17)
β_{LL}	0.038	0.081
	(2.03)	(5.00)
β_{EE}	0.176	0.178
	(6.83)	(10.79)
β_{KL}	–0.018	–0.011
	(–1.01)	(–0.749)
β_{KE}	–0.159	–0.107
	(–6.05)	(–7.48)
β_{LE}	–0.020	–0.070
	(–2.08)	(–6.30)

Test statistics for restrictions on economies of scale[b]

Statistic	Homotheticity	Homogeneity
1955	78.22	102.27
1970	57.91	157.46
Critical value (1%)	9.21	11.35

[a] *Source*: Christensen and Greene (1976, table 4, p. 666).
[b] *Source*: Christensen and Greene (1976, table 5, p. 666).

scale. For both 1955 and 1970 data sets this parameter is significantly different from unity.

Christensen and Greene employ the fitted cost functions presented in table 1.2 to characterize scale economies for individual firms in each of the two cross sections. For both years the cost functions are U-shaped with a minimum point occurring at very large levels of output. In 1955, 118 of the 124 firms have significant economies of scale; only six firms have no significant economies or diseconomies of scale, but these firms produce 25.9 percent of the output of the sample. In 1970, ninety-seven of the 114 firms have significant economies of scale, sixteen have none, and one has significant scale diseconomies.

Econometric modeling of economies of scale in the U.S. electric power industry has generated a very extensive literature. The results through 1978 have been surveyed by Cowing and Smith (1978). More recently, the Christensen-Greene data base has been extended by Greene (1983) to incorporate cross sections of individual electric utilities for 1955, 1960, 1965, 1970, and 1975. By including both the logarithm of output and time as an index of technology in the translog total cost function (1.5.15), Greene is able to characterize economies of scale and technical change simultaneously.

Stevenson (1980) has employed a translog total cost function incorporating output and time to analyze cross sections of electric utilities for 1964 and 1972. Gollop and Roberts (1981) have used a similar approach to study annual data on eleven electric utilities in the United States for the period 1958–1975. They use the results to decompose the growth of total cost among economies of scale, technical change, and growth in input prices. Griffin (1977b) has modeled substitution among different types of fuel in steam electricity generation using four cross sections of twenty OECD countries. Halvorsen (1978) has analyzed substitution among different fuel types, using cross-section data for the United States in 1972.

Cowing, Reifschneider, and Stevenson (1983) have employed a translog total cost function similar to that of Christensen and Greene to analyze data for eighty-one electric utilities for the period 1964–1975. For this purpose they have grouped the data into four cross sections, each consisting of three-year totals for all firms. If disturbances in the equations for the cost shares (1.5.19) are associated with errors in optimization, costs must increase relative to the minimum level given by the cost function (1.5.15). Accordingly, Cowing, Reifschneider and Stevenson employ a disturbance for the cost function that is constrained to be positive.[52]

An alternative to the Christensen-Greene model for electric utilities has been developed by Fuss (1977b, 1978). In Fuss's model, the cost function is permitted to differ *ex ante*, before a plant is constructed, and *ex post*, after the plant is in place.[53] Fuss employs a generalized Leontief cost function with four input prices—structures, equipment, fuel, and labor. He models substitution among inputs and economies of scale for seventy-nine steam generation plants for the period 1948–1961.

We have observed that a model of the behavior of a regulated firm based on cost minimization must be carefully distinguished from the model originated by Averch and Johnson (1962). In addition to allowing a given rate of return, regulatory authorities may permit electric utilities to adjust the regulated price of output for changes in the cost of specific inputs. In the electric power industry a common form of adjustment is to permit utilities to change prices with changes in fuel costs.

Peterson (1975) has employed a translog cost function for the electric utility industry to test the Averch-Johnson hypothesis. For this purpose he introduces three measures of the effectiveness of regulation into the cost function: a one-zero dummy variable distinguishing between states with and without a regulatory commission, a similar variable differentiating between alternative methods for evaluation of public utility property for rate making purposes, and a variable representing differences between the rate of return allowed by the regulatory authority and the cost of capital. He analyzes annual observations on fifty-six steam generating plans for the period 1966 to 1968.

Cowing (1978) has employed a quadratic parametric form to test the Averch-Johnson hypothesis for regulated firms. He introduces both the cost of capital and the rate of return allowed by the regulatory authority as determinants of input demands. Cowing analyzes data on 114 steam generation plants constructed during each of three time periods—1947–1950, 1955–1959, and 1960–1965. Gollop and Karlson (1978) have employed a translog cost function that incorporates a measure of the effectiveness of regulatory adjustments for changes in fuel costs. This measure is the ratio of costs that may be recovered under the fuel cost adjustment mechanism to all fuel costs. Gollop and Karlson analyze data for cross sections of individual electric utilities for the years 1970, 1971, and 1972.

Atkinson and Halvorsen (1980) have employed a translog parametric form to test the effects of both rate of return regulation and fuel cost adjustment mechanisms. For this purpose they have analyzed cross-section data for electric utilities in 1973. Gollop and Roberts (1983) have studied the effectiveness of regulations on sulfur dioxide emissions in the electric utility industry. They employ a translog cost function that depends on a measure of regulatory effectiveness. This measure is based on the legally mandated reduction in emissions and on the enforcement of emission standards. Gollop and Roberts analyze cross sections of fifty-six electric utilities for each of the years 1973–1979 and employ the results to study the impact of environmental regulation on productivity growth.

1.6.2 Multiple Outputs

Brown, Caves, and Christensen (1979) have introduced a model for joint production of freight and transportation services in the railroad industry based on the translog cost function (1.5.15).[54] A cost flexibility (5.4) can be defined for each output. Scale biases and derivatives of the cost flexibilities with respect to each output can be taken to be constant parameters. The resulting cost function depends on logarithms of input prices and logarithms of the quantities of each output. Caves, Christensen, and Trethaway (1980) have extended this approach by introducing Box-Cox transformations of the quantities of the outputs in place of logarithmic transformations. This generalized translog cost function permits complete specialization in the production of a single output.

The generalized translog cost function has been applied to cross sections of Class I railroads in the United States for 1955, 1963, and 1974 by Caves, Christensen, and Swanson (1981). They consider five categories of inputs: labor, way and structures, equipment, fuel, and materials. For freight transportation services they take ton-miles and average length of freight haul as measures of output. Passenger services are measured by passenger-miles and average length of passenger trip. They employ the results to measure productivity growth in the U.S. railroad industry for the period 1951–1974. Caves, Christensen, and Swanson (1981) have employed data for cross sections of Class I railroads in the United States to fit a variable cost function, treating way and structures as a fixed input and combining equipment and materials into a single variable input. They have employed the results in measuring productivity growth for the period 1951–1974.

Friedlaender and Spady (1981) and Harmatuck (1979) have utilized a translog total cost function to analyze cross-section data on Class I railroads in the United States. Jara-Diaz and Winston (1981) have employed a quadratic cost function to analyze data on Class III railroads, with measures of output disaggregated to the level of individual point-to-point shipments. Bräutigan, Daugherty, and Turnquist (1982) have used a translog variable cost function to analyze monthly data for nine years for a single railroad. Speed of shipment and quality of service are included in the cost function as measures of the characteristics of output.

The U.S. trucking industry, like the U.S. railroad industry, is subject to price regulation. Spady and Friedlaender (1978) have employed a translog cost function to analyze data on a cross section of 168 trucking firms in 1972. They have disaggregated inputs into four categories—labor, fuel, capital, and purchased transportation. Freight transportation services are measured in ton-miles. To take into account the heterogeneity of freight transportation services, five additional characteristics of output are included in the cost function—average shipment size, average length of haul, percentage of less than truckload traffic, insurance costs, and average load per truck.

Friedlaender, Spady, and Chiang (1981) have employed the approach of Spady and Friedlaender (1978) to analyze cross sections of 154, 161, and 47 trucking firms in 1972. Inputs are disaggregated in the same four categories, while an additional characteristics of output is included, namely, terminal density, defined as ton-miles per terminal. Separate models are estimated for each of the three samples. Friedlaender and Spady (1981) have employed the results in analyzing the impact of changes in regulatory policy. Harmatuck (1981) has employed a translog cost function to analyze a cross secton of 100 trucking firms in 1977. He has included data on the number and size of truck load and less-than-truckload shipments and average length of haul as measures of output. He disaggregates input among five activities—line haul, pickup and delivery, billing and collecting, platform handling, and all other.

Finally, Chiang and Friedlaender (1984) have disaggregated the output of trucking firms into four categories—less than truckload hauls of under 250 miles, between 250–500 miles, and over 500 miles, and truck load traffic—all measured in ton miles. Inputs are disaggregated among five categories—labor, fuel, revenue equipment, "other" capital, and purchased transportation. Characteristics of output similar to those included in earlier studies by Chiang, Friedlaender, and Spady are incorporated into the cost function, together with measures of the network configuration of each firm. They have employed this model to analyze a cross section of 105 trucking firms for 1976.

The U.S. air transportation industry, like the U.S. railroad and trucking industries, is subject to price regulation. Caves, Christensen, and Trethaway (1983) have employed a translog cost function to analyze a panel data set for all U.S. trunk and local service airlines for the period 1970–1981. Winston (1985) has provided a survey of econometric models of producer behavior in the transportation industries, including railroads, trucking, and airlines.

In the United States the communications industries, like the transportation industries, are largely privately owned but subject to price regulation. Nadiri and Schankerman (1981) have employed a translog cost function to analyze time-series data for 1947–1976 on the U.S. Bell System. They include the operating telephone companies and Long Lines, but exclude the manufacturing activities of Western Electric and the research and development activities of Bell Laboratories. Output is an aggregate of four service categories; inputs of capital, labor, and materials are distinguished. A time trend is included in the cost function as an index of technology; the stock of research and development is included as a separate measure of the level of technology.

Christensen, Cummings, and Schoech (1983) have employed alternative specifications of the translog cost function to analyze time-series data for the U.S. Bell System for 1947–1977. They employ a distributed lag of research and development expenditures by the Bell System to represent the level of technology. As alternative representations they consider the proportion of telephones with access to direct distance dialing, the percentage of telephones connected to central offices with modern switching facilities, and a more comprehensive measure of research and development. They also consider specifications with capital input held fixed and with experienced labor and management held fixed. Evans and Heckman (1983, 1984) have provided an alternative analysis of the same data set. They have studied economies of scope in the joint production of telecommunications services.

Bell Canada is the largest telecommunications firm in Canada. Fuss and Waverman (1981) have employed a translog cost function to analyze time series data on Bell Canada for the period 1952–1975. Three outputs are distinguished: message toll service, other total service, and local and miscellaneous service. Capital, labor, and materials are treated as separate categories of input. The level of technology is represented by a time trend. Denny, Fuss, Everson, and Waverman (1981) have analyzed time series data for the period 1952–1976. The percentage of telephones with access to direct dialing and the percentage of telephones connected to central offices with modern switching facilities are incorporated into the cost function as measures of the level of technology. Kiss, Karabadjian, and Lefebvre (1983) have compared alternative specifications of output and the level of technology. Fuss (1983) has provided a survey of econometric modeling of telecommunications services.

1.7 Conclusion

The purpose of this concluding section is to suggest possible directions for future research on econometric modeling of producer behavior. We first discuss the application of econometric models of production in general equilibrium analysis. The primary focus of empirical research has been on the characterization of technology for individual producing units. Application of the results typically involves models for both demand and supply for each commodity. The ultimate objective of econometric modeling of production is to construct general equilibrium models encompassing demands and supplies for a wide range of products and factors of production.

A second direction for future research on producer behavior is to exploit statistical techniques appropriate for panel data. Panel data sets consist of observations on several producing units at many points of time. Empirical research on patterns of substitution and technical change has been based on time series observations on a single producing unit or on cross-section observations on different units at a given point of time. Research on economies of scale has been based primarily on cross-section observations.

Our exposition of econometric methods has emphasized areas of research where the methodology has crystallized. An important area for future research is the implementation of dynamic models of technology. These models are based on substitution possibilities among outputs and inputs at different points of time. A number of promising avenues for investigation have been suggested in the literature on the theory of production. We conclude the chapter with a brief review of possible approaches to the dynamic modeling of producer behavior.

1.7.1 General Equilibrium Modeling

At the outset of our discussion it is essential to recognize that the predominant tradition in general equilibrium modeling does not employ econometric methods. This tradition originated with the seminal work of Leontief (1951), beginning with the implementation of the static input-output model. Leontief (1953) gave a further impetus to the development of general equilibrium modeling by introducing a dynamic input-output model. Empirical work associated with input-output analysis is based on estimating the unknown parameters of a general equilibrium model from a single interindustry transactions table.

The usefulness of the "fixed coefficients" assumption that underlies input-output analysis is hardly subject to dispute. By linearizing technology it is possible to solve at one stroke the two fundamental problems that arise in the practical implementation of general equilibrium models. First, the resulting general equilibrium model can be solved as a system of linear equations with constant coefficients. Second, the unknown parameters describing technology can be estimated from a single data point.

The first successful implementation of a general equilibrium model without the fixed coefficients assumption of input-output analysis is due to Johansen (1974). Johansen retained the fixed coefficients assumption in modeling demands for intermediate goods, but employed linear logarithmic or Cobb-Douglas production functions in modeling the substitution between capital and labor services and technical change. Linear logarithmic production functions imply that relative shares of inputs in the value of output are fixed, so that the unknown parameters characterizing substitution between capital and labor inputs can be estimated from a single data point.

In modeling producer behavior Johansen employed econometric methods only in estimating constant rates of technical change. The essential features of Johansen's approach have been preserved in the general equilibrium models surveyed by Fullerton, Henderson, and Shoven (1984). The unknown parameters describing technology in these models are determined by "calibration" to a single data point. Data from a single interindustry transactions table are supplemented by a small number of parameters estimated econometrically. The obvious disadvantage of this approach is that arbitrary constraints on patterns of production are required in order to make calibration possible.

An alternative approach to modeling producer behavior for general equilibrium models is through complete systems of demand functions for inputs in each industrial sector. Each system gives quantities demanded as functions of prices and output. This approach to general equilibrium of modeling producer behavior was originated by Berndt and Jorgenson (1973). As in the descriptions of technology by Leontief and Johansen, production is characterized by constant returns to scale in each sector. As a consequence, commodity prices can be expressed as functions of factor prices, using the nonsubstitution theorem of Samuelson (1951). This greatly facilitates the solution of the econometric general equilibrium model constructed by Hudson and Jorgenson (1974) by permitting a substantial reduction in dimensionality of the space of prices to be determined by the model.

The implementation of econometric models of producer behavior for general equilibrium analysis is very demanding in terms of data requirements. These models require the construction of a consistent time series of interindustry transactions tables. By comparison, the noneconometric aproaches of Leontief and Johansen require only a single interindustry transactions table. Second, the implementation of systems of input demand functions requires methods for the estimation of parameters in systems of nonlinear simultaneous equations. Finally, the restrictions implied by the economic theory of producer behavior require estimation under both equality and inequality constraints.

Jorgenson and Fraumeni (1983) have constructed an econometric model of producer behavior for thirty-five industrial sectors of the U.S. economy. The next research objective is to disaggregate the demands for energy and materials by constructing a hierarchy of models for allocation within the energy and materials aggregates. A second research objective is to incorporate the production models for all thirty-five industrial sectors into an econometric general equilibrium model of production for the U.S. economy along the lines suggested by Jorgenson (1983b, 1984a). A general equilibrium model will make it possible to analyze the implications of sectoral patterns of substitution and technical change for the behavior of the U.S. economy as a whole.

1.7.2 Panel Data

The approach to modeling economies of scale originated by Christensen and Greene (1976) is based on the underlying assumption that individual producing units at the same point of time have the same technology. Separate models of production are fitted for each time period, implying that the same producing unit has a different technology at different points of time. A more symmetrical treatment of observations at different points of time is suggested by the model of substitution and technical change in U.S. agriculture developed by Binswanger (1974c, 1978c). In this model technology is permitted to differ among time periods and among producing units.

Caves, Christensen, and Trethaway (1983) have employed a translog cost function to analyze a panel data set for all U.S. trunk and local service airlines for the period 1970–1981. Individual airlines are observed in some or all years during the period. Differences in technology among years and among producing units are incorporated through one-zero dummy variables that enter the cost function. One set of dummy variables corresponds to the individual producing units. A second set of dummy variables corresponds to the time periods.

Although airlines provide both freight and passenger service, the revenues for passenger service greatly predominate in the total, so that output is defined as an aggregate of five categories of transportation services. Inputs are broken down into three categories—labor, fuel, and capital and materials. The number of points served by an airline is included in the cost functions as a measure of the size of the network. Average stage length and average load factor are included as additional characteristics of output specific to the airline.

Caves, Christensen, and Trethaway introduce a distinction between economies of scale and economies of density. Economies of scale are defined in terms of the sum of the elasticities of total cost with respect to output and points served, holding input prices and other characteristics of output constant. Economies of density are defined in terms of the elasticity of total cost with respect to output, holding points served, input prices, and other characteristics of output constant. Caves, Christenen, and Trethaway find constant returns to scale and increasing returns to density in airline service.

The model of panel data employed by Caves, Christensen, and Trethaway in analyzing air transportation service is based on “fixed effects." The characteristics of output specific to a producing unit can be estimated by employing one-zero dummy variables for each producing unit. An alternative approach based on "random effects" of output characteristics is utilized by Caves, Christensen, Trethaway, and Windle (1984) in modeling rail transportation service. They consider a panel data set for forty-three Class I railroads in the United States for the period 1951–1975.

Caves, Christensen, Trethaway, and Windle employ a generalized translog cost functon in modeling the joint production of freight and passenger transportation services by rail. They treat the effects of characteristics of output specific to each railroad as a random variable. They estimate the resulting model by panel data techniques originated by Mundlak (1963, 1978). The number of route miles served by a railroad is included in the cost function as a measure of the size of the network. Length of haul for freight and length of trip for passengers are included as additional characteristics of output.

Economies of density in the production of rail transportation services are defined in terms of the elasticity of total cost with respect to output, holding route miles, input prices, firm-specific effects, and other characteristics of output fixed. Economies of scale are defined holding only input prices and other characteristics of output fixed. The impact of changes in outputs, route miles, and firm specific effects can be estimated by panel data techniques. Economies of density and scale can be estimated from a single cross section by omitting firm-specific dummy variables.

Panel data techniques require the construction of a consistent time series of observation on individual producing units. By comparison, the cross-section methods developed by Christensen and Greene require only a cross section of observations for a single time period. The next research objective in characterizing economies of scale and economies of density is to develop panel data sets for regulated industries—electricity generation, transportation, and communications—and to apply panel data techniques in the analysis of economies of scale and economies of density.

1.7.3 Dynamic Models of Production

The simplest intertemporal model of production is based on capital as a factor of production. A less restrictive model generates costs of adjustment from changes in the level of capital input through investment. As the level of investment increases, the amount of marketable output that can be produced from given levels of all inputs is reduced. Marketable output and investment can be treated as outputs that are jointly produced from capital and other inputs. Models of production based on costs of adjustment have been analyzed, for example, by Lucas (1967b) and Uzawa (1969).

Optimal production planning with costs of adjustment requires the use of optimal control techniques. The optimal production plan at each point of time depends on the initial level of capital input, so that capital is a "quasi-fixed" input. Obviously, labor and other inputs can also be treated as quasi-fixed in models of production based on costs of adjustment. The optimal production plan at each point of time depends on the initial levels of all quasi-fixed inputs.

The optimal production plan with costs of adjustment depends on all future prices of outputs and inputs of the production process. Unlike the prices of outputs and inputs at each point of time employed in the production studies we have reviewed, future prices cannot be observed on the basis of market transactions. To simplify the incorporation of future prices into econometric models of production, a possible aproach is to treat these prices as if they were known with certainty. A further simplification is to take all future prices to be equal to current prices, so that expectations are "static."

Dynamic models of production based on static expectations have been employed by Denny, Fuss, and Waverman (1981), Epstein and Denny (1983), and Morrison and Berndt (1981). Denny, Fuss, and Waverman have constructed models of substitution among capital, labor, energy, and materials inputs for two-digit industries in Canada and the United States. Epstein and Denny have analyzed substitution among these same inputs for total manufacturing in the United States. Morrison and Berndt have utilized a similar data set with labor input divided between blue collar and white collar labor. Berndt, Morrison, and Watkins (1981) have surveyed dynamic models of production.

The obvious objection to dynamic models of production based on static expectations is that current prices change from period to period, but expectations are based on unchanging future prices. An alternative approach is to base the dynamic optimization on forecasts of future prices. Since these forecasts are subject to random errors, it is natural to require that the optimization process take into account the uncertainty that accompanies forecasts of future prices. Two alternative approaches to optimization under uncertainty have been proposed.

We first consider the approach to optimization under uncertainty based on certainty equivalence. Provided that the objective function for producers is quadratic and constraints are linear, optimization under uncertainty can be replaced by a corresponding optimization problem under certainty. This gives rise to linear demand functions for inputs with prices replaced by their certainty equivalents. This approach has been developed in considerable detail by Hansen and Sargent (1980, 1981) and has been employed in modeling producer behavior by Epstein and Yatchew (1985), Meese (1980), and Sargent (1978).

An alternative approach to optimization under uncertainty is to employ the information about expectations of future prices contained in current input levels. This approach has the advantage that it is not limited to quadratic objective functions and linear constraints. Pindyck and Rotemberg (1983a) have utilized this approach in analyzing the Berndt-Wood (1975) data set for U.S. manufacturing, treating capital and labor input as quasi-fixed. They employ a translog variable cost function to represent technology, adding costs of adjustment that are quadratic in the current and lagged values of the quasi-fixed inputs. Pindyck and Rotemberg (1983b) have employed a similar approach to the analysis of production with two kinds of capital input and two types of labor input.

Notes

1. This approach to production theory is employed by Carlson (1939), Frisch (1965), and Schneider (1934). The English edition of Frisch's book is a translation from the ninth edition of his lectures, published in Norwegian in 1962; the first edition of these lectures dates back to 1926.
2. These studies are summarized by Douglas (1948). See also: Douglas (1967, 1976). Early econometric studies of producer behavior, including those based on the Cobb-Douglas production function, have been surveyed by Heady and Dillon (1961) and Walters (1963). Samuelson (1979) discusses the impact of Douglas's research.
3. Econometric studies based on the CES production function have been surveyed by Griliches (1967), Jorgenson (1974), Kennedy and Thirlwall (1972), Nadiri (1970), and Nerlove (1967).
4. Hotelling (1932) and Samuelson (1954) develop the dual formulation of production theory on the basis of the Legendre transformation. This approach is employed by Jorgenson and Lau (1974a,b) and Lau (1976, 1978b).
5. Shephard utilizes distance functions to characterize the duality between cost and

production functions. This approach is employed by Diewert (1974a, 1982), Hanoch (1978), McFadden (1973), and Uzawa (1964).

6. Surveys of duality in the theory of production are presented by Diewert (1982) and Samuelson (1983).

7. This approach to the selection of parametric forms is discussed by Diewert (1974a), Fuss, McFadden, and Mundlak (1978), and Lau (1974).

8. A more detailed discussion of this measure is presented in section 1.2.2.

9. An alternative approach, originated by Diewert (1971, 1973, 1974b), employs the square roots of the input prices rather than the logarithms and results in the "generalized Leontief" parametric form.

10. Surveys of parametric forms employed in econometric modeling of producer behavior are presented by Fuss, McFadden, and Mundlak (1978) and Lau (1986).

11. Methods for estimation of nonlinear multivariate regression models are summarized by Malinvaud (1980).

12. Nonlinear two and three stage least squares methods are also discussed by Amemiya (1977), Gallant (1977), and Gallant and Jorgenson (1979).

13. Constrained estimation is discussed in more detail in section 1.3.3.

14. Surveys of methods for estimation of nonlinear multivariate regressions and systems of nonlinear simultaneous equations are given by Amemiya (1983) and Malinvaud (1980), esp. Chs. 9 and 20. Computational techniques are surveyed by Quandt (1983).

15. Time-series and cross-section differences in technology have been incorporated into a model of substitution and technical change in U.S. agriculture by Binswanger (1974a,c, 1978c). Binswanger's study is summarized in section 1.4.2.

16. The dual formulation of production theory under constant returns to scale is due to Samuelson (1954).

17. The share elasticity was introduced by Christensen, Jorgenson, and Lau (1971, 1973) and Samuelson (1973).

18. This definition of the bias of technical change is due to Hicks (1932). Alternative definitions of biases of technical change are compared by Binswanger (1978b).

19. Alternative definitions of substitution and complementarity are discussed by Samuelson (1974).

20. The concept of separability is due to Leontief (1947a,b) and Sono (1961).

21. The concept of homothetic separability was introduced by Shephard (1953, 1970).

22. A proof of this proposition is given by Lau (1969, 1978b).

23. A proof of this proposition is given by Lau (1978b).

24. This characterization of price and quantity indexes was originated by Shephard (1953, 1970).

25. Gorman (1959) has analyzed the relationship between aggregation over commodities and two-stage allocation. A presentation of the theory of two-stage allocation and references to the literature are given by Blackorby, Primont, and Russell (1978).

26. Share elasticities were introduced as constant parameters of an econometric model of producer behavior by Christensen, Jorgenson, and Lau (1971, 1973). Constant share elasticities, biases, and deceleration of technical change are employed by Jorgenson and Fraumeni (1983) and Jorgenson (1983b, 1984b). Binswanger (1974a,c, 1978c) uses a different definition of biases of technical change in parameterizing an econometric model with constant share elasticities.

27. Arrow, Chenery, Minhas, and Solow (1961) have derived the CES production function as an exact representation of a model of producer behavior with a constant elasticity of substitution.

28. An alternative approach to the generation of the translog parametric form for the production function by means of Taylor's series was originated by Kmenta (1967). Kmenta employs a Taylor's series expansion in terms of the parameters of the CES

production function. This approach imposes the same restrictions on patterns of production as those implied by the constancy of the elasticity of substitution. The Kmenta approximation is employed by Griliches and Ringstad (1971) and Sargan (1971), among others, in estimating the elasticity of substitution.

29. This approach to global concavity was originated by Jorgenson and Fraumeni (1983). Caves and Christensen (1980) have compared regions where concavity obtains for alternative parametric forms.

30. The Cholesky factorization was first employed in imposing local concavity restrictions by Lau (1978b).

31. Different stochastic specifications are compared by Appelbaum (1978), Burgess (1975), and Geary and McDonnell (1980). The implications of alternative stochastic specifications are discussed in detail by Fuss, McFadden, and Mundlak (1978).

32. This approach to estimation is presented by Jorgenson and Fraumeni (1983).

33. Maximum likelihood estimation by means of the "seemingly unrelated regressions" model analyzed by Zellner (1962) would not be appropriate here, since the symmetry constraints we have described in section 1.2.4 cannot be written in the bilinear form considered by Zellner.

34. Computational techniques for constrained and unconstrained estimation of nonlinear multivariate regression models are discussed by Malinvaud (1980). Techniques for computation of unconstrained estimators for systems of nonlinear simultaneous equations are discussed by Berndt, Hall, Hall, and Hausman (1974) and Belsley (1974, 1979).

35. The method of nonlinear three-stage least squares introduced by Jorgenson and Laffont (1974) was extended to nonlinear inequality constrained estimation by Jorgenson, Lau, and Stoker (1982), esp. pp. 196–204.

36. A nonstatistical approach to testing the theory of production has been presented by Afriat (1972), Diewert and Parkan (1983), Hanoch and Rothschild (1972), and Varian (1984).

37. Statistics for testing linear inequality restrictions in linear multivariate regression models have been developed by Gourieroux, Holly, and Montfort (1982); statistics for testing nonlinear inequality restrictions in nonlinear multivariate regression models are given by Gourieroux, Holly, and Monfort (1980).

38. Restrictions on patterns of substitution implied by homothetic separability have been discussed by Berndt and Christensen (1973a), Jorgenson and Lau (1975), Russell (1975), and Russell and Blackorby (1976).

39. The methodology for testing separability restrictions was originated by Jorgenson and Lau (1975). This methodology has been discussed by Blackorby, Primont and Russell (1977) and by Denny and Fuss (1977). An alternative approach has been developed by Woodland (1978).

40. The advantages and disadvantages of summarizing data from process analysis models by means of econometric models have been discussed by Maddala and Roberts (1980, 1981) and Griffin (1980, 1981c).

41. A review of the literature on induced technical change is given by Binswanger (1978a).

42. The model of capital as a factor of production was originated by Walras (1954). This model has been discussed by Diewert (1980) and by Jorgenson (1973a, 1980).

43. Myopic decision rules are derived by Jorgenson (1973b).

44. Data on energy and materials are based on annual interindustry transactions tables for the United States compiled by Jack Faucett Associates (1977). Data on labor and capital are based on estimates by Fraumeni and Jorgenson (1980).

45. Parameter estimates are given by Jorgenson and Fraumeni (1981b), pp. 255–264.

46. The implications of patterns of biases of technical change are discussed in more detail by Jorgenson (1981).

47. A survey of the literature on frontier representations of technology is given by Forsund, Lovell, and Schmidt (1980).

48. Duality between cost and production functions is due to Shephard (1953, 1970).

49. The concept of homotheticity was introduced by Shephard (1953). Shephard shows that homotheticity of the cost function is equivalent to homotheticity of the production function.

50. A model of a regulated firm based on cost minimization was introduced by Nerlove (1963). Surveys of the literature on the Averch-Johnson model have been given by Bailey (1973) and Baumol and Klevorick (1970).

51. Christensen and Greene have assembled data on cross sections of individual firms for 1955 and 1970. The quantity of output is measured in billions of kilowatt hours (kwh). The quantity of fuel input is measured by British thermal units (Btu). Fuel prices per million Btu are averaged by weighting the price of each fuel by the corresponding share in total consumption. The price of labor input is measured as the ratio of total salaries and wages and employee pensions and benefits to the number of full-time employees plus half the number of part-time employees. The price of capital input is estimated as the sum of interest and depreciation.

52. Statistical methods for models of production with disturbances constrained to be positive or negative are discussed by Aigner, Amemiya and Poirier (1976) and Greene (1980).

53. A model of production with differences between *ex ante* and *ex post* substitution possibilities was introduced by Houthakker (1955). This model has been further developed by Johansen (1972) and Sato (1975) and has been discussed by Hildenbrand (1981) and Koopmans (1977). Recent applications are given by Forsund and Hjalmarsson (1979, 1983), and Forsund and Jansen (1983).

54. A review of the literature on regulation with joint production is given by Bailey and Friedlaender (1982).

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