Studying Attention Dynamics of a Predator in a Prey-Predator System

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Abstract

Mathematical treatment of attention dynamics of a predator chasing prey is studied. In our model, prey and a predator move around a two-dimensional surface. Real numbers that correspond to individuals of prey are used. A predator selects prey of the highest number. If prey's motion is ordered, the predator selects the nearest individual. If prey's motion is disordered, the predator tends to select the individual which has split away from a collective, ie. alone. It is emphasized that partially disordered motion, is rather difficult for the predator to select an individual.

Introduction

Many animal species show group motion. In the ocean, millions of fish make a school in which almost all individuals have the same heading direction. On the other hand, in the sky a lot of birds fly collectively, as if they dance. It is no doubt that flocking or schooling reduces dangerous accidents for prey species. Many researchers believe that collective behavior of prey functions as a protection against predators(Breder, 1959; Partridge, 1982; Cusing and Jones, 1968). One effective method of protection is to break predators' decision making. Some reports say that predators invest a lot of energy into deciding their target. For example, Kruuk studied spotted hyenas over years(Kruuk, 1972). When hunting, hyenas spend a lot of time in deciding a target. Either an old, or a weak individual often becomes the prey of hyenas. But old and weak features are not the reason they become prey. When Kruuk puts an arbitrary marker on an individual prey (e.g. eland), the individual is almost always taken as a target for hyenas. Thus it is emphasized that easily recognizable prey can easily become a target for predators. This is why prey all of the same size (especially fish) gather, since predators lose criteria of targeting(Peuhkuri, 1997). They have to look for small differences among their prey.

In this paper, we study one predator's selection mechanism of prey gathering. What mechanism of targeting is effective for a predator? How does it depend on the prey's grouping behavior? We tend to try to find mechanisms independent of a predator's internal structure, for instance, structure of eyes, a brain, and so on. This paper does not address the issues of how to catch prey and how to escape from predators. All problems of a predator's selection mechanism will be thought of as problems of selecting "priority functions" introduced in the next section.

A Predator's Selection Mechanism

For mathematical treatment of a predator's selection mechanism of prey, we use real numbers: each number corresponds to each individual of prey. Such a number gives "priority" for a predator attacking prey. As the most natural definition, a predator attracts attention to the prey individual corresponding to the highest number. We called the individual the "candidate" and the action "attention". If a candidate changes one after another, a predator might hardly catch any prey. We define that a predator decides to chase an individual, (which is merely called the "target") only if it attracts attention to the same candidate in a certain interval, I_a without a break. This action is called "locking on".

This definition reflects a problem of how to calculate priority. Before discussing this, we have to clarify treatment of prey. Roughly speaking, there are two categories of properties that prey have. One is static properties, namely, shape, color, smell, and so on. The other is dynamical properties, for example, position, velocity, acceleration and so on. Only considering the latter, makes our investigation rather simplified. In this paper, all individuals of prey and a predator are thought of as "material points", which have been widely discussed in many papers, especially fish schooling studies (Breder, 1954; Shimoyama et al., 1996; Sannomiya et al., 1996; Sannomiya and Dousrari, 1996; Niwa, 1994; Aoki, 1980; Nishimura and Ikegami, 1998). Along the above line, priority of prey becomes computed from position, velocity and acceleration of prey. More details will be discussed in the next section.

Assume many individuals of prey are chased by one predator. In general, the priority for the ith individual,

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p_i can be written as follows:

$$p_i = F(w_i : w_1, w2, ..., w_{i-1}, w_{i+1}, ..., w_n), \quad (1)$$

where w_j is a function of the i individual and the predator. F is called a "priority function".

Note that the function $F(\cdot : \cdot)$ is invariable against permutation of variables after a colon, that is,

$$\begin{aligned} & F(w_i : ..., w_j, ..., w_k, ..., w_n) \\ = \; & F(w_i : ..., w_k, ..., w_j, ..., w_n) \quad , \end{aligned} \quad (2)$$

with regard to any pair of numbers, j and k. This means that the function is independent of correspondence of prey individuals to natural numbers. Destruction of the invariability causes unnatural results, for example, results depending on programming techniques (cf. a usage of memories).

In this paper, we assume that the priority function F is linear because of its simplicity. The linear function has terms about current position of prey as well as delayed data, given by the following:

$$p_i = \sum_{l=0} s_l |\vec{r}_i(t_l) - \vec{r}_a(t_l)|, \quad (3)$$

where $\vec{r}_i$ indicates the position of the ith individual of prey, $\vec{r}_a$ does that of the predator and t_l does the time step labeled l. t_0 means the current step, whereas $t_l : l > 1$ does the delayed steps. s_l are constants. Comparing Equation (3) with (1), there is no variable after a colon in Equation (3), that is the ith priority depends on only the ith position. This definition automatically preserves the independence of permutation as written above. In this paper, the max of l is two, which means total terms equal three.

Note that non-linear functions, such as neural networks, will be discussed in **Discussion** briefly.

Equation of Motion

All individuals in the model move around a two-dimensional, unbounded surface. First we discuss motion of prey. We desire a model in which many behaviors emerge with only a few parameters. Fortunately, we have a good model studied by (Shimoyama et al., 1996), who have studied collective motion of many animal species. We use the model with a small modification:

$$\begin{aligned} \frac{d\vec{r}_i}{dt} &= b \sum \frac{1 + e \cdot \cos \psi_{ij}}{1+e} \vec{f}_{ij} + d_p \vec{n}_i & (4) \\ \frac{d\theta_i}{dt} &= h_p \cdot \sin(\phi_i - \theta_i), & (5) \end{aligned}$$

where $\vec{n}_i = (\cos\theta_i, \sin\theta_i)$, n_i indicates the heading direction of the i individual, θ_i does the angle notion of the heading direction, ϕ_i does the angle notation of the velocity direction and ψ_{ij} does the relative angle between the ith heading direction and the direction from the ith to jth individuals. b, e, d_p and h_p are constant parameters. The term $\frac{1+e\cdot\cos\psi_{ij}}{1+e}$ indicates a visual field of the ith individual. The larger the parameter e, the smaller the visual field is. When e equals zero, the individual's visual field becomes a disc. We omit an inertial term from the original equation by Shimoyama et al., since the term seems to be ineffective, which is also reported in their paper. It is noted that by choosing appropriate parameters, only searching the parameter e leads to almost all patterns in this equation. $\vec{f}_{ij}$ indicates mutual interaction given by the following:

$$\vec{f}_{ij} = (\frac{1}{r_{ij}} - \frac{1}{r_{ij}^2})\frac{\vec{r}_{ij}}{r_{ij}}, \quad (6)$$

where $\vec{r}_{ij} = \vec{r}_j - \vec{r}_i$ and $r_{ij} = |\vec{r}_{ij}|$. This interaction is called Lenard Jones potential, introduced by Breder(Breder, 1954) for fish schooling, and has an effect of maintaining a distance between the ith and jth individuals.

As has been written, the predator chases a target after it locks on to the target. We can express the behavior of the predator by the following equation:

$$\begin{aligned} \frac{d\vec{r}_a}{dt} &= g \cdot (\vec{r}_t - \vec{r}_a)/|\vec{r}_t - \vec{r}_a| + d_a \vec{n} & (7) \\ \frac{d\theta_a}{dt} &= h_a \cdot \sin(\phi_a - \theta_a), & (8) \end{aligned}$$

where $\vec{r}_t$ is the position of the target, g, d_a and h_a are constants. Roughly, the value of d_a and g mean the velocity before and after locking on to a target, respectively. Before locking on, g is set to zero. If distance between a target locked on and the predator becomes smaller than a certain value, the score of the predator rises by one, and the target is removed from the surface. The predator becomes the state of attention and finds a target.

Collective Motion of Prey

Except the parameter e, all parameters of prey's motion defined in the previous section are fixed, since only searching the parameter e is enough to find all patterns of motion. The parameters are set as follows: $b = 0.9$, $d_p = 7.0$ and $h_p = 100$.

The following four distinct collective behavior are found by the dynamics which were reported by (Shimoyama et al., 1996).

- **Marching**
  The elements form a regular triangular crystal, moving at a constant velocity. The formation is stable against disturbance. (Figure 1-(1))
- **Oscillation**
  Several group motions exhibit regular oscillations: (i)

Wavy motion of the cluster along a linear trajectory. (ii) A cluster circling a center outside the cluster. (iii) A cluster circling a center inside the cluster. (Figure 1-(2))

- **Wandering**
  Although the lattice-like order inside the cluster persists, the center of the cluster can wander quite irregularly. Chaotic intermittency of motion is found. (Figure 1-(3))
- **Swarming**
  The most irregular motions are found in swarming. Although the cluster persists, lattice-like order is broken completely. The velocity of the elements has a large distribution, and the mobility of the cluster is small. (Figure 1-(4))

In this paper, we call marching and oscillation ordered motion, wandering partially disordered motion, and swarming disordered motion.

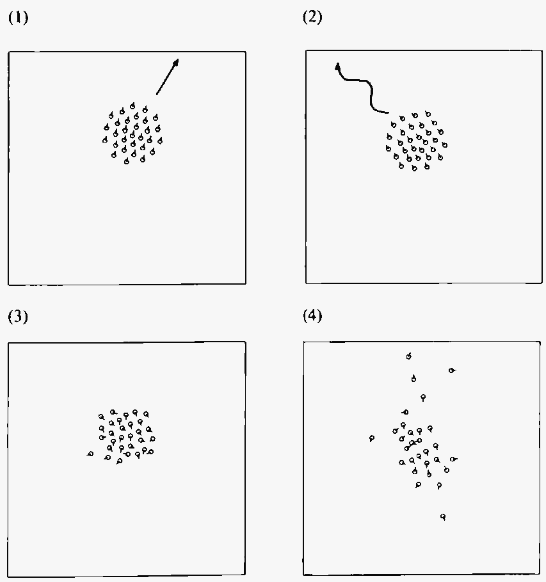

Figure 1: An individual of prey is marked by a circle with a line that is the heading direction. (1)Marching: $e = 0$ (2)Oscillation: $e = 0.2$ (3)Wandering: $e = 0.5$ (4) Swarming: $e = 0.9$. Details are written in the body of the paper.

Results of Prey with a Predator

We investigate the characteristics of this model, by searching parameters of a priority function of a predator, and the parameter e of its prey. Although there are three parameters of a priority function, we can reduce one degree of freedom. Instead of any three real numbers, s_0, s_1 and s_2, we can use s_0/S ...

where $S = \sqrt{s_0^2 + s_1^2 + s_2^2}$ since a priority function is linear. It is convenient to adopt the following notation:

$$
\begin{aligned}
s_0 &= \sin\alpha \\
s_1 &= \cos\alpha\sin\beta \\
s_2 &= \cos\alpha\cos\beta
\end{aligned} \tag{9}
$$

since $\sqrt{s_0^2 + s_1^2 + s_2^2}$ equals 1 automatically. Note that the pair, (α, β) indicates a point of a sphere with the radius of 1. We run a simulation for 1000 steps. We run 100 simulations for one set of parameters, and average the results to avoid dependence on initial conditions.

Parameters are set as follows: $I_a = 0.08$, $d_a = d_p = 7.0$ and $g = 10$. It is important that g is greater than d_p since the predator always catches a target.

The relation between averaged scores of a predator and e is shown in Figure 2. Averaging was the over all (discrete) values of α and β. At the point $e = 0.5$, the averaged score was the smallest. At both ends, the scores are the maxima.

We pick three values for e: $e = 0$, $e = 0.5$ and $e = 0.9$. At the those three points, we calculate Lyapunov spectrums of prey's (not including predator's) motion shown in Figure 3. [1]

At the value of $e = 0$, all Lyapunov exponents are less than zero, since the prey's motion is ordered. At the value of $e = 0.5$, some larger exponents are greater than zero. This is relatively a low dimensional chaotic motion. Finally, at the value of $e = 0.9$, more exponents are larger than zero, that is, high dimensional chaos. From Figures 2 and 3, it is asserted that when prey's motion becomes low dimensional chaos, the predator gets the smallest score rather than during high dimensional chaos. This result does not necessarily depend on the magnitude of the largest Lyapunov exponent.

In relation to the above three points, we also show the relation between parameters of the priority function and scores of the predator. Figure 4 consists of three contour plots. Corresponding to the value of $e = 0$, one peak ridge of scores can be found in Figure 4-(1). Figure 4-(2) corresponds to the value of $e = 0.5$, it is comparatively flat. Figure 4-(3) with regard to the value of $e = 0.9$ has two peak ridges, one of which is nearly the same as Figure 4-(1), whereas the other is new.

Let us look at snapshots of prey and a predator for Figure 4. Figure 5 and 6 correspond to Figure 4-(1) and 4-(2), respectively. In Figure 5, we set parameters from the point $(0, \pi)$ in the ridge of Figure 4-(1) marked by a cross. It is clear that a predator locks on to the relatively nearest individual of prey. On the other hand, we use parameters from the point $(0, 2\pi)$ in the second ridge of Figure 4-(3) (also marked by a cross) for Figure 6. With contrast to Figure 5, the predator chases the farthest individual.

[1] How to calculate Lyapunov exponents was written in (...al., 1980; Shimada and Nagashima, 1979).

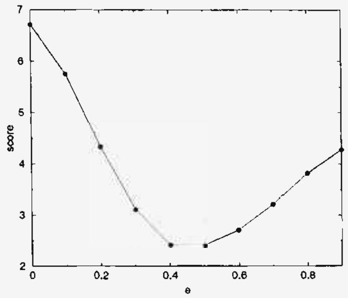

Figure 2: The horizontal axis is e and the vertical axis is the scores of a predator.

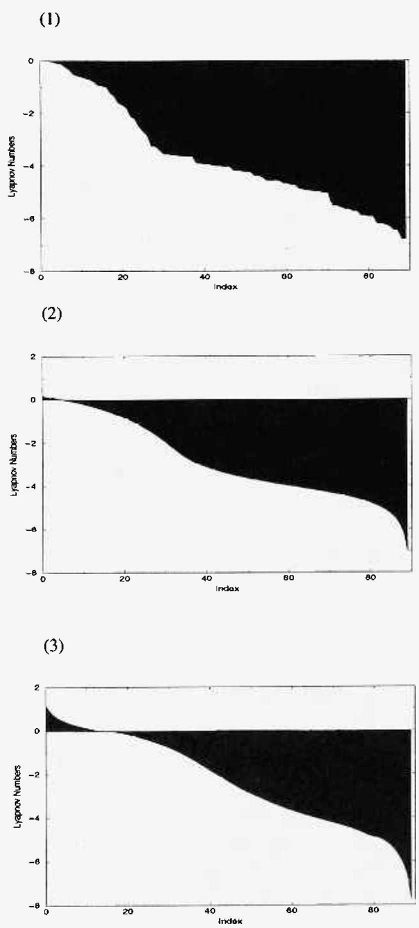

Figure 3: Lyapunov spectrums of prey. Horizontal axis is indices and vertical axis is Lyapunov exponents arranged in order of decreasing size. The graphs are filled in black from a horizontal line of zero to Lyapunov exponents. Sub-figures (1), (2) and (3) correspond to the values of $e = 0$, $e = 0.5$ and $e = 0.9$, respectively.

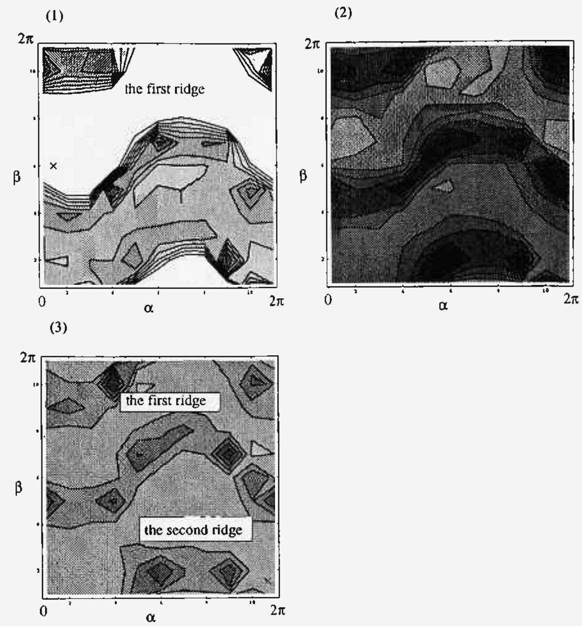

Figure 4: Contour plots of predator's scores. Horizontal and vertical axis are α and β. $0 < \alpha, \beta < 2\pi$. The lighter area is the higher score. (1)$e = 0$. One ridge can been seen that is called the first ridge. (2)$e = 0.5$. The contour is rather flat compared to the others. (3) $e = 0.9$. There are two ridges, one of which is nearly the same as the first ridge of (1), and the other is new, called the second ridge.

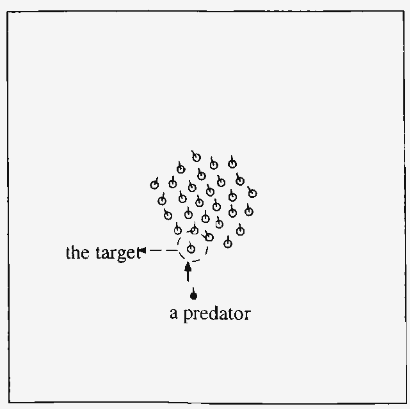

Figure 5: A snapshot of prey and a predator. White circles are the prey and the black circle is a predator. A dashed circle indicates the target of the predator. $\alpha = 0$ and $\beta = \pi$.

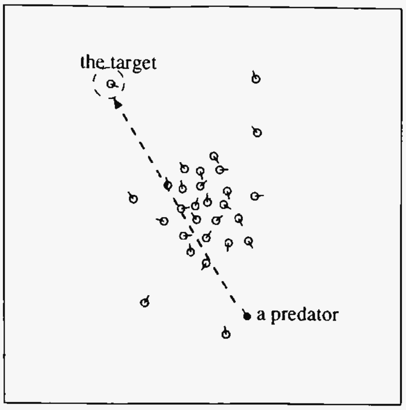

Figure 6: A snapshot. $\alpha = 0$ and $\beta = 2\pi$.

Discussion

The summary of all the results are as follows. If prey's motion is ordered, in other words, if all Lyapunov exponents are negative, the predator chases the nearest individual. However, if its motion is disordered, that is, a lot of Lyapunov exponents are positive, the predator locks on to the farthest one as well. The predator gets the smallest score when prey's motion is partially disordered, that is, when a few Lyapunov exponents are positive.

It is trivial that the predator chases the nearest individual. Rather we are interested in chasing the farthest individual. In the most disordered pattern, one individual often rushes out from a cluster. The class of priority functions given in this paper cannot directly detect such an individual. However, in many cases, the individual goes often far away from a cluster of prey. Consequently, the predator catches the individual rushing out from a cluster. It is very dangerous for an individual to rush out or split away from a cluster in the natural world. For instance, when there were two prey fish as well as a predator fish in a tank, the predator fish caught neither fish. However, if the researcher removed one prey fish, the predator instantly ate the remaining prey(Partridge, 1982).

Difficulty of partially disordered patterns for the predator, is an interesting topic. A cluster of those patterns has few crevices. In addition, this cluster moves in a disordered manner. The order of the priority easily changes, since the priority is a function of position of prey. Thus, the predator has to have a sufficiently precise priority function. In the natural world, similar flocking behavior can be found. One good example is starlings(Edmunds, 1974; Wilson, 1995). When they are attacked by a falcon, they gather into one flock and fly in zigzags. The falcon often runs against the flock haphazardly. This motion of the starlings may be partially disordered.

We address the feature works. In this paper, priority functions are linear. However, nonlinear functions are also interesting. What priority function is required corresponding to partially disordered patterns? What about chaotic functions? Possibly, chaotic functions have an affinity to partially disordered motion. Observations of flocks or schools attacked by a predator must be important. Of cause, it is difficult to calculate Lyapunov exponents from only data. In addition, two dimensional data in a video film transformed into tree dimensional data is also complicated. It is expected that similar calculations to Lyapunov exponents, which is somewhat easy to be computed from two dimensional data in a video film.

One effective strategy of prey is "dispersing". Herds of many prey species radially disperse when a predator approaches. After that, prey make a herd again. It is believed that this behavior confuses a predator's selection mechanism(Driver and Humphries, 1988; Edmunds, 1974). I speculate that dispersing is an easier task than zigzagging since many species, birds, insects, fish, and so on, show dispersing behavior, whereas only some bird species fly in zigzags. The reason is that, since a nervous system has time delay effects, it seems to be difficult for birds to avoid splitting away from a herd without prediction of other individuals' motion. In order to disperse however, individuals do not need prediction but to detect a predator approaching. My future paper will investigate effects of dispersing.

Conclusion

We studied attention dynamics of a predator through priority functions. Although the results of this paper were not novel but predictable, we succeeded in mathematically formulating those results. In order to obtain generic results, all we have to do is to investigate more classes of priority functions, since we studied one special class of function in this paper. Nevertheless, future works will be based on the results of the paper.

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