The Iconic Logic of Peirce's Graphs

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For Henry

Contents

Acknowledgments ix

1 Introduction 1

2 The Birth of Graphical Systems 13

2.1 Preliminaries 14

2.1.1 Graphs and the model-theoretic tradition 14

2.1.2 Graphs as a formal system 17

2.2 Diagrammatic reasoning 19

2.3 The theory of signs 22

2.3.1 Symbols, indices, and icons 23

2.3.2 Symbolic versus iconic signs 27

2.4 Heterogeneous formal systems 31

3 Existential Graphs as a Heterogeneous System 37

3.1 Introduction of Existential Graphs 37

3.1.1 Alpha graphs 37

3.1.2 Beta graphs 39

3.2 The symbolicity of Existential Graphs 48

3.3 The iconicity of Existential Graphs 53

3.3.1 Lines of identity 53

3.3.2 Existential versus universal quantifiers 56

3.3.3 Scope of quantifiers 57

4 The Alpha System Reconsidered 59

4.1 "Endoporeutic" reading 61

4.2 "Negation normal form" reading 63

4.2.1 Conjunctive and disjunctive juxtapositions 64

4.2.2 Semantics 68

4.3 Multiple readings 71
4.3.1 Scroll as conditional 72
4.3.2 The Multiple Carving Principle 76
4.4 Transformation rules 80
4.4.1 A natural deductive system versus the Alpha system 80
4.4.2 The rules reformulated 83
4.4.3 The rules reinterpreted 88
4.4.4 Efficacious graphical systems 90
4.5 Sentences versus graphs 93
4.5.1 Translation of sentences into graphs 93
4.5.2 Applications: logical equivalence and NNF 95
4.5.3 Visual efficiency 97

5 The Beta System Reconsidered 99
5.1 Preliminaries 100
5.1.1 Zeman's reading 100
5.1.2 Roberts' reading 111
5.2 A new reading 118
5.3 Transformation rules 134
5.4 Appendix: direct semantics 150

6 Logical System versus Calculus 163

7 Conclusion 171

Notes 175
Bibliography 197
Index 205

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1
Introduction

A fresh approach to reasoning and representation has been emerging at the crossroads of the philosophy of mind, cognitive science, logic, and computer science.[1] Our ordinary reasoning typically involves information obtained through more than one medium—sentences, diagrams, smells, sounds, and so on. Recognizing the actual practice of this *multi-modal* reasoning, researchers have started focusing on multi-modal, or *heterogeneous*, representation systems, which employ both symbolic and diagrammatic elements. This is a clear departure from the major direction taken by logicians and mathematicians since the development of modern logic: For more than a century, symbolic representation systems have been the exclusive subject for formal logic.

This book analyzes a well-known, but much-criticized, non-symbolic representation system, Peirce's system of Existential Graphs (henceforth, 'EG'), and presents a new approach to EG based on the discovery of its unique nature and on the reconstruction of Peirce's theory of representation. I will explore specific differences between symbolic and diagrammatic systems both in reading-algorithms and in the formulation of inference rules, and I will draw out the implications of these results for several long-standing debates in various disciplines that study multi-modal systems. Before locating this work within the overall picture of research into heterogeneous reasoning, it will be useful to re-examine two related major assumptions which underlie the general heterogenous reasoning project. One is the shared view that there has until quite recently been a long-standing prejudice against non-symbolic representation in logic, mathematics, and computer science. The other is the assumed distinction between symbolic and diagrammatic systems. I do share the view that there is a prevailing bias against non-symbolic systems, and I do assume that symbols are different from diagrams. However, I believe that in order to achieve further fruitful results the time has come to raise more fundamental questions about the main assumptions behind this fast-growing body of research into heterogeneous systems.

In the present work, I would like to emphasize the following circular relation between these two points—the strong preference for symbolic over diagrammatic systems and the distinction between these two types of systems. Without a solid theoretical background for the distinction between symbolic and diagrammatic systems, we easily overlook different kinds of strength and weaknesses that each type of system possesses. Then, given the general predominance of symbolic systems in the study of logic, we tend to try to understand and evaluate diagrammatic systems against the criteria of a symbolic system. As a result, without their own strengths being discovered, diagrammatic systems have been criticized mainly because they lack the properties of a symbolic system. In turn, this unfortunate result—stemming partly from a prejudice against diagrams and partly from an unclear distinction between symbols and diagrams—only reinforces the existing prejudice against non-symbolic systems. I claim that these two main phenomena reinforce each other in a vicious circle.

Where do we need to step in to break this circle? First of all, we should stop evaluating diagrammatic systems in terms of symbolic systems. This would be possible only if we could develop an independent way of approaching diagrams. Again, an independent method cannot become available without identifying the positive properties that diagrammatic systems uniquely possess.

In this book, I examine Charles S. Peirce's "Existential Graphs" as a case study to support my claim that a prejudice against diagrammatic systems reinforced a superficial distinction between symbols and diagrams and vice versa, and to show how this vicious circle can be broken. Let me briefly explain below why Peirce's graphical system provides us with an exceptionally fitting case study for our present inquiry.

At the dawn of modern logic, Charles S. Peirce invented two different types of logical systems, one symbolic and the other graphical. Why has the symbolic system absolutely dominated the other in the subsequent development of logic, despite the fact that Peirce himself considered EG superior to his symbolic logical system?[2] We should be careful not to answer this question by citing the existing prejudice against diagrammatic systems. That would not explain much about the nature of the prejudice we are interested in investigating. More importantly, complaints against EG are different from complaints against the use of diagrams in proofs. Diagrams, critics say, are not rigorous enough to be used in a proof or tend to mislead us in a proof. Accordingly, diagrams are used with caution and only as heuristic aids. However, in the case of EG, logicians accept that EG is sound and complete. In spite of its rigor, EG has not been adopted for deductive reasoning. In this case, the criticism is more specific: The graphs of EG are difficult to read off, and the rules of EG difficult to use.

Why does a translation of a graph of EG almost always result in a complicated-looking sentence? Why is EG harder to use than natural deductive systems in spite of the apparent similarity between how these two systems are set up? Surprisingly, I find the same source responsible for these two puzzles: Despite the fact that EG is known to be a graphical system, the iconic features of EG have not been fully implemented either in any existing reading algorithm or in how the transformation rules are stated. This deficiency has yielded complicated and unintuitive reading methods and produced transformation rules that are not fine-grained enough to be used conveniently. A somewhat ironic, trivial-sounding but unobserved lesson follows: When a system is heterogeneous or diagrammatic, its iconic features should be fully utilized both in its syntax and semantics. Otherwise, many kinds of inefficiency will arise. Therefore, the existing criticisms against EG are the necessary product of the traditional, less then fully iconic way of approaching EG, and I argue that my new ways of understanding EG disarm the main criticisms of EG.

What, then, is the main difference between the traditional method and the new method I will present in this book? I will show that a strong dominance of symbolic over diagrammatic systems is present in how we traditionally have understood EG: Graphs of EG are read off very much as symbolic sentences are, and the inference rules of EG are understood as being like the rules of natural deductive systems. This misguided assimilation of EG into a symbolic system, I claim, is the most fundamental source for the existing complaints against EG, which, in turn, supports a bias against non-symbolic systems. Here we confirm the existence of the vicious circle pattern. In order to correct this situation, I will rely on my discovery of the fundamental differences between EG and a logically equivalent symbolic system in two major respects.

First, I demonstrate how differently a meaningful unit of each system can be read off. In order to prevent ambiguity, the semantic interpretation of a symbolic sentence requires its unique readability, and hence no possibility of multiple readings. On the other hand, in the case of EG, multiple readings do not generate ambiguity. Moreover, multiple readings for one and the same graph are the most *natural* since we perceive a graph differently depending on how we carve it up. Importantly, we may obtain a comprehensive algorithm of multiple readings when we utilize more fine-grained visual features present in graphs. Second, I claim that the *naturalness* of the inference rules of a system is a relative concept. In the case of a natural deductive system, inference rules are stated so as to reflect the derivational history of a formula. However, in the case of EG, since so many different derivations are available for a graph, we should free the transformation rules of EG from any derivational history of a graph and look for a type of naturalness appropriate to a graphical system.

To find this type of naturalness both in the reading of graphs and in how inference rules are set up, the current work presents a new reading method of graphs and a newly formulated system of EG. Hence, we may meet the traditional objections to EG—that the system is hard to read off and hard to manipulate. Furthermore, this case study offers a new approach to non-symbolic systems in general. It urges heterogenous-system researchers to ask whether they fully utilize visual aspects of a non-symbolic system for a direct and natural reading algorithm, for an efficient formulation of inference rules, and for an intuitive interpretation of these rules. If we do not take advantage of visual distinctions already present in a system, we can only expect a graphical system to be less useful or less intuitive than the symbolic system whose criteria have been used to interpret the graphical system. Unless we adopt independent methods or criteria for a graphical system, we cannot challenge the long-standing prejudice against non-symbolic systems, but only reinforce it.

This project draws on results from various disciplines concerned with diagrams. A distinct role that diagrams or pictures—as opposed to traditional linguistic forms—play in our cognition has been extensively discussed in several different areas, including philosophy, cognitive science, logic, artificial intelligence, and design theory. After outlining how various disciplines pursue this topic from slightly different points of view and how the relation between diagrams and representation (or reasoning) has been emerging as an interdisciplinary topic out of these different approaches, I explain how the project of this book fits in this overall picture.

Among many different approaches to the relation between diagrams (or pictures) and our cognition, I would like to draw attention to an interesting and useful distinction between those approaches that focus on diagrams as internal representations and those that treat them as external representations. In the Introduction to one of the most comprehensive anthologies on this subject, Chandrasekaran, Glasgow, and Narayanan (1995) make the following distinction between internal versus external diagrammatic representations:

- *External diagrammatic representations* These are constructed by the agent in a medium in the external world (paper, etc), but are meant as representations by the agent.
- *Internal diagrams or images* These comprise the (controversial) internal representations that are posited to have some pictorial properties.[3]

From now on, unless I specify otherwise, I will use the words 'diagrams' or 'pictures' to refer to external representations, and 'images' to refer to internal representations. Of course, many do not doubt that these two different levels of representation are closely related to each other.[4] Moreover, this distinction is not needed at all for certain projects, and hence some researchers conflate internal and external representations.[5] However, this distinction will nicely serve as a useful framework which I would like to use to show how different areas falling under the category of research on diagrammatic reasoning are related to one another.

The imagery debate between pictorialists and descriptionalists, one of the most time-honored controversies in psychology, focuses on our internal mental representation: It is about whether picture-like images exist as mental representations.[6] Kosslyn and other pictorialists[7] present a series of experimental data to support their position that some of our mental images are more like pictures than a linear form of language (for example, natural languages or artificial symbolic languages) in some important aspects, even though not all visual mental images and pictures are exactly of the same kind. By contrast, Pylyshyn and other descriptionalists[8] raise questions about the status of picture-like mental images and argue that mental images are formed out of structured descriptions. To them, mental images represent in the manner of language rather than pictures, and hence no picture-like images play an important role in our cognition.[9]

At this point we are not far from philosophical territory—the philosophy of mind. Philosophers' deep interest in mental representation easily goes back to ancient times.[10] Nobody would be surprised to realize that mental images were heatedly discussed during the heyday of ideas.[11] As we know, Hobbes', Locke's, Berkeley's, and Hume's writings concern themselves in large part with mental discourse, the meaning of words, mental images, particular ideas, abstract ideas, impressions, etc. Descartes' well-known distinction between imagining and conceiving something has generated much discussion about the unique role of visual images in mental representations. In the twentieth century, pictorialists in the imagery debate found the modern sense-datum theory in philosophy quite close to their point of view. By the same token, the critics of the sense-datum theory argued that the mistaken pictorial view of mental images arises mainly from our confusion about ordinary language. Not surprisingly, they are sympathetic to the view that mental images are epiphenomena.[12] Contemporary philosophers, mainly in the philosophy of mind,[13] have participated in a recent imagery debate among cognitive scientists.[14]

Being slightly distant from the imagery debate itself, some cognitive scientists have concentrated on the functions of mental images or diagrams in our various cognitive activities, for example, memory,[15] imagination, perception, navigation,[16] inference, problem-solving,[17] etc., instead of exploring the ontological status of internal visual images.[18] Here the distinct nature of "visual information," which is obtained either through internal mental images or through externally drawn diagrams, has become a major topic of research. In particular, research on heterogeneous reasoning focuses on how visual information plays a unique role in our reasoning process. In the following, let me outline three important aspects of this research, all of which are crucial to understand the background of my project.

First, when the distinct role of visual information in inference or reasoning becomes a main question, we find an interesting shift of focus among researchers from internal to external representations, while we do not find a similar tendency in other subareas of research on images and cognition. For example, the study on the relation between memory and visual information focuses on internal as well as external visual information. However, in studies of inference, 'images' or 'diagrams' mainly refers to external representations, i.e., drawn pictures, graphs or diagrams, rather than picture-like mental representations.[19] In Larkin and Simon's classic paper "Why a diagram is (sometimes) worth ten thousand words" (1987), this change was made very clear: "Although our discussion [about the unique nature of diagrams in problem solving] may be relevant to this current controversy about the distinguishability of different internal representations, our analysis explicitly concerns external representations."[20] One benefit from this shift is that although consensus is lacking on whether there are different kinds of internal mental representations (as the long history of the imagery debate shows), everyone agrees that there are different forms of external representations.[21]

Second, research on heterogenous reasoning has attracted much more attention from various different disciplines than has research on the relation between imagery and other cognitive activities—for example, memory or perception—mainly for two reasons. One is that human reasoning is a common topic among cognitive science, logic, mathematics, and artificial intelligence. The other reason for interdisciplinary interest in imagery and reasoning is that externally drawn diagrams or graphs, on which researchers have focused can, be a subject of all of these disciplines, unlike mental images.

Third, research on multi-modal representation has led researchers to explore the differences among different forms of (external) representations, but mainly between diagrammatic and symbolic representations.[22] A strong dominance of symbolic languages in the study of representation systems (since the dawn of modern logic) compels anyone who is working on the relation between diagrams and inference to compare two different kinds of languages, that is, graphical and symbolic systems. In spite of a common interest in heterogeneous representation, various disciplines have pursued the same topics—here the relation between diagrams and reasoning, and the comparison between symbolic and graphical systems—from different points of view. After briefly summarizing the slightly different agendas of various disciplines, I will show how the work presented in this book serves to bridge the gap among them.

It has been a while since cognitive scientists started paying attention to how different forms of representation vary in their cognitive effects on human inference. Many important results have been produced along these lines. Based on Simon's distinction between informational and computational equivalence among representations,[23] Larkin and Simon (1987) present a case study in which two informationally equivalent systems, one sentential and the other diagrammatic, are shown to be computationally non-equivalent. Lindsay (1988) makes a related point by specifying where this computational difference lies. Claiming that an important role of diagrammatic representation[24] in inference is not its expressive power, but its efficiency, he showed that this efficiency is obtained through the special properties that diagrams possess. Constraints built into the diagram-construction processes rule out many trivial cases, and, after the construction is completed, conclusions are directly read off from a diagram.[25] Shimojima (1996) uses term 'free ride' to refer to an inference in which the conclusion seems to be read off almost automatically from the representation of premises. Gurr, Lee, and Stenning (1998) argue that the semantics of a diagrammatic system is more "direct" than the semantics of a symbolic system and that this crucial difference explains their characteristic low cost of reading off a conclusion. They also correctly point out that directness is relative, and hence, some rides are cheaper than others. Having a distinct role of graphs in mind, Wang and Lee (1993) present a formal framework as a guideline for correct visual languages. This impressive work provides design theorists and AI researchers with computational support for visual reasoning.

Not surprisingly, AI researchers, one of whose main concerns is the heuristic power of a representation system in addition to its expressive power, have been debating for decades about different forms of representation.[26] Hence, they have welcomed discussions of the distinct role of visual reasoning and have recently hosted interdisciplinary symposiums on diagrammatic reasoning at AI conferences.[27] At the same time, realizing that human beings adopt different representation forms depending on the kinds of problems they face, some AI researchers and design theorists have practiced domain-specific approaches to bringing in problem-tailored representation forms.[28] Harel's invention of higraphs (1988) is an excellent example of obtaining practical results without being bogged down in a more abstract and theoretical controversy. However, it is likely that both top-down and bottom-up approaches to the distinct role of diagrams in reasoning are necessary for the project to accomplish its goal.

Heterogeneous reasoning has also been taken up by logicians. It is important to note that logicians bring slightly different concerns to this project from those of cognitive scientists or of AI researchers. First, logicians' main interest is exclusively in externally drawn representation systems, as opposed to internal mental representations. Second, differences in cognitive effects or in heuristic power among different forms of representation are not at the top of logicians' agenda. The first and necessary test for any representation system is to prove that the system is correct. The next important question is the expressive power of the system. If a language fails to justify its logical status or if its expressive power is too limited, logicians' interest in that language will fade. Accordingly, facing a strong prejudice against diagrams, which is more deeply rooted in the modern history of logic than in any other area, some logicians put their priority in examining whether there is any intrinsic reason why symbolic systems, but not diagrams, could provide us with a rigorous proof. I took up this question with Venn diagrams and showed that this system is not only sound and complete, but a slightly modified version of the Venn system is logically equivalent to a monadic language.[29] Hammer and Shin (1998) modified Euler diagrams to show that their modified version can be used in rigorous proof as well. Earlier works on Peirce's Existential Graphs[30] (in which graphical reasoning itself was not the main topic) can be re-evaluated to find significant contributions to the understanding of heterogeneous reasoning from logicians' point of view. Based on existing case studies, Barwise and Etchemendy, the first two logicians who launched the inquiry into diagrammatic proofs in logic, conclude that "there is no principled distinction between inference formalisms that use text and those that use diagrams. One can have rigorous, logically sound (and complete) formal systems based on diagrams."[31] This conviction was necessary for the birth of their innovative computer program Hyperproof, which adopts both first-order languages and diagrams (in a multi-modal system) to teach elementary logic courses.[32]

Each of these various research agendas is important to understanding heterogenous reasoning and needs to be taken seriously. With many concrete results in diagrammatic reasoning in hand, now is a good time to bridge the gap among the different strands of the research, which is one of the main goals of this work. First of all, EG is a sound and complete diagrammatic system, which makes logicians' main worry about its logical status disappear. And EG is logically equivalent to a first-order language, which meets the criticism among logicians and mathematicians that a diagrammatic system cannot express as much as a symbolic system does. Also, unlike with domain-specific diagrams or pictures,[33] this system is comprehensive and general enough to have the wider application required by AI researchers and design theorists.[34] Moreover, this book will show how to make the existing system more efficacious and more natural by a new way of understanding graphs on their own terms, which is directly related to AI researchers' and design theorists' agendas. At the same time, abundant comparisons between EG and a logically equivalent symbolic language[35] provide useful material to satisfy cognitive scientists' main interests—cognitive differences between symbolic and diagrammatic systems.[36]

The study of Peirce's EG could not only bridge a gap among multiple agendas in contemporary research on multi-modal representation, but also provide us with a chance to explore a rather forgotten historical fact that a diagrammatic representation system was invented at almost the same time as a modern symbolic system was. Moreover, the first comprehensive non-symbolic system, EG, was devised by Peirce, who is one of the founders of modern symbolic logic. This interesting historical fact encourages us to inquire into the philosophical motivation behind Peirce's invention of a graphical system.

Chapter 2 will reconstruct Peirce's view of different logical notations and different representations systems, which I claim to be the first comprehensive theory of heterogeneous reasoning. The evaluation of Peirce's work as a theory of heterogenous reasoning is bound to be new for at least two reasons. First of all, while many important works are available about Peirce's theory of signs and logical notation,[37] the topic of heterogeneous reasoning has not been explored in the context of logical theory, of formalization, or of the philosophical motivation behind EG. More importantly, the area in which I claim Peirce's contribution should be recognized, i.e., heterogeneous reasoning, is a new interdisciplinary research area, as explained above. I will show in chapter 3 that Peirce's view—the advocacy of a system with more than one kind of sign—is implemented in his own graphical system. That is, chapter 3 presents the thesis that EG is a heterogeneous representation system, by examining both symbolic and iconic elements of the system.

Chapters 4 and 5 turn to more specific aspects of EG. Granting the legitimacy of logicians' criticisms against EG, in these two chapters I present new methods of understanding the Alpha and Beta systems, respectively. We can meet existing complaints against EG by discovering more visual features present in graphs and by identifying the unique nature of graphs. The fundamental differences between symbolic and diagrammatic systems are discussed in detail in these two chapters.

Chapter 6 raises a question as to the relation between Peirce's theory and his own practice: How did Peirce himself, who laid out the philosophical groundwork for heterogeneous systems as shown in chapters 2 and 3, fail to obtain better ways of understanding his own graphical system?[38] For this curious historical question, I explore assumptions behind Peirce's distinction between logical systems and calculi and his intention to make EG a logical system, not a calculus. Chapter 7 concludes with a short summary of the book and suggestions for future research on heterogeneous reasoning.

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