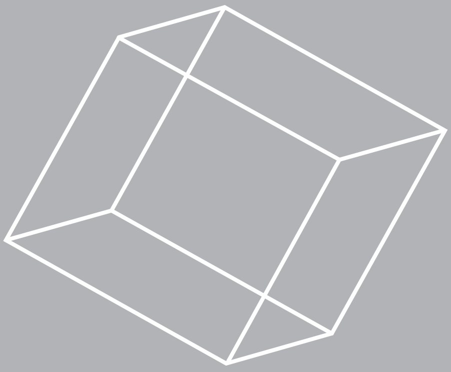
ZYGMUNT PIZLO

3D SHAPE

Its Unique Place in Visual Perception



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This book is dedicated to Prof. Robert M. Steinman, teacher, collaborator, and friend, whose questions and suggestions made this book possible.

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Preface

This book is the very first devoted exclusively to the perception of shape by human beings and machines. This claim will surely be surprising to many, perhaps most, readers, but it is true nonetheless. Why is this the first such book? I know of only one good reason. Namely, the fact that shape is a unique perceptual property was not appreciated, and until it was, it was not apparent that shape should be treated separately from all other perceptual properties, such as depth, motion, speed, and color. Shape is special because it is both complex and structured. These two characteristics are responsible for the fact that shapes are perceived veridically, that is, perceived as they really are "out there." The failure to appreciate the unique status of shape in visual perception led to methodological errors when attempts were made to study shape, arguably the most important perceptual property of many objects. These errors resulted in a large conflicting literature that made it impossible to develop a coherent theoretical treatment of this unique perceptual property. Even a good working definition of shape was wanting. What got me interested in trying to understand this unique, but poorly defined, property of objects?

My interest began when I was working on an engineering application, a doctoral project in electrical engineering that involved formulating statistical methods for pattern recognition. Pattern recognition was known to be an important tool for detecting anomalies in the manufacture of integrated circuits. The task of an engineer on a production line is like the task of a medical doctor; both have to diagnose the presence and the nature of a problem based on the pattern of data provided by "signs." I realized shortly after beginning to work on this problem that it was very difficult to write a pattern recognition algorithm "smart" enough to accomplish what an engineer did very easily just by looking at histograms and scatter plots. It became obvious to me that before one could make computers discriminate one pattern from another, one might have to understand how humans manage to do this so well. This epiphany came over me on the night before I defended my first doctoral dissertation. My interest in studying human shape perception started during the early morning hours of that memorable day as I tried to anticipate issues likely to come up at my defense.

Studying pattern and shape perception requires more than a cursory knowledge of geometry, both Euclidean and projective. It also requires the ability to apply this knowledge to a perspective projection from a three-dimensional (3D) space to a two-dimensional (2D) image. I had a reasonable background in electrical engineering, but it did not include projective geometry. I had to learn it from scratch. It took both time and effort, but it paid off. At the time I did not realize that this was unusual. It never occurred to me that anyone would try to study shape, the topic that served for my second doctoral degree, without knowing geometry quite well.

My formal study of human shape perception was done in the Sensori-Neural and Perceptual Processes Program (SNAPP) of the Psychology Department at the University of Maryland at College Park where Robert M. Steinman served as my doctoral advisor. My dissertation also benefited a great deal from interactions with several members of the Center for Automation Research and Computer Science at this institution. My independent study of projective geometry was greatly facilitated by numerous discussions with Isaac Weiss. Realize that I was starting from scratch. I was analyzing known properties of geometrical optics simultaneously with learning about groups, transformations, and invariants. Here, my limited formal background in geometry led me to stumble onto some new aspects of projective geometry that had not been explored before. I was encouraged to pursue this path by Azriel Rosenfeld, my second doctoral mentor, who was affiliated with SNAPP. Azriel Rosenfeld, who was well-known for his many contributions to machine vision, was a mathematician by training. He was always interested in exploring the limits of mathematical knowledge and of mathematical formalisms, and he, Isaac Weiss, and I published some of our insights about a new type of perspective invariants that grew out of my dissertation. After mastering what I needed to understand in projective geometry, and after developing the new geometrical tools needed for a model of the perspective projection in the human eye, I realized that I should also learn regularization theory with elements of the calculus of variations. Learning this part of mathematics was facilitated by interactions with Yannis Aloimonos, who was among the first to apply this formalism in computer vision. He asked me, now almost 20 years ago, whether regularization theory is the right formalism for understanding human vision. I answered then that I was not sure. My answer now is "Yes" for reasons made abundantly clear in this book. My interactions and learning experiences during my graduate education at the University of Maryland at College Park were not limited to geometry and regularization theory. From Azriel Rosenfeld I learned about pyramid models of figure–ground organization, and I learned about computational applications of Biederman's and Pentland's theories of shape from Sven Dickinson. Both figure prominently in my treatment of shape presented in this book. Now that the reader knows the circuitous route that led me to study human shape perception, I will explain why I decided to write this book.

The primary motivation for writing it grew out of my teaching obligations. When I began to teach, I tried to present the topic called "shape perception" as if it were a traditional topic within the specialty called "perception." As such, shape perception, like other topics such as color perception, should be taught on the basis of the accumulation of specialized knowledge. Clearly, the history of a topic in a scientific specialty, such as shape perception, should be more than a collection of names, theories, and experimental results. The history of the topic should reveal progress in our understanding of the relevant phenomena. I found it impossible to demonstrate the accumulation of knowledge in the area called "shape perception." The existing literature did not allow a coherent story, and I decided to try to figure out what was going on. Knowing this was important for doing productive research, as well as for teaching. How do you decide to take the *next step* toward understanding shape when where the last step left you was unclear? Recognizing that shape is a special perceptual property did the trick. It made both teaching and productive research possible. This book describes how much we currently understand about shape and how we came to reach the point that we have reached. It is a long story with many twists and turns. I found it an exciting adventure and hope that the reader experiences it this way, too.

By trying to maintain the focus of my presentation, I deliberately left out material that ordinarily would have been included if I were writing a comprehensive review of visual perception, rather than a book on the specialized topic called “shape perception.” Specifically, I did not include a treatment of the neuroanatomy or neurophysiology of shape perception. Little is known about shape at this level of analysis because we are only now in a position to begin to ask appropriate questions. The emphasis of the book is on understanding perceptual mechanisms, rather than on brain localization. For example, the currently available knowledge of neurophysiology cannot inform us about which “cost function” is being minimized when a 3D shape percept is produced. I also did not include a large body of evidence on the perception of 2D patterns and 3D scenes that is only tangentially relevant to our understanding of the perception of 3D shapes.

The text concentrates on the discussion of the main concepts; technical material has been reduced to a minimum. This made it possible to tell the “story of shape” without interruption. A full understanding of the material contained in this book, however, requires understanding the underlying technical details. The appendices provide the basic mathematical and computational information that should be sufficient for the reader to achieve a technical understanding of the infrastructure that provided the basis for my treatment of shape. The references to sources contained in these appendices can also serve as a starting point for more in-depth readings in geometry and computational vision, readings that I hope will encourage individuals to undertake additional work on this unique perceptual property. Much remains to be done.

I had six goals when I began writing this book, namely, I set out to (i) critically review *all* prior research on shape; (ii) remove apparent contradictions among experimental results; (iii) compare several theories, computational and noncomputational, to each other, as well as to dozens of psychophysical results; (iv) present a new theory of shape; (v) show that this new theory is consistent with all prior and new results on shape perception; and (vi) set the stage for *meaningful* future research on shape. My choice of these particular goals and the degree to which I have been successful in reaching each of them can only be evaluated by reading the book. Obviously, my success with each goal is less important than my success in (i) encouraging the reader to think deeply about the nature and significance of shape perception and (ii) stimulating productive research on this fundamental perceptual problem.

The new theory presented in this book shows how a 3D shape percept is produced from a 2D retinal image, assuming only that the image has been *organized* into 2D shapes. One can argue that this new theory is able to solve the most difficult aspect of 3D shape perception. What remains to be done is to explain how the 2D shapes on the retina are *organized*. The process that accomplishes this, called "figure–ground organization" by the Gestalt psychologists, is not dealt with in great detail in this book, simply because not much is known about it at this writing. It is likely, however, that now that I have called attention to the importance of this critical organizing process in shape perception, it will be easier to (i) expand our understanding of how it works and (ii) formulate plausible computational models of the mechanisms that allow human beings to perceive the shapes of objects veridically.

I will conclude this preface by acknowledging individuals who contributed to this book and to the research that made it possible, beginning with the contributions of my students: Monika Salach-Golyska, Michael Scheessele, Moses Chan, Adam Stevenson, and Kirk Loubier worked with me on shape perception and figure–ground organization; Yunfeng Li designed and conducted recent psychophysical experiments on a number of aspects of shape and helped me formulate and test the current computational model; and he, along with Emil Stefanov and Jack Saalweachter, helped prepare the graphical material used in this book.

I also acknowledge the contributions of the late Julie Epelboim, who was a valuable colleague at the University of Maryland, where she served as a subject in my work on pyramid models and perspective invariants. My son, Filip Pizlo, contributed to a number of aspects of my shape research. He helped write programs for our psychophysical experiments and was instrumental in designing demos illustrating many of the key concepts. Interactions with my colleagues, Charles Bouman, Edward Delp, Sven Dickinson, Gregory Francis, Christoph Hoffmann, Walter Kropatsch, Longin Jan Latecki, Robert Nowack, Voicu Popescu, and Karthik Ramani contributed to my understanding of inverse problems, regularization theory, shape perception, geometrical modeling, and figure–ground organization. I also acknowledge the suggestion and encouragement to write a book like this that I received from George Sperling and Misha Pavel after a talk on the history of shape research that I gave at the 25th Annual Interdisciplinary Conference at Jackson Hole in 2000. None of these individuals are responsible for any imperfections, errors, or omissions present in this book.

I acknowledge support from the National Science Foundation, National Institutes of Health, the Air Force Office of Scientific Research, and the Department of Energy for my research and for writing this book. I thank Barbara Murphy, Kate Blakinger, Meagan Stacey, and Katherine Almeida at MIT Press for editorial assistance.

Finally, I thank my family for their understanding and support while my mind was bent out of shape by concentrating excessively on this unique perceptual property.

3D Shape

1 Early Theories of Shape and the First Experiments on Shape Constancy

1.1 Shape Is Special

This book is concerned with the perception of shape. “Perception” can be defined simply—namely, as becoming aware of the external world through the action of the senses. “Shape,” unlike perception, cannot be defined in such simple terms, and much of this book is devoted to explaining why this is the case, how it came to pass, and how we have finally reached a point where we can discuss and study shape in a way that captures the significance of this critical property of objects. When we refer to the “shape” of an object, we mean those geometrical characteristics of a specific three-dimensional (3D) object that make it possible to perceive the object veridically from many different viewing directions, that is, to perceive it as it actually is in the world “out there.” Understanding how the human visual system accomplishes this is essential for understanding the mechanisms underlying shape perception. Understanding this is also essential if we want to build machines that can see shapes as humans do.

Understanding shape perception is of fundamental importance. Why? Shape *is* fundamental because it provides human beings with accurate information about objects “out there.” Accurate information about the nature of objects “out there” is essential for effective interactions with them. An object’s shape is a *unique* perceptual property of the object in the sense that it is the *only* perceptual property that has sufficient complexity to allow an object to be identified. Furthermore, shape’s high degree of complexity makes it quite different from *all* other perceptual properties. For example, color varies along only three dimensions: hue, brightness, and saturation. Many objects “out there” will have the same color. Other perceptual properties are even simpler: An object's size and weight can vary only along a single dimension, and many objects will have the same size or weight. Shape is unlike all of these properties because it is much more complex. An object's shape can be described along a large number of dimensions. Imagine how many points on the contour of a circle would have to be moved to transform the circle into the outline of a human silhouette or how many points on the outline of the silhouette would have to be moved to change its outline into a circle. When two shapes are very different, as they are in figure 1.1, the position of almost all points along their contours would have to be changed to change the shape of one to the shape of the other. The circle and the inscribed silhouette of a human being are about as different as any two shapes can be. All of the points except those where the human silhouette touches the circle (the tips of the fingers and the soles of the feet) would have to be moved to change one to the other. Theoretically, the number of points along an outline is infinite, so the number of dimensions characterizing an arbitrary shape is, theoretically, infinitely large. Fortunately, in the world of living things like ourselves, one need not deal with an infinite number of dimensions because

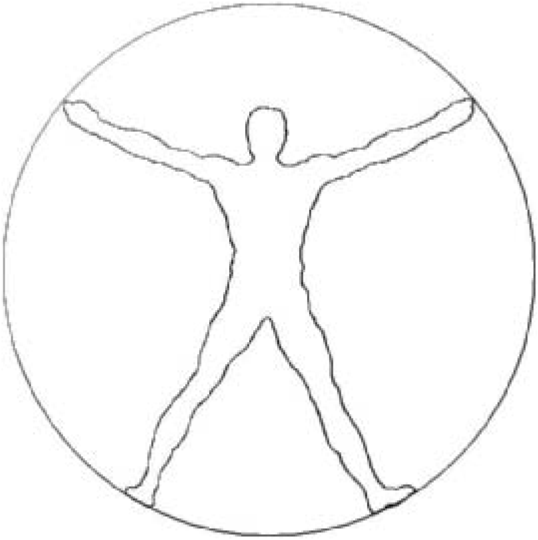

Figure 1.1
A human silhouette and a circumscribed circle (after Leonardo DaVinci).

the human being's sensory systems are constrained. Even in the fovea, where the highest density of cells in the retina is found, there are only about 400 receptor cells per millimeter (Polyak, 1957). Thus, when a circular shape with a diameter of 1 deg of visual angle is projected on the fovea, only 300 or 400 receptors would receive information about the circle's contour. It is clear, however, that despite such constraints, sufficient information would remain to disambiguate all objects human beings have encountered within the environment in which they evolved and are likely to encounter in the future. Once this is appreciated, it becomes clear that what we call "shape" has considerable evolutionary significance because the function of very many objects is conveyed primarily by their shape.

Naturally occurring objects tend to fall into similarly shaped groups, and this makes it convenient to deal with them as members of families of similar shapes. Most apples look alike, and most cars look alike. Note that when you view your car from a new angle, its image on your retina changes, but it is perceived as the same car. This fact defines what is called "shape constancy." Formally, "shape constancy" refers to the fact that the percept of the shape of a given object remains constant despite changes in the shape of the object's retinal image. The shape of the retinal image changes when the viewing orientation changes.[1] Shape constancy is a fundamental perceptual phenomenon, and much of this book is devoted to explaining conditions under which shape constancy can be reliably achieved and the mechanisms underlying this accomplishment. Shape constancy has profound significance because the perceived shape of a given object is veridical (the way it is "out there") despite the fact that its shape on the retina, the plane in which it stimulates our visual receptors, has changed. These considerations apply to many shape families. Figure 1.2 shows two views of the same scene, each taken from a different viewpoint. It is easy to recognize all of the individual objects in each view. Determining which contours and which regions of an image correspond to a single object is called "figure–ground organization." This terminology and its role in shape constancy was introduced by the Gestalt psychologists. It will be discussed later when their contributions are described. Interestingly, both figure–ground organization and shape constancy can be achieved when only the contours of objects are visible, as can be seen in figure 1.3. Surface details and structure are not needed to recognize a

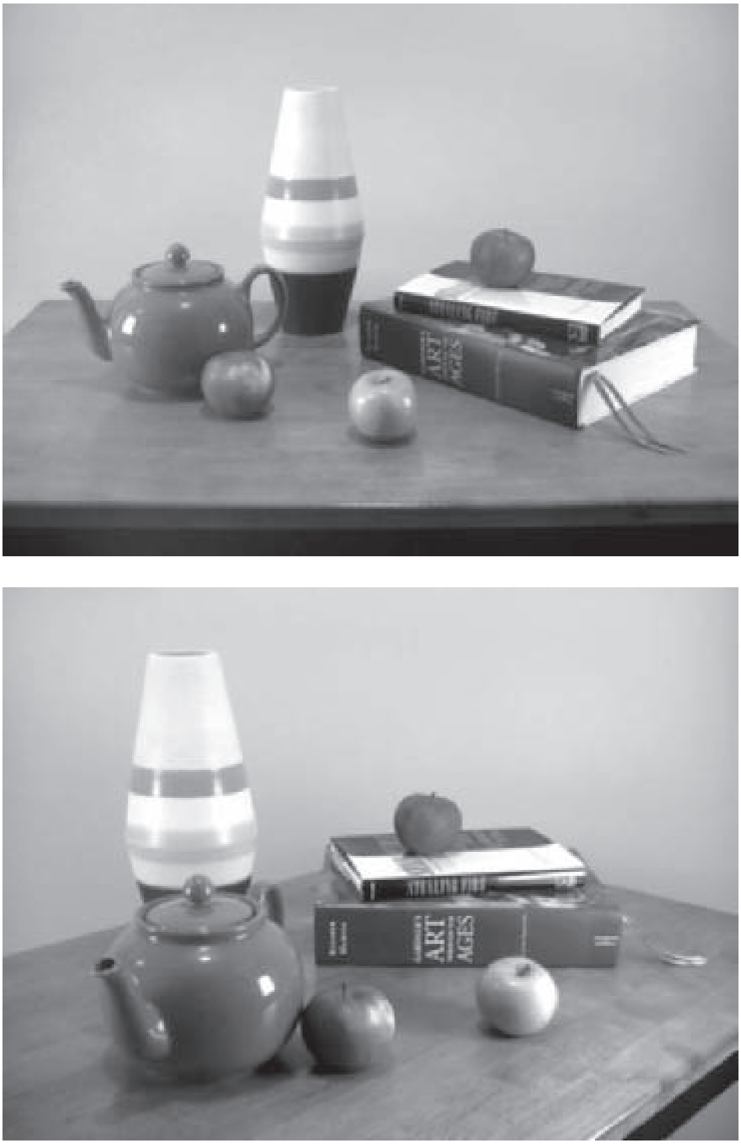

Figure 1.2
Two views of an indoor scene illustrating two fundamental perceptual phenomena. “Figure–ground organization” is illustrated by the fact that it is easy to determine which regions and contours in the image correspond to individual objects. Note, also, that the contour in the image belongs to the region representing the object. “Shape constancy” is illustrated by the fact that it is easy to recognize the shapes of objects regardless of the viewing direction (photo by D. Black).

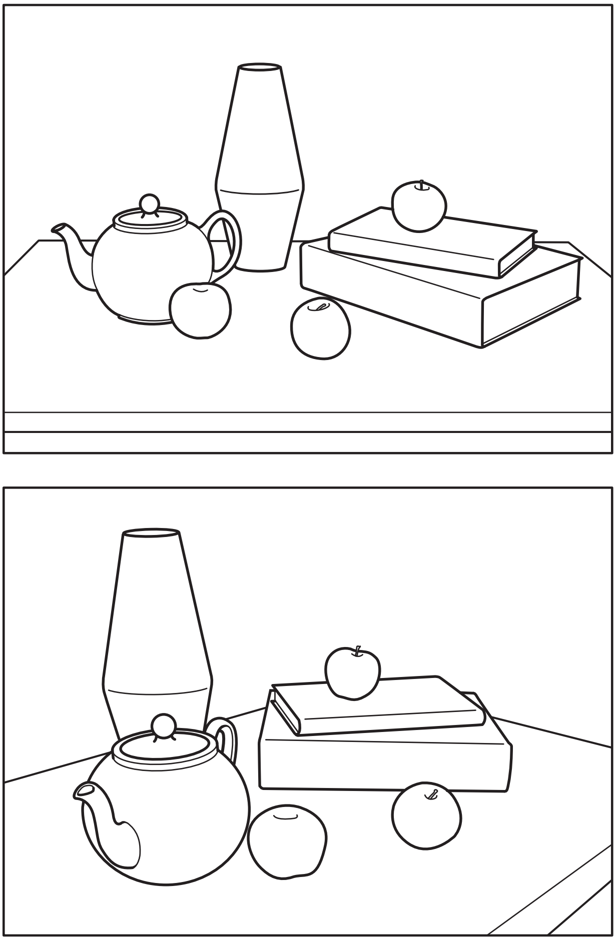

Figure 1.3
Line drawing version of the previous figure (prepared by D. Black).

variety of individual objects. Retinal shape, alone, is sufficient for shape recognition and shape constancy.

Note, however, that two shape families, ellipses and triangles, are quite different, and, as you will see, failure to appreciate this difference can make a lot of trouble. Ellipses and triangles are very much simpler than all other shapes. They do not offer the degree of complexity required by the visual system to achieve shape constancy. A shape selected from the family of ellipses requires only one parameter, its aspect ratio (the ratio of the lengths of the long and short axis), for a unique identification of a particular ellipse. Changing the magnitude of the two axes, while keeping their ratio constant, changes only the size of an ellipse, not its shape. The family of triangular shapes requires only two parameters (triangular shape is uniquely specified by two angles because the three angles in a triangle always sum to 180 deg). Note that the number of parameters needed to describe shape within these two families (ellipses and triangles) is small, similar in number to the parameters required to describe color, size, and weight. Much was made above about how a high degree of complexity makes shape special in that it can provide a basis for the accurate identification of objects. Clearly, using ellipses and triangles to study shape might present a problem because their shapes are characterized by only one or two parameters. It has. It held the field back for more than half a century (1931–1991).

Why do ellipses and triangles present problems? They present problems because the 3D world is represented in only two dimensions on the retina. The Bishop Berkeley (1709) emphasized that a perspective transformation from the world to the retina reduces the amount of information available for the identification of both objects and depth. Note that this loss affects ellipses and triangles profoundly. Any ellipse "out there" will, at various orientations, be able to produce any ellipse on the retina. This fact is illustrated in figure 1.4a. Here two ellipses with different shapes are shown at the top, and their retinal images are shown at the bottom. The retinal images have identical shapes because the taller ellipse was slanted more. Similarly, any triangle "out there" can produce any triangle on the retina. Note that these are the *only* two families of shapes that confound the shape itself with the viewing orientation. They do this because a perspective transformation from 3D to two dimensions (2D) changes the shape of a 2D (flat or planar) shape with only two degrees of freedom (see appendix A, section A.1). It follows that if the shape itself is characterized by only one or two parameters (as ellipses and triangles are), the information about their shape is completely lost during their projection to the retina and shape constancy may become difficult, even impossible, to achieve. However, if the shape of a figure is characterized by more than two parameters, perspective projection does not eliminate all of the shape information, and shape constancy can almost always be achieved. This is true for any family of shapes, other than ellipses and triangles. The simplest family in which constancy can be achieved reliably is the family of rectangles. In

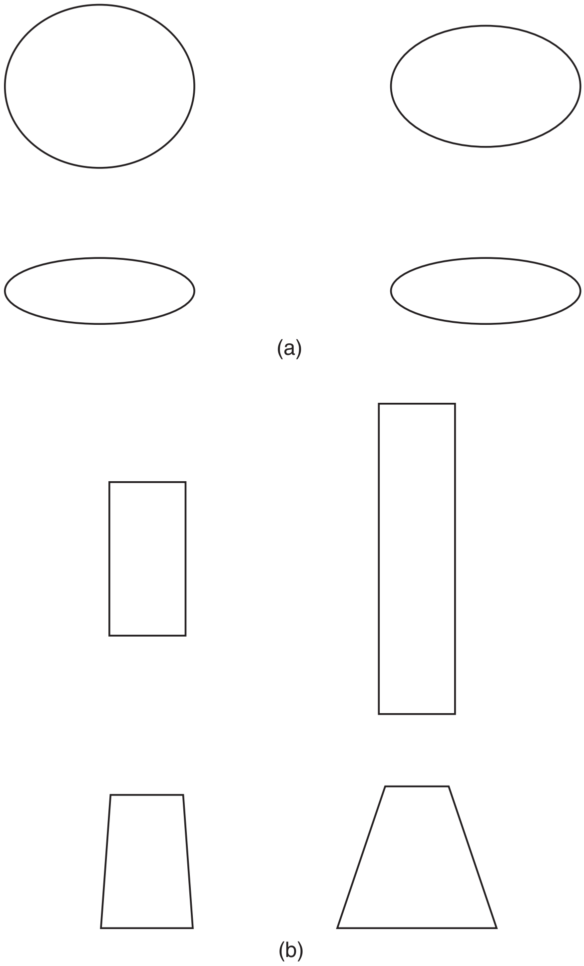

Figure 1.4
(a) Ellipses with different shapes (top) can produce identical retinal images (bottom). The ellipse on the top left was slanted around the horizontal axis more than the ellipse on the top right. As a result, their retinal images (bottom) are identical. (b) Rectangles with different shapes cannot produce identical retinal images. The rectangle on the top right was slanted around the horizontal axis more than the rectangle on the top left. As a result, the heights of their retinal images (bottom) are identical, but their shapes are not. Specifically, the angles in the two retinal images are different. If the slant of the rectangle on the top right were equal to that of the rectangle on the top left, the angles in the retinal images would be identical, but the heights would be different. This means that the shapes of the retinal images would be different, as well.

figure 1.4b, two rectangles with different shapes are shown at the top, and their retinal images are shown at the bottom. The taller rectangle had to be slanted more than the shorter one, to produce images with the same heights, but despite the fact that the heights of the retinal images are the same, the angles are not. In fact, two rectangles with different shapes can *never* produce identical retinal images. More generally, if two figures or objects have different shapes, they are very unlikely to produce identical retinal images, as long as the figures are not ellipses or triangles. It follows that understanding shape constancy cannot be based on experiments in which ellipses or triangles were used. This fact, which was overlooked until very recently, has led to a lot of confusion in the literature on shape perception. Note that this confusion might have been avoided because a formal treatment of the rules for making perspective projections (rules that reveal the confound of shape and viewing orientation) had been used by artists since the beginning of the fifteenth century (see Kemp, 1990), and the mathematics of projective geometry had been worked out quite completely by the end of the nineteenth century (Klein, 1939). Why was this confound ignored until recently by those who studied shape perception? The answer lies in the fact that the people who made this mistake did not come to their studies of shape from art or mathematics. They came from a quite different tradition, a tradition that will be described next.

1.2 Explaining Visual Constancies with a “Taking into Account” Principle

Formal research on shape did not start until the beginning of the twentieth century, after the Gestalt Revolution had been launched. By that time, the perception of other important properties of objects such as color, size, lightness, and motion had been studied intensively and very successfully for almost 100 years. For each of these properties a perceptual “constancy” had been defined: The percept of a surface’s lightness and color, of an object’s size, and of its speed, had been shown to remain approximately constant despite changes in its retinal image. These changes of the retinal image could be brought about by changes in the spectrum and intensity of the illuminating light, and by changes of the viewing distance. The conceptual framework and research questions adopted for the study of shape constancy were based on these successful studies of other perceptual constancies. However, generalizing existing knowledge and borrowing an experimental methodology from simple perceptual properties such as color and size to shape, which is a complex multidimensional property, was unwarranted, and dangerous as well. Could this mistake have been avoided? Perhaps. When the formal study of shape started with Thouless' (1931a, b) experiments, existing experimental results had already suggested that the mechanisms underlying shape perception were likely to be different from those underlying size, speed, and lightness, but many students of shape perception mistakenly assumed that shape is like all other visual properties. This *encouraged* them to try to confirm, rather than to question, their theory of shape constancy, which made use of other perceptual properties, when they began to do experiments on shape perception. Their commitment to this assumption caused them to ignore some important aspects of their results. Assuming that shape was like other perceptual properties prevented them from appreciating what was actually going on in their experiments. Had they considered the possibility that shape is fundamentally different from the simpler perceptual properties, they probably would have noticed important, unusual patterns in their data.

The conceptual framework for Thouless' (1931a, b) study of shape can be traced back a long way. His approach was derived from philosophical discussions of epistemological problems reaching back to Alhazen (1083) in the eleventh century. Highlights of these discussions will be presented here because they will allow the reader to appreciate why Thouless and many other modern researchers adopted the particular type of explanation of the perceptual constancies they did. They adopted "taking into account" explanations of lightness, color, and size and expected to be able to extend this approach to their studies of shape constancy as well. Traditionally, all of these perceptual constancies were explained by "taking into account" contextual information present in the viewing conditions. For example, size constancy was "explained" by taking viewing distance into account. Lightness constancy was explained by taking cues to illumination into account, and so forth. Contextual information was critical because the retinal image was *ambiguous*.

The recorded history of the perceptual constancies began long ago with Alhazen (1083), whose book was the first work known to the author to raise the problem of shape constancy. Alhazen, who lived in the second half of the tenth and the first half of the eleventh centuries, is generally viewed as representing a bridge between the science of the ancient Greek philosophers and the precursors of modern science following the European Renaissance. Alhazen made many fundamental contributions to the study of vision. Unfortunately, most were either overlooked during the development of modern science in Europe, which took place between the seventeenth and twentieth centuries, or are not mentioned in contemporary reviews of the history of the subject (Sabra, 1989, 1994; Howard, 1996). To illustrate, Alhazen performed the first systematic observations of afterimages (Alhazen, 1083, p. 51). He also reported the dependence of visual acuity on luminance (p. 54). In addition, he described mixing colors (pp. 144–5) with a precursor of Maxwell's top. He also described color constancy (pp. 141–2), shape constancy (p. 279), and position constancy (pp. 193–4). He conjectured that what came to be called "unconscious inference" in the nineteenth century explained all of these important perceptual phenomena (p. 136). He also discussed the perceived size/perceived distance relationship and its role in size constancy (p. 177). Alhazen even described what we now call "Panum's fusional area" in his discussion of binocular vision (p. 240).[2] Alhazen did not perform systematic experiments to verify his claims, but he described many important perceptual phenomena and recognized the operation of several perceptual mechanisms. Most subsequent writers seldom credited his contributions.

In Europe, the thirteenth century marks the revival of philosophy and the beginnings of what came to be called "science" in Europe. This revival was facilitated by the founding of the first European universities in Bologna, Paris, and Oxford in eleventh and twelfth centuries and a number of others soon after. Philosophers and mathematicians, such as Grosseteste, Bacon, and Peckham in England, Witelo in Poland, and Aquinas and Bonaventure in Italy, stimulated interest in natural sciences by translating old works from Arabic into Latin, as well as contributing new ideas (Hamlyn, 1961; Howard & Rogers, 1995). However, modern philosophy and the scientific study of perception did not start until the seventeenth century when Descartes (1596–1650) came on the scene. Descartes contributed to several areas of knowledge. In philosophy, he offered a dualistic, interactionist interpretation of the mind–body problem and a nativistic view of the origin of our knowledge about the external world (time, space, and motion). In mathematics, he founded analytic geometry. In physiology, he introduced the concept of reflex action and distinguished what came to be called "sensory and motor mechanisms" in the nervous system. Only his contributions to the psychology of visual perception will be discussed here.

Descartes distinguished the mental faculties called “perception” (becoming aware), “cognition” (knowing and understanding), and “conation” (willing). The shapes of objects are, according to Descartes, perceived intuitively in an essentially passive act. The rules of geometrical optics are also intuited. Descartes (1637) published his views on spatial vision in Discourse on Method, Optics, Geometry, and Meteorology. In this book, Descartes discussed the problem presented by the inversion of the retinal image produced by the eyes’ lens, cues to depth, and size and shape constancy. For these constancies he, like Alhazen, offered a “taking into account” explanation, the explanation that will dominate virtually all thinking about perceptual constancies in the nineteenth and twentieth centuries. His treatment of “taking into account” goes as follows: It begins with a discussion of Kepler’s (1604) book *Comments on Witelo*, in which the rules of image formation predicted that the retinal image was inverted. Kepler’s prediction was verified empirically by Scheiner in 1625 and by Descartes in 1637. It raises a problem, namely, we perceive an object as “right side up” despite the fact that its retinal image is “upside down.” Descartes analyzed this problem by using an analogy from tactual perception. When a blind man holds a stick in each hand, and when he knows that the sticks form an *X*, the man not only has knowledge of the positions of his hands but he also can infer knowledge of the positions of the ends of the sticks. Once he knows that the sticks are crossed, he knows that the tip of the stick on the right is on the left side of his body and that the tip of the stick on the left is on the right side (see figure 1.5). According to Descartes, having such knowledge (the rules of geometry) a priori is critical for solving this problem. It allows the blind man to draw the correct inference about the spatial position of the ends of the sticks. Thus, for example, while keeping the two sticks crossed, if he touches an object with a stick that he holds in his right hand, he would naturally know that the object is on the left. Here, the perception of left and right in physical space is not determined by the positions of the left and right parts of the body (hands in this example). Thus, it was not surprising to Descartes that the mind perceives up versus down, as well as right versus left, in the physical world correctly, despite the fact that the retinal image is inverted. This visual example is clearly analogous to the example of the blind man holding

Figure 1.5
A blind man using sticks can correctly judge left and right "out there" despite the fact that left "out there" is actually sensed by his right hand (after Descartes, 1637).

sticks because the visual rays intersect within the eye before they hit the retina. However, note that Descartes adopted a view that perception of the location of an object "out there" involves inferences or thinking. The question remains how the visual system knows that the visual rays intersect before they hit the retina without reading Kepler's book. For Descartes, this did not present a problem because he considered such knowledge to be innate.

Descartes went on to describe ocular vergence as a cue to distance. Again, he used an analogy of a blind man who, with two sticks, can judge the distance of an object by triangulation. The man does this by means of a "natural geometry" made possible by the fact that he knows the distance between his hands and the angles each stick makes with the line connecting his hands. In the case of visual triangulation, the distance between the hands is analogous to the distance between the two eyes, and the angles between the sticks are analogous to the angles formed by the line of sight of each eye with the line connecting the two eyes. Specifically, the length of one side in a triangle, together with sizes of two angles, allow solving the triangle, including the computation of its height, which in this case corresponds to the viewing distance. Descartes goes on to give another example of how the blind man, who represents the visual system, can solve the triangulation problem in the case of motion parallax, that is, when an observer moves relative to some object. Note that in both cases, Descartes, like Alhazen, proposed that these problems were solved by *unconscious* thought processes.

Locke (1690), along with Hobbes (1651), is credited with founding British empiricism. Locke formulated an alternative view of perception, namely, he, in contrast to Descartes, conjectured that all knowledge is derived from experience. He completely rejected Descartes' assumption of innate ideas. He held that a human being's mind begins as a "tabula rasa" (a blank page), and experience with recurring sensations leads to the learning of simple ideas, which are then elaborated into complex ideas by additional associations. For Locke, perceptions of such basic things as shape and motion were complex ideas. Locke claimed that the rules of association, described by Aristotle, provide the mechanisms underlying perception. For Locke, unlike Descartes, perceptual constancies have to be learned.

Molyneux (1692), a friend of Locke, shared his rejection of innate ideas. He supported this claim by posing the following problem. Assume that a person born blind learned to identify and discriminate among objects by the sense of touch. In particular, assume that the person can correctly identify a sphere and a cube. Now suppose that the blind person is made to see. Will the person be able to tell which object is a sphere and which is a cube using vision alone, without touching the objects? Molyneux claimed that the person will not be able to identify these objects. The reason, according to Molyneux, is that the blind person did not have a chance to learn how to see. Molyneux's thought experiment was to receive a lot of attention in 1960s when von Senden's (1932/1960) book on the vision of newly sighted patients came under critical review (Zuckerman & Rock, 1957).

Berkeley, in his *A New Theory of Vision*, published in 1709, elaborated Locke's and Molyneux's empiricism. For Berkeley, vision was always uncertain because it, like hearing, sensed things at a distance. The shapes and sizes of objects had to be learned by comparing visual sensations to touch sensations, which provided a direct and, therefore, reliable source of information. *Only* tactual perception, along with the sensations from the muscles that moved the hands during tactual exploration, can provide direct information about the environment. He illustrates this by pointing out that the perspective projection from the 3D environment to the 2D retina does not preserve information about depth: A point on the retina could be an image of any of the infinitely many points along the line emanating from the point on the retina and proceeding to the object. A newborn human being has no way of judging distances given visually. In essence, according to Berkeley, the visual perception of distance is learned by forming sensorimotor associations. Specifically, when an observer looks at an object binocularly, the line of sight of each eye is directed toward the object, forming an angle called "vergence." The observer is aware of the angle by feeling the state of his eye muscles, and when the observer walks toward the object, the angle changes and the sensations associated with change are noticed. The relation between the sensations from the eye muscles and the number of steps required to reach the object is learned, stored, and used later by means of what we would call today a "look-up table" to provide the mechanism underlying the perception of distance. Similarly, haptics (movements, positions, and orientations of the hands) associated with manipulating objects can provide a basis for creating look-up tables for the shapes of different objects and for the orientation of surfaces. In other words, the individual need not solve geometrical problems to "take into account" environmental characteristics once appropriate look-up tables have been established by associative learning. Berkeley's suggestion has become the standard way of formulating "taking into account" explanations by empiricists ever since his day.

1.3 Helmholtz' Influence When the Modern Era Began

The next important development appeared about 150 years later when Helmholtz published his *Treatise on Physiological Optics* (1867/2000), in which he takes on these problems at what is generally accepted as the beginning of the modern scientific era: He addressed the question of how sensations produced by stimulation of the retina lead to perceptions of 3D space. Helmholtz' approach, like Berkeley's, was empiristic. He supported his teacher's, Johannes Müller's, claims about case histories of persons who were born blind and whose vision was restored by surgery (Helmholtz, 1867/2000, volume 3, pp. 220–7). Such persons, who did not have any prior visual experience, were said to be unable to discriminate among shapes and spatial relations. Helmholtz confirmed these claims and concluded that these patients, like newborn babies, had to learn how to see. He suggested that learning how to see was accomplished by making repetitive eye movements along contours of shapes, an idea that was to be used almost a century later by Hebb (1949).

How did Helmholtz apply his empiristic views to the perceptual constancies? According to Helmholtz, visual perception is derived from "unconscious conclusions" about the external world. These conclusions are reached by means of associations of sensations and memory traces. For example, we come to learn to appreciate the locations of objects in space in the following way:

> When those nervous mechanisms whose terminals lie on the right-hand portions of the retinas of the two eyes have been stimulated, our usual experience, repeated a million times all through life, has been that a luminous object was over there in front of us on our left. We had to lift the hand toward the left to hide the light or to grasp the luminous object; or we had to move toward the left to get closer to it. Thus while in these cases no particular conscious conclusion may be present, yet the essential and original office of such a conclusion has been performed, and the result of it has been attained; simply, of course, by the unconscious process of association of ideas going on in the dark background of our memory. (Helmholtz, 1867/2000, volume 3, p. 26, translated by Southall)

The concept of "unconscious conclusion" is perhaps *the* critical concept in Helmholtz' theory of perception.[3] Binocular depth perception can provide another example of how it was used by Helmholtz. Namely, each point in the environment produces a retinal image in the observer's left and right eye. Assume that the observer's visual system knows accurately and precisely the orientation and position of one eye relative to the other. In such a case, the 3D position of the physical point can be computed as an intersection of the visual rays emanating from the retinal points (volume 3, p. 155). This should remind the reader of Descartes' explanation described above. The difference between Helmholtz' and Descartes' formulation was that Helmholtz does not subscribe to Descartes' notion that the human being has an innate understanding of geometry. Instead, he adopts Berkeley's approach in which a look-up table is established between sensations and their significance "out there."

Now, let us examine Helmholtz' views on shape perception. They are probably best expressed in the following paragraph from his "Review of the Theories" section of his *Treatise*:

> an idea of an individual object . . . includes all the possible single aggregates of sensation which can be produced by this object when we view it on different sides and touch it or examine it in other ways. This is the actual, the real content of any such idea of a definite object. It has no other; and on the assumption of the data above mentioned, this content can undoubtedly be obtained by experience. The only psychic activity required for this purpose is the regularly recurrent association between two ideas which have often been connected before. The oftener this association recurs, the more firm and obligatory it becomes. (volume 3, pp. 533–4).

Thus, according to Helmholtz, the memory of a 3D shape (its mental representation) involves a collection of 2D images of the shape (plus tactual sensations) obtained from different viewing directions. Subsequent recognition of the shape involves matching the current view with the stored views (volume 3, p. 23). There is very little additional discussion of shape perception in Helmholtz' three-volume *Treatise*, at most a paragraph or two.

Now that we have an idea of the prevailing views when the modern study of shape perception began, we can turn to a discussion of the first experimental study of shape perception. It was performed in a period in which Helmholtz' ideas were taken very seriously.

1.4 Thouless' Misleading Experiments

Thouless' two papers, published in 1931, were the most influential, albeit misleading, contributions in the history of shape constancy (Thouless, 1931a, b). These papers are cited in all textbooks of perception known to the author. The significance of these papers stems from the fact that Thouless concluded, and was widely believed to have demonstrated, that shape constancy involves "taking slant into account" ("slant" is defined as the angle between the frontal plane and the plane containing the test figure). He actually did not do this. This claim requires a detailed description of Thouless' papers. Once this is done, Thouless' "contribution" will be evaluated.

In his first experiment, Thouless used two figures, a circle and a square, and tested the accuracy of shape perception of each figure when the figure was presented at a slant (Thouless, 1931a). One *should* expect different outcomes with these two shapes. Remember that the family of ellipses to which the circle belongs (a circle is an ellipse with aspect ratio of one), is completely characterized by only *one* parameter. You must also keep in mind that the family of perspective projections changes the shape of a figure with two degrees of freedom. It follows that in the case of ellipses (one of the two stimuli used by Thouless), the retinal image completely confounds the shape of the figure with its viewing orientation. That is, *any* ellipse "out there" can produce *any* ellipse on the retina (figure 1.4a). Squares, which are in the family of "quadrilaterals," are very different. They are characterized by *four* parameters (ratio of lengths of two sides, plus three angles). As a result, even though the retinal image of a rectangle is affected by slant, its image does not confound the shape of a rectangle with its slant (figure 1.4b). Clearly, ellipses and rectangles should lead to very different results in a shape constancy experiment.

The test figure (a square or a circle) was put on a table. The subject viewed the figure binocularly and was asked to draw its shape. If the percept of the slanted figure were veridical, the reproduced and the presented shapes would have been identical. In particular, their aspect ratios would be the same. The aspect ratios produced were greater than the retinal aspect ratio, but lower than the physical aspect ratio. Thus, perfect shape constancy was not obtained with either figure, but shape constancy was less accurate (larger systematic error) and less reliable (more variable across trials) with the circle than with the square. The fact that shape constancy was not perfect was to be expected—similar results had already been obtained in size, lightness, and color constancy experiments. What was not expected was the difference in the amount of constancy between the circle and the square. This result cannot be easily explained by the "taking slant into account" theory because in this theory, the perceived slant and, hence, the degree of shape constancy do not depend on the shape itself. Unfortunately, instead of studying this unexpected and important result, that is, the difference between the amount of shape constancy observed with a circle and a square, Thouless concentrated on the less interesting and already well-known result, which was the fact that shape constancy was not perfect with either shape.

In his second paper (Thouless, 1931b), Thouless performed additional experiments to try to explain the failure of shape constancy he observed. This time he used *only* ellipses, the family of stimuli that, because of its simplicity, is most likely to support the "taking into account" principle. In the first experiment, he tested the effect of reducing cues to depth on the accuracy of shape perception. Accuracy was evaluated by varying the aspect ratio of the ellipse (recall that the aspect ratio of an ellipse is the only parameter characterizing its shape). Three viewing conditions were used: (i) binocular, (ii) binocular through a pseudoscope (a pseudoscope reverses the sign of binocular disparities), and (iii) monocular. The results were as follows: Monocular perception of a slanted ellipse was slightly less accurate than binocular perception. Similarly, binocular (direct) viewing led to somewhat more accurate perception than binocular viewing through a pseudoscope. Based on these results, Thouless concluded (p. 4) that

(i) phenomenal regression in the perception of shapes (i.e., shape constancy) is, at least in a large part, determined by the actual presence of cues to slant (i.e., cues that determine the perceptions of the relative positions of the near and far edges of the ellipse);

(ii) when cues to slant are partially eliminated, shape constancy is reduced.

Thouless considered next which factor (familiarity or availability of depth cues) was responsible for the fact that constancy was *not* eliminated completely by the partial elimination of cues to slant. He considered the following two possibilities: (i) Either the subject was able to use the remaining cues to slant, or (ii) the subject relied on the memory of the actual object (figure).

To decide between these two possibilities, Thouless performed an experiment in which the subject viewed a circle under three slants, producing three different ellipses on the retina. Three viewing conditions were used: (i) binocular with the knowledge that the stimulus shape was a circle, (ii) monocular with the knowledge of the stimulus' shape, and (iii) monocular without the knowledge of the stimulus' shape. Thouless tried to remove all depth cues (except, of course, for binocular disparity, in the case of binocular viewing). In binocular viewing, the perceived aspect ratio was slightly greater than the retinal aspect ratio. In the two monocular conditions, however, the perceived aspect ratio was equal to the retinal aspect ratio. That is, shape constancy completely failed in monocular viewing. This result led Thouless to the following conclusions (p. 7):

(iii) Shape constancy is not dependent on the subject's previous knowledge of the actual shape;

(iv) shape constancy depends only on the presence of cues to slant;

(v) in the presence of cues to slant, the percept is equivalent neither to the retinal image nor to the actual shape of the figure but is a compromise between them.

These five conclusions can be generalized as follows: *Cues to slant are both necessary and sufficient for (approximate) shape constancy*. This statement has been commonly accepted by perceptionists as *the* explanation of shape constancy. It is widely cited in introductory psychology and perception texts. There is, however, a fundamental methodological flaw in Thouless' experiments. Recognizing this flaw drastically changes the conclusions that can be drawn legitimately from Thouless' results.

Thouless used the simplest family of shapes (ellipses), and as has been pointed out repeatedly above, the shape of an ellipse is completely characterized by a single parameter, its aspect ratio. It is also important to remember that a perspective projection of an ellipse is also an ellipse. Furthermore, a perspective projection of any 2D shape on the retina affects the shape with two degrees of freedom for a given retinal position and size (see appendix A, section A.1). From these three facts, *it follows that any ellipse can produce any other ellipse at any given place on the retina*. The family of triangles, which is characterized by only two parameters, is the only other family of shapes for which this statement is true. This statement is not true for any other family of 2D (or 3D) shapes, including a relatively simple family like quadrilaterals, which is characterized by four parameters.

What methodological implication follows from using ellipses to study shape constancy? The answer is simple. *Ellipses must not be used to study shape constancy*. The best way to understand this claim is to begin by assuming that shape constancy is a *problem* that has to be solved by the visual system. Consider first the case of a 2D figure slanted in 3D space (the case of a 3D object will be discussed below). A given 2D figure can produce a large number of different retinal images when the figure is presented with different slants. To solve the shape constancy problem, the observer must recognize that these different retinal images can be produced by the same figure. There is a complementary problem. It is called the "shape ambiguity" problem. In this problem, two or more 2D figures, having different shapes and presented at different slants, produce identical retinal images. The observer's problem is to try to recognize which figure produced a given image. Figure 1.4 illustrates that in the case of ellipses, but not in the case of rectangles (quadrilaterals), shape constancy is confounded with shape ambiguity. It is clear that the only way one can solve the shape ambiguity problem is by taking the slant of the figure into account. In other words, if ellipses are used as stimuli, one is forced to employ a "taking into account" mechanism.[4] Clearly, Thouless' subjects had no choice but to "take slant into account," so it is not surprising that Thouless was able to conclude that this was necessary, but note that he did not realize that his subjects *had* to solve the shape *ambiguity* problem, not the shape *constancy* problem. Once this critical distinction is understood the question is whether his conclusions about the importance of slant generalize to shape constancy when the confound with shape ambiguity is removed by using appropriate stimuli. This issue was neither appreciated nor addressed by Thouless. It is worth noting that shape ambiguity, unlike shape constancy, is probably very rare in everyday life because the shapes of many objects are quite different from each other. To the extent that this is true, it seems unlikely that two (or more) different objects, which are not elliptical or triangular, will give rise to identical retinal images. More than a decade would pass after Thouless published his study before a shape experiment would be published that did not confound shape ambiguity with shape constancy. More than half a century would pass before attention would be called to the problems inherent in Thouless' influential, but misleading, experiments. Most of the intervening experiments on shape contained Thouless' methodological flaw. They tested shape ambiguity rather than shape constancy. All of these studies used either ellipses as stimuli (e.g., Leibowitz & Bourne, 1956; Meneghini & Leibowitz, 1967; Leibowitz, Wilcox, & Post, 1978), triangles as stimuli (e.g., Gottheil & Bitterman, 1951; Beck & Gibson, 1955; Epstein, Bontrager, & Park, 1962; Wallach & Moore, 1962), or trapezoids, chosen in such a way that they were perspectively equivalent (Beck & Gibson, 1955; Kaiser, 1967). Not surprisingly, all of these studies confirmed Thouless' result that cues to slant are necessary and sufficient for solving the *shape ambiguity* problem. These authors, like Thouless, thought, erroneously, that their results were relevant to the phenomenon of shape constancy. They were not.

Shape ambiguity can lead to problems in experiments not only with planar (2D) but also with solid (3D) stimuli (see chapter 4). In fact, confusing shape ambiguity with shape constancy when 3D stimuli are used leads to a further, even more serious problem. Specifically, once shape ambiguity is erroneously assumed to be the same phenomenon as shape constancy, it becomes possible for a researcher to completely change the definition of shape constancy. This actually happened. Recall that shape ambiguity is observed when two or more objects having different shapes produce *the same retinal shape*. Obviously, this is not shape constancy. "Shape constancy" refers to the fact that the percept of the shape of a given object is constant despite *changes in the shape of the object's retinal image, caused by changing the viewing direction* (see endnote 1). In order to solve the shape ambiguity problem, the visual system must make use of information other than the *retinal shape*, as the retinal shape is useless because it is the same for all objects. It provides no useful information whatsoever. Shape constancy is different from shape ambiguity because retinal shape is sufficient to solve the constancy problem. Nothing else is needed. Shape ambiguity is completely different. Retinal shape cannot be used to solve the ambiguity problem, so it is not surprising that concentrating on performing shape ambiguity experiments encouraged studying the efficacy of depth cues, context, and familiarity on the percept of the shape of 3D surfaces. The authors of these experiments mistakenly thought that they were studying shape constancy. They were not. These authors thought that they could study shape constancy by trying to find out whether *perceived shape was constant* when they varied illumination, texture, binocular disparity, context, or familiarity (e.g., Johnston, 1991; Doorschot et al., 2001; Nefs et al., 2005; Scarfe & Hibbard, 2006). However, this approach, keeping viewing direction and thus the *retinal shape* unchanged while varying other properties of the visual stimulus, has nothing to do with shape constancy. This mistake shows that these authors did not realize that they were changing the conventional definition of "shape constancy." It should not be surprising, then, that the results of all of these experiments are not relevant to the study of the well-established phenomenon called "shape constancy." Studying shape constancy requires manipulating the viewing direction, which changes the shape of the test stimuli on the retina. One cannot claim to be studying shape or shape constancy when the viewing direction and the retinal shape of the stimuli are kept constant. Shape ambiguity experiments, in which the viewing direction and the retinal shape of the stimuli are kept constant, like those listed above, can only demonstrate the degree to which depth cues and context provide support for using a "taking slant into account" explanation of a subject's behavior in a shape ambiguity experiment. They have no significance, whatsoever, for understanding shape constancy. Unfortunately, this fact is still not generally appreciated, resulting in considerable confusion in the shape literature. Failure to appreciate the constancy–ambiguity distinction has been one of the major millstones on the road to making progress in the study of shape.

1.5 Stavrianos' (1945) Doctoral Dissertation Was the First Experiment to Show that Subjects Need Not Take Slant into Account to Achieve Shape Constancy

Stavrianos did her dissertation under Woodworth's direction. For most of his career, Woodworth had subscribed to the operation of a "taking into account" mechanism (Woodworth, 1938). When Stavrianos published her dissertation, both experimental results and existing theories implied that the perception of a shape is related to the perception of its orientation. Details of the proposed mechanisms differed among researchers, but it was widely held that there was a relationship between the perceived shape and the orientation of an object. Recall that Thouless (1931a, b) claimed that the percept of the shape of an object depends on the perception of the object's orientation (its slant). Others (Eissler and Klimpfinger—see Stavrianos, 1945) also subscribed to this view, but these authors emphasized that the observer does not have conscious access to the slant of the object. In other words, its orientation is automatically registered and used in determining the perception of shape, but its orientation is not "perceived." This emphasis is closely related to Helmholtz' use of the idea of an unconscious conclusion to "explain" a number of perceptual constancies. One implication of this kind of explanation is that there may be no correlation between the perceived shape and the perceived orientation of an object. Koffka (1935), however, claimed that these two properties of a percept must be correlated: "if two equal retinal shapes give rise to two different perceived shapes, they will at the same time produce the impression that these two shapes are differently oriented" (p. 229). Evidence was available to support Koffka's claim, for example, experiments on size perception showed that cues that affect perceived distance also affect perceived size (e.g., Holway & Boring, 1941). Stavrianos assumed, as Koffka had, that a similar relation would exist between the perceived shape of an object and cues to its slant. She did not know whether to expect an exact, as opposed to an approximate, relation or whether the observer would have conscious access to the percept of slant. Stavrianos designed three experiments to answer these questions. Her subjects were required to make explicit judgments about both the shape and the slant of an object (she used the term “tilt” and “inclination” for what we call “slant”). Her study, specifically her Experiment 1, provides a fundamental contribution to our understanding of shape perception, a contribution that has been largely neglected. A relatively detailed description of Stavrianos’ watershed experiment will be provided next. This experiment should have been more influential than it proved to be.

Stavrianos managed to avoid several methodological problems that were inherent in Thouless’ experiments. Even though there is good reason to believe that she did not have a full grasp of the differences between the designs of Thouless’ and her own experiments (see her discussion of her and Thouless’ experiments), she was a much more thorough and systematic experimenter. These admirable traits proved to be critical. On each trial, the subject was presented with a standard rectangle and two comparison rectangles. The comparison rectangles were used to adjust slant and shape to that of the standard rectangle. Specifically, the *slant-variable* rectangle had a constant shape, but its slant could vary. The *shape-variable* rectangle, on the other hand, was always presented in the frontal plane (slant zero), but its shape could vary. The slant of the standard rectangle changed randomly from trial to trial. This rectangle was presented under three “reduction” conditions. Each provided a different number of depth cues. The viewing conditions were (i) normal binocular, (ii) binocular with reduction tubes, and (iii) monocular with a reduction tube. Stavrianos expected, based on preliminary experiments, that reducing cues to depth would substantially harm the accuracy of slant perception. The main question was whether the accuracy of shape perception would deteriorate correspondingly. The subject was asked to adjust first the slant of the slant-variable rectangle and then the aspect ratio of the shape-variable rectangle. By using this order, Stavrianos was trying to facilitate the process of “shape perception by taking slant into account,” as would be the case if such a process actually operated in human perception. The adjustments of the slant-variable and shape-variable rectangles were done under normal binocular viewing. The subjects were asked to adjust the shape-variable rectangle so that it represented the “best bet” as to the actual shape of the standard rectangle. She adopted this instruction from Brunswik (1944), who reported that such an “object-directed” attitude leads to the most reliable and accurate results in perceptual constancy experiments.

Stavrianos’ Experiment 1, unlike Thouless’, did not confound shape constancy with shape ambiguity. That is, her stimuli, rectangles with different shapes, could not produce identical retinal images. This claim follows from the known fact that a single-perspective image of a rectangle, obtained by a calibrated camera (i.e., a camera with known focal length), is sufficient to uniquely reconstruct this rectangle (Perkins & Cooper, 1980, p. 113; Haralick & Shapiro, 1993, volume 2, pp. 80–1). Once this is known, it follows that in Stavrianos’ experiment the shape ambiguity problem did not exist. This geometrical fact has a very important implication: It means that *in Stavrianos’ experiment, the information about the orientation of the standard rectangle was not needed to match the rectangle’s shape accurately; the retinal shape of the rectangle was sufficient to solve the shape constancy problem.*[5]

Note that if Stavrianos had allowed her subjects to adjust more than one parameter of the shape-variable stimulus (as others did—e.g., Kaiser, 1967), she would have introduced the shape ambiguity problem into the experiment and cues to the orientation of shape would have become critical. This follows from the trivial fact that a given retinal image (say, a trapezoid) can be produced by infinitely many different quadrilaterals. Interestingly, it is exactly this aspect of Stavrianos’ experiment, that is, eliminating shape ambiguity, that was criticized by others. Gottheil and Bitterman (1951) said that this was a methodological flaw when, in fact, it represented a fundamental improvement over all prior and many subsequent experiments.

Stavrianos first computed correlation coefficients for the relation between the observed error in slant and the error in shape, for each subject and each of the twenty-four experimental conditions used. She found that “no close relation between the deviations for [slant] and for shape is observable for the separate pairs of judgments” (p. 50). She suggested that the relations may have been weak because, within a single reduction condition, the range of errors was not very large. This led her to analyze the accuracy and variability of slant and shape judgments as a function of her three reduction conditions.

The effect of the reduction condition on the mean of slant and shape judgments is shown in her figures 5 and 6 (pp. 52–3). To avoid the confounding effect of day-to-day variability, Stavrianos plotted only means from judgments where a given slant was presented to the subject for all three reduction conditions on the same day. Stavrianos found that reducing cues to depth led to a systematic deterioration in the accuracy of the slant judgments. All differences in the average adjusted slant between the two extreme reduction conditions (normal binocular minus monocular) were positive and substantially larger than the standard errors, indicating that these differences were statistically significant. The direction of the deterioration in accuracy of slant perception was exactly as expected, that is, reducing depth cues led to a greater underestimation of slant.[6] Now, if the percept of shape was based on information about slant, as prior theories and experiments implied, reducing cues to depth should have led to a systematic deterioration of the accuracy of shape judgments. Specifically, the subjects would have been expected to produce smaller aspect ratios in monocular viewing than in normal binocular viewing. This did not happen. The effect of the reduction conditions on the accuracy of shape judgments was small and not systematic.

Next, consider the effect of reducing cues to depth on the variability (precision) of slant and shape judgments. These results are presented in her figures 11 and 12 (pp. 69 and 70), which show that reducing cues to depth led to poorer precision (higher standard deviations) of the slant judgments. Again, this deterioration of slant perception was not accompanied by a corresponding deterioration of shape perception.

These results on the effect of reducing cues to depth on the accuracy and precision of slant and shape perception are the main contribution of Stavrianos. *They clearly show that perceived shape is not systematically related to perceived slant.* This result contradicts both theories of shape perception popular at that time.

Finally, Stavrianos checked whether there is any similarity between the effect of slant on systematic errors in slant judgments and the effect of slant on systematic errors in shape judgments. She found some similarity only in the most extreme case in which viewing was monocular with a reduction tube. She conjectured that shape is one of several cues to slant (p. 65).[7] When other cues are missing, as was presumably the case in monocular viewing, shape remained the only cue. If slant is perceived by taking shape into account, a correlation between shape and slant judgments is expected. It was found. This important observation has been overlooked by all perceptionists until recently (Pizlo, 1994).

To summarize, Stavrianos, in her main experiment, showed that shape constancy does not involve cues to the depth or to the orientation of the figure presented. This result contradicts any theory in which shape constancy depends on context, including all theories of shape perception that claim that perceived shape is based on "taking slant into account." It also contradicts Koffka's claim of an invariant relation between perceived shape and perceived slant. These important implications of Stavrianos' results were overlooked by many perceptionists, including Stavrianos herself (see Hochberg, 1972, for one of very few exceptions). Thouless' results on the perception of ellipses were widely, perhaps even universally, considered to have established the fact that shape constancy requires depth cues. Stavrianos' results clearly contradicted the established wisdom about the relation of shape and slant. However, note that there was actually no contradiction because Thouless had actually studied shape *ambiguity*, showing the trivial fact that information about depth is critical for disambiguating shapes under his conditions. Stavrianos, on the other hand, actually studied shape *constancy*, showing that information about depth is not important for veridicality. Stavrianos' failure to draw appropriate conclusions from her results, and her failure to reject existing shape theories, could be, at least in part, related to the absence of an alternative plausible theory at the time. But at least one thing is clear—namely, that Stavrianos did not understand the nature of the differences between her and Thouless' experiments. This limitation is not as disturbing as one might think because even if she had understood this difference and presented it clearly, progress in the study of shape perception would probably not have been possible in her day.

What alternative theories of shape were available in this period? The philosopher Cassirer (1938) and the mathematicians Courant and Robbins (1941) were the first to conjecture that the perception of shapes from perspective images can be explained by projective invariants (see sections A.2–3).[8] Nothing in her published work suggests that Stavrianos was aware of the idea of geometrical invariants. However, even if she had been, this, in itself, would probably not have led her to propose a new theory of shape constancy because projective (or any other conventional) invariants cannot account for her results. In projective geometry, all rectangles (and in fact all quadrilaterals) are equivalent. It follows that if shape perception were based on projective invariants, all rectangles would have been perceptually equivalent and, thus, Stavrianos, like Thouless, would have ended up studying shape ambiguity, rather than constancy. In short, her results would have been identical to Thouless'! They were very different.[9] In order to explain Stavrianos' results in terms of geometrical invariants, a new class of invariants had to be discovered. These invariants, called "perspective invariants," were first formulated by Pizlo and Rosenfeld (1990, 1992; see section C.9). They can account for existing and new results on shape constancy of 2D figures (Pizlo, 1994). Perspective invariants belong to a more general class of model-based invariants that can be applied to other cases of transformations, which do not form groups (Weinshall, 1993; Rothwell et al., 1993; Pizlo & Loubier, 2000). Thus, Stavrianos was about fifty years ahead of her time. This probably explains why her experiment did not have the influence it deserved.

1.6 Contributions of Gestalt Psychology to Shape Perception (1912–1945)

A great deal of progress toward understanding shape perception has been made since the late 1960s. This progress derived, in large part, from the contributions of the Gestalt psychologists after their work had been incorporated into the Cognitive Revolution. Their main contribution was providing compelling evidence that the perception of shape is produced automatically from the relations among the elements present in the retinal image. They did this by applying what they called the "Laws of Perceptual Organization" (Wertheimer, 1923). The most important of these laws of organization was called "figure–ground organization." It refers to the fact that closed contours establish special closed regions in the percept that correspond to objects in the visual scene. These regions, which were called "figures," are perceived as lying in front of the "background." N.B. that contours always belong to the objects (figures), never to the background. Thus, objects have shapes; backgrounds do not. Figure–ground organization is illustrated in figures 1.2 and 1.3. Clearly, it is easy to see individual objects. They are not confounded with each other or with the background. Furthermore, figure–ground organization is as obvious in the line drawings as in the photographs. Also, note that not only individual objects are seen. The 3D spatial relations within and among the objects are seen, as well. Realize that the 2D image present on your retina when you view figures 1.2 and 1.3 is sufficient to establish veridical 3D percepts of this natural scene. Thus, once the contours of objects are identified in the image, their shapes can be described and then used to solve the shape constancy problem. It will be shown later that the percept of a 3D shape is produced by applying invariants and constraints to the 2D retinal shape. This means that the percept of a 3D shape can be produced only after the contours of the figures have been established. This is why figure–ground organization is so critical. *If the figure–ground organization process fails, invariants and constraints cannot be applied and shape constancy must fail*. This is arguably the most fundamental characteristic of human shape perception. It will be discussed in considerable detail in this book. Unfortunately, the Gestalt psychologists did not elaborate on the relation between shape perception and figure–ground organization. This oversight probably explains why they did not do much to advance the study of shape perception.

The Gestalt psychologists also offered support for their claim that the Laws of Perceptual Organization are innate by showing that one need not learn to perceive such fundamental properties as motion, lightness, or shape. Note that shape, unlike other stimulus properties, is a "Gestalt quality" and, as such, does not depend on the nature of the elements producing it. In fact, the perception of shape, because it is an emergent perceptual property, is arguably the best way to illustrate the uniqueness of the Gestalt contribution to the study of perception. However, Gestalt psychologists, as everyone else at the time, did not appreciate that shape, unlike other perceptual characteristics, has a high degree of complexity. This property permits different shapes to be discriminated easily. Instead of exploring the ways in which shape is special, they concentrated on relational characteristics within the percept, for example, they studied perceived size as a function of perceived distance, and perceived shape as a function of perceived slant. This emphasis on "higher order" variables prevented the Gestalt psychologists from noticing the fundamental problems with Thouless' experiments, an oversight that, in turn, prevented them from rejecting Thouless' "taking slant into account" explanation of shape constancy. Had they appreciated this problem, they might have tried to develop a theory of shape perception based on existing knowledge of projective geometry. Despite this failure, the Gestalt psychologists did pave the way for an explanation of shape perception. Appreciating how they did this requires more than a fleeting discussion of the origin and nature of the Gestalt approach to perception.

The main idea of Gestalt psychology was anticipated by the philosopher Christian von Ehrenfels (1890), who introduced the terminology, as well as the concept, called *Gestaltqualität* (form quality). He pointed out that as soon as there are three elements in the visual field that do not fall on a line, a Gestalt quality emerges, namely, the shape we call “a triangle.” The nature, size, or orientation of the three elements does not matter. Once there are three noncollinear elements, a triangle is perceived. If there are four elements, a quadrilateral will be perceived, and so forth. Von Ehrenfels not only pointed this out, and gave this phenomenon the name that was adopted by his students, the Gestalt psychologists, but he also introduced the use of the term “transposition” into the study of perception. This term refers to the fact that a Gestalt quality, such as a triangle, remains when the elements producing it are replaced with other elements or when the elements are translated or rotated. Thus, the properties of the elements composing a shape are not important; only relations among the elements are.

The Gestalt school of perception emphasized these relations and called them “configurations.” This is in stark contrast to the Associationists’ approach to shape perception. The Associationists emphasized the importance of mental elements they called “sensations.” They claimed that the shapes of objects, revealed by the relations among elements, had to be learned by manipulating objects while they were viewed. This claim was at the center of the Gestalt psychologists’ attack on the Associationists represented by Helmholtz’ student Wilhelm Wundt and his English-speaking student Edward Bradford Titchener, who led the Associationist approach in Europe and in America when Max Wertheimer launched the Gestalt Revolution in 1912 (see Heidbreder, 1933, for a good description of these “schools of psychology” and their controversies). Koffka, one of the three founding “fathers” of the Gestalt Revolution called the Associationists’ rejection of the possibility of relations (interactions) among elements in the retinal image the “constancy hypothesis.” In his words, this hypothesis *"implies that all locally stimulated excitations run their course without regard to other excitations"* (Koffka, 1935, pp. 96–7).[10]

Gestalt psychologists contributed another idea that occupies a prominent place in contemporary research on shape perception, specifically, they placed emphasis on the importance of simplicity in determining the nature of a given percept. Their simplicity principle is analogous to what is called "a minimum principle" in physics. They embodied their simplicity principle in their Law of Prägnanz (*Prägnanz* in German means "succinctness, conciseness, or terseness"). Koffka (1935, p. 110) described this law as follows: *"perceptual organization is always as good as the prevailing conditions allow,"* where good means regular, symmetrical, or simple. An example illustrating the operation of this simplicity principle is shown in figure 1.6. Here, at least two different interpretations are possible. One is a pair of vertical lines with a symmetrical figure between them. The other is a superposition of the letters *M* and *W*. The letters are superimposed in such a way that parts of the *M* and parts of the *W* form longer, continuous lines and a closed form. Despite our familiarity with the letters *M* and *W*, and indeed, despite our knowledge that this interpretation is possible, it is much easier and more natural to see two longer lines and a closed figure between them than the letters *M* and *W* (after Koffka, 1935, p. 155). The "good figure" prevails. The Gestalt psychologists called the perceptual organizing principles responsible for this result "good continuation" and "closure."

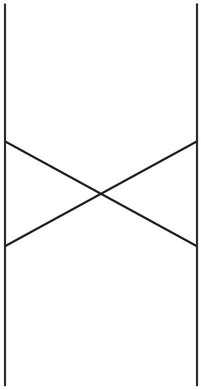

Figure 1.6
It is not easy to see the letters *M* and *W*. Good continuation and symmetry lead to a different (simpler) perceptual organization (after Koffka, 1935).

The Gestalt psychologists also introduced a number of other ideas that influenced subsequent theories of shape perception in more indirect ways, for example, Blum's (1973) symmetry axes and "grassfire" model, Grossberg's (Grossberg & Mingolla, 1985) neural network model, and Ginsburg's (1986) spatial filter model. All of these approaches derive from simple assumptions about relationships between the nature of the percept and its underlying physiological cause. The Gestalt psychologists took this on very early in the development of their theory. Their interest is embodied in what they called "psychophysiological isomorphism." For example, if one perceives an object as moving, one could claim, as Wertheimer (1912) did, that the underlying brain process consists of moving excitation within the brain tissue.[11] Gestalt psychologists claimed that the relations between the percept and its physiological correlate were topological, not metric. That is, only neighborhood relations are preserved; the distances between elements are not. Gestalt psychologists embodied their isomorphism principle in an electrical brain currents model. Specifically, a minimum state of the brain currents was supposed to provide a physiological explanation of the simplicity principle responsible for perceptual organization (Köhler, 1920). Köhler, who was the third founding father of Gestalt psychology, made extensive use of the simplicity principle in developing the concept of psychophysiological isomorphism. He had a background not only in psychology but also in physics and mathematics. He was quite familiar with the use of a minimum principle, the kind of principle that allowed elegant formulations of laws in physics (Lanczos, 1970). For example, the Fermat principle, according to which light travels along the path that *minimizes* the time of travel,[12] allows derivation of the laws of reflection and refraction in optics. Another example is Kirchhoff's laws for electrical circuits, that is, given a circuit with a voltage source and resistors, the currents in the branches of the circuit will be such that the total amount of heat generated in the resistors is *minimal*. According to Köhler, similar simplicity mechanisms were likely to apply in perception. For Köhler, the steady state of electrical brain currents was a plausible physiological counterpart (a cause) of perceptual simplicity (Köhler, 1920). Köhler's claims, which may seem far-fetched to many today, fitted in well with the zeitgeist of his time. We are now sure that electrical brain currents do not provide a plausible physiological model of perception as Köhler proposed, but in his day, such speculations were quite reasonable.[13] The Gestalt psychologists' insistence on the importance of inherited, built-in, perceptual organizing principles and on simplicity constraining the nature of perceptual organization continue to be important. Next, we will consider how their simplicity principle fared following its introduction.

1.6.1 The Role of Simplicity in the Perception of Shape

If we accept the claim that the simplicity principle is critical in determining the percept, it becomes important to know what we mean by "simplicity." It is one thing to state that a given percept is simple, after the percept has been observed, and quite another to predict what the percept will be like before it occurs. A precise formulation of simplicity was not available until information theory was formulated by Shannon in 1948. Until then, attempts to provide a Gestalt-inspired theory of shape perception were doomed to fail despite the fact that the Gestalt emphasis on inbuilt automatic perceptual organizing processes had promise. Their attempts to develop a theory of shape perception will be illustrated by describing how Koffka dealt with the fundamental question about shape first posed by Wertheimer in 1923.

1.6.1.1 Why Do Things Look as They Do Koffka's (1935) approach to perception, in general, and to shape perception, in particular, begins by raising this question (p. 75). He discusses two commonsensical answers: (i) Things look as they do because they are what they are, and (ii) things look as they do because their retinal images are what they are. Neither answer is accepted by Koffka, who turns next to the answer proposed by the Associationists, who held that the nature of the percept is determined by an unconscious conclusion in which learning and associations play the crucial role. Koffka does not accept this empiristic answer on the grounds that it does not offer an adequate explanation of the fact that human and animal infants, who have had very little time for learning, demonstrate many perceptual constancies.

Koffka's "true answer" to his question "Why do things look as they do?" is that things look as they do because this is the simplest interpretation of their retinal image (p. 98). Koffka's answer begs the question because he could not provide a formal operational definition of simplicity. This forced him to speculate about the physiological basis of simplicity. He did this by invoking what the Gestalt psychologists called "field forces." These forces were associated with the electrical activities of the brain. The brain field, according to Koffka, involves the operation of two forces, one internal and one external. The external force is produced by the proximal stimulus (the retinal image). The internal force is produced by the brain's tendency toward simple percepts. Each force is represented by electrical brain currents whose steady state (the minimum state) is the physiological correlate of the percept. The two forces interact. If both forces act in the same direction, as would presumably be the case when the retinal image is simple—for example, a circle—the percept is stable. When the two forces are in strong conflict, as would be the case when the retinal image is an irregular shape, the percept should be unstable (Koffka, 1935, pp. 138–9). For Gestalt psychologists, the circle was special. It was the simplest, most regular shape. It was the simplest 2D shape because the given surface area was enclosed by the minimal-length, therefore simplest, contour. In other words, the circle is the most "compact" 2D figure, because a contour of a given length encloses the maximal area. This property of "compactness" will take on considerable significance later when we move to consideration of 3D shapes, where a sphere will become the simplest figure. An example of a simplicity principle operating in three dimensions can be illustrated by a soap bubble: A soap bubble will always take on a spherical shape because a sphere requires the minimum amount of energy to maintain its integrity. Note that it is difficult to demonstrate the operation of a simplicity (minimum) principle with 2D shapes, the case most often discussed by Koffka (1935). Koffka, and others who were sympathetic to Gestalt ideas, had little success when they tried to demonstrate the operation of a minimum principle by measuring absolute thresholds of 2D shapes. It is much easier to demonstrate the application of a minimum principle with 3D shapes. This is how it can be done.

1.6.1.2 Perception of 3D Shape from 2D Images *Koffka's treatment of the perception of 3D shapes from 2D images begins with the following statement:* "three-dimensional shapes are matters of organization in the same way as two-dimensional ones, depending on the same kind of laws" (p. 161). *What Koffka is saying is that 3D percepts, like 2D percepts, derive from built-in automatic organizing processes. They do not depend on learning. The 3D percept, like the 2D percept, reflects the operation of a simplicity principle.* He supports this claim with a discussion of Kopfermann's (1930) and Schriever's (1925)

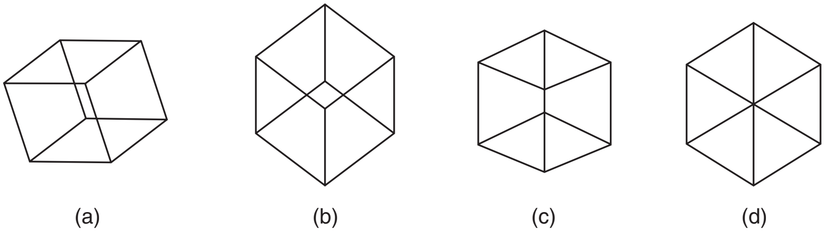
(a) (b) (c) (d)

Figure 1.7
Kopfermann's stimuli (from Kopfermann, 1930, p. 298, figure 1—with kind permission of Springer Science and Business Media).

experiments. They studied 2D representations of solid objects whose percepts were "simpler" when they were viewed as 3D objects.

Kopfermann used a set of orthographic projections of a cube. The subjects were shown line drawings like those in figure 1.7. The four figures are different with respect to regularity or simplicity as measured by topological and metric properties. Specifically, these figures differ with respect to the number of points of intersection of the lines contained in each figure, as well as with respect to the lengths of their line segments and the sizes of their angles. The subjects in Kopfermann's (1930) study were asked whether they saw a 2D (planar) figure or a 3D (solid) object. The figures in (c) and (d) usually led to the percept of a planar (flat) figure, identical with the figure itself. The figures in (a) and (b), on the other hand, usually led to the percept of a cube.[14] These observations allowed her to conclude that the percept is equivalent to the 2D projection when this projection is simple as in (c) and (d). However, when the 2D projection is more complex, as in (a) and (b), the "simpler" 3D object is seen. In fact, the percept produced by (a) or (b) corresponds to a cube, the most symmetrical 3D object consistent with these figures' retinal images. This fits nicely with Koffka and his Gestalt colleagues' conjectures about the important role of simplicity in shape perception. Note also that, as Koffka pointed out, the percept in figure 1.7a corresponds to a 3D regular (simple) figure despite a powerful conflicting cue (external force), namely, binocular disparity, which should tell the observer that the stimulus is actually flat (Koffka, 1935, p. 161).

A related experiment was performed by Schriever in 1925 (see Koffka, 1935, pp. 274–5). He used a meaningless 3D object (figure 1.8) composed of three connected bars located in different depth planes. He then

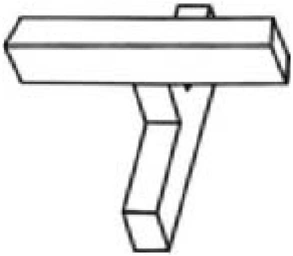

Figure 1.8
Schriever's stimulus (after Koffka, *Principles of Gestalt psychology*, Harcourt Brace 1935, figure 82, p. 274—with kind permission of the publisher).

introduced a conflict between binocular disparity and perceptual organization of the meaningless 3D object's retinal image. He did this by photographing the object from slightly different viewing directions and then using a stereoscope to reverse the bar's order in depth by presenting the left image to the right eye and the right image to the left eye. If the binocular percept of this 3D meaningless object were determined by binocular disparity, the subject should have perceived the parts of the object with their order in depth reversed. Instead, the subject perceived the depth order given by the available monocular perceptual grouping principles, namely, good continuation and occlusion. Binocular disparity had little, if any, effect. This is a nice example of the way Gestalt psychologists determined the relative importance of various cues and rules of perceptual organization. Here, good continuation (internal force) was found to be more important than binocular disparity (external force). Another way of stating Schriever's result is to say that *the perceived 3D shape had more to do with the 2D shape on the retina than with the depth relations indicated by binocular disparity*.

The Gestalt psychologists did not say much more about the perception of 3D shape produced by 2D retinal images than described above. This is unfortunate because the relation between a percept of a 3D object and its 2D retinal image is critical, especially when the role of a simplicity principle in shape perception is considered. Once the 2D shape on the retina is established through the operation of figure–ground organization, the simplicity principle is involved in producing the 3D shape percept. The simplicity principle is not needed to *simplify* the percept of the 2D retinal shape. It follows that invocation of a simplicity principle is not (i) needed or (ii) productive when one confines discussion to the perception of 2D (planar) shapes, which are abstractions. N.B., *there are no 2D objects, no matter how thin*. Unfortunately, this canon has not been and is not widely appreciated, an oversight that has led to a great many pointless experiments, controversies, and erroneous conclusions. Its neglect has been as big an obstacle to the development of an adequate theory of shape as Thouless' naiveté with respect to the projective properties of triangles and ellipses proved to be in the study of shape constancy.

1.6.1.3 Studies of Shape Thresholds The interest in shape thresholds was initiated by Goethe, who reported that an afterimage of a square becomes more and more circular with the passage of time (Koffka, 1935, p. 143). Koffka made note of this and went on to claim that internal forces, which bias a percept toward simplicity, compete with external forces produced by the square because it is not as simple as the "simplest form," that is, a circle. In other words, the afterimage of the square weakens over time and the resulting percept becomes more distorted in the direction of greater simplicity. It comes to be perceived more and more like the simpler, "best," figure, the circle.

Gestalt psychologists used several methods to test the conflict between internal and external forces by using (i) short exposure times, (ii) low contrast stimuli, (iii) small targets, and (iv) afterimages. These methods were believed to provide a means of studying how competition between internal and external forces determined the percept. In other words, these methods allowed them to study the relative importance of the simplicity principle vis-à-vis properties of the retinal image.

Studying weak external forces lends itself naturally to the measurement of both detection and identification thresholds. The main prediction of the Gestalt theory was that simple shapes would be easier to detect and identify than complex shapes. This line of research led to a large number of studies published over the span of more than four decades, beginning in the early 1920s. The interested reader should consult reviews by Hochberg (1972, pp. 443–4), who was involved in this research himself, and also Zusne (1970, pp. 265–9, 304–7), who was not. The results are, at best, inconclusive: (i) Decreasing the "strength" of the stimulus does not necessarily lead to simpler percepts, (ii) the luminance threshold for detecting shapes does not depend systematically on the nature of the shape itself, and (iii) complex shapes do not necessarily require longer exposure duration for successful shape identification. Today, no one studies shape thresholds. This is not because the questions leading to these experiments have been answered, but because these experiments have not been productive. These experiments were based on an implausible assumption, namely, that the percept arises from a conflict between simplicity constraints and properties of the retinal image. This kind of psychodynamical approach has fallen out of favor. We now make a quite different assumption, namely, that *the simplicity principle is incorporated in perceptual mechanisms in order to make up for information lost due to (i) the projection from the distal to the proximal stimulus and (ii) the presence of noise in the visual system*. This assumption leads to different kinds of experiments than those performed by the Gestalt psychologists (see section 4.3).

To summarize, the Gestalt psychologists' preference for studying 2D shapes, as well as their commitment to their physiological model of Prägnanz, did not produce useful results on shape perception beyond the work of Kopfermann and Schriever. The Gestalt psychologists, as well as everyone else until recently, simply did not appreciate the fact that the only way to study the role of simplicity in shape perception is to use 3D objects (or 2D images of 3D objects). Stimuli with only two dimensions are too impoverished to reveal how simplicity constraints can compensate for information lost in the projection from distal to proximal stimulus. Gestalt psychologists were on the right track in emphasizing the role of simplicity in perception, but they failed to provide a formal definition of what they meant by "simplicity." Obviously, they cannot be faulted for failing to do so because information theory, the theory which deals with the concept of simplicity, was not formulated until 1948 (Shannon, 1948). A revival of the Gestalt approach took place in the early 1950s. This revival was stimulated by the formulation of information theory and by the shift of emphasis from research with 2D shapes to research with 3D shapes. Once these were in place, it became possible to make major advances in the study of perception and cognition. These changes led to what had been called the "Cognitive Revolution."

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