



A PRIMER ON
Auction Design, Management,
and Strategy

DAVID J. SALANT

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David J. Salant

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Preface

My work on auctions started in 1994 when I was a Senior Member of Technical Staff at GTE Laboratories (now Verizon Laboratories) and was asked to make a one-hour presentation on auctions to some executives involved in the planning for the first US spectrum auctions. That one-hour talk evolved into twenty years of auction consulting assignments.

This work evolved still further in mid-2007, when Patrick Rey asked if I might be interested in visiting the Toulouse School of Economics (TSE) and teaching a course on auctions. I accepted Patrick's offer, and this book is a compilation of the lecture notes I put together in teaching this class for each of the past six years at TSE. This work has benefited enormously from the hospitality I received at Toulouse and the feedback of my students.

The TSE has been the ideal place for me to develop the material contained in this Primer. My goal has been to provide a rudimentary introduction to auctions to someone who is potentially considering running an auction or bidding in one. This does require some familiarity with basic concepts of game theory and some results in auction theory.

There are a number of existing books that provide excellent coverage of auction theory—notably Milgrom's *Putting Auction Theory to Work*, Klemperer's *Auctions: Theory and Practice*, and Krishna's *Auction Theory*. I felt that there was still a large gap for the practitioner. Notably, it seems, at least based on my experience, that it is all too easy to misinterpret or misapply some of the theory, or all too difficult to apply that theory appropriately. These other works, while providing very valuable structures and case studies, leave a lot of gaps, requiring an auctioneer or a bidder to fully understand the limits of this theory in interpreting and in applying this theory. I have tried, as much as

possible to help bridge these gaps. This Primer doesn't cover as much of the theory, or in as much detail. However, I try to explain in some detail how the theory has been or can be applied to the decision problems that auction participants face. To this end, I have included game theory at a level needed to use and avoid abusing the main basic results from auction theory, and at a level for the practitioner to be able to critically evaluate prior auction experience.

A step-by-step manual to run or bid in an auction was never my intent in writing this book. It was instead my intent to explain how to go about planning to run or bid in an auction. As the book grew in part out of a course I have taught at the TSE, it is probably more of a textbook than I had originally intended. It can probably serve as a textbook on auctions for graduate students in economics and other areas, as well as upper level undergraduates. And perhaps a good text is the best form of Primer.

This book has also benefited enormously from my work on consulting projects. I have had the privilege of being able to advise bidders and auctioneers in dozens of auctions over nearly twenty years. This experience has been a source of many of the examples used and influenced the material in the book. Indeed some assignments have motivated new research—for example, the “chopstick” paper of Rosenthal and Szentcs (2003) was motivated by the Dutch 2G auction. And much of the section on sequential auctions was motivated by my work on the New Jersey BGS auction (and published in Loxley and Salant 2004). The experience is also a great source of examples of how auctions can fail, because of design flaws and possible bidding miscalculations, and not just a lack of interest from bidders.

One of the goals of the book is to explain the decision problems facing bidders; so while the revenue equivalence theorem states that a bidder should expect to pay the same amount in a first-price and second-price sealed-bid auction, the decision problem facing bidders is not at all the same. Bidders should bid values in a second-price auction for a single object. In contrast, in first-price auctions bidders need to calculate an optimal bidding strategy—which is not all that simple and requires some assumptions about rivals.

I have benefited from comments of Elmar Wolfstetter who has provided comments on the entire manuscript. My work on chapter 10 benefited a great deal from a consulting project with Andy Skrzypacz and Jon Levin. Paul Klemperer read through the manuscript and provided many helpful comments. In fact this Primer owes an enormous

intellectual debt to Klemperer's work, and especially his *Auctions: Theory and Practice*. I have also had the truly distinct privilege to work with and learn from Paul Milgrom. Milgrom has made enormous efforts to develop auction designs that improve market efficiency. His uncanny ability to express the most complex ideas in easy to grasp concrete terms has been pivotal in getting new types of auctions introduced for new types of transactions. Over time, my appreciation of his skill and the effort he makes to find the best way to explain a difficult concept has grown.

The past twenty years have been a golden age for game theory. When I finished graduate school in 1979, game theory was just starting to become incorporated into standard economics curriculum. Until the first Federal Communications Commission (FCC) auctions in 1994, game theorists, and even more generally mathematical economics, had little impact outside of academe. The FCC auctions changed all that. For the first time, mathematical economics and game theory had a direct value to very real problems. Since the first FCC auctions, auction theorists have been involved in the design of auctions in a great many sectors—from airwaves to milk powder, gas and electricity to pollution rights. However, there is still much to be done I hope this Primer will expand the scope of the application of game theory to address practical problems.

Still there are a few significant obstacles to the application of concepts from modern auction theory to practical problems. One is that there are often parties that have a vested interest in markets and who would oppose new auctions that would improve market efficiency. For example, an auction can reduce the value of brokers or totally eliminate the need for brokers. Thus brokers who have the best access to the information needed to design and set up an auction may also very much oppose it. Another main impediment to the introduction of new types of auctions is normal business inertia. If an existing system for trade seems to work reasonably well, then buyers and sellers may not see a need to try to improve it. This is partly due to the costs of learning a new system. And inertia can set in even where auctions have had some success. For example, I introduced the simultaneous descending clock auction (SDCA), a variant of the simultaneous multi-round auction (SMRA) for energy procurement. These auctions typically take several days to complete—but given the experience with the SMRA and SDCA, these auctions can be completed within hours and not days. For the larger energy procurement auctions, the transaction costs of

bidders are significantly higher when their offers have to be held open several days. But the electric utilities that run the auctions and the regulatory agencies that oversee have no desire to fix an auction that is not broken. The best opportunities for introducing new auction formats are often immediately after there has been some sort of problem, such as an energy crisis, or one with mortgage-backed securities.

Last, I wish to express my appreciation to my family. My wife Deborah, and my children Jacob, Rebecca, Ilana, and Max have had to put up with my long absences and constant traveling during the time I was accumulating much of the experience that is the source of much of the material in this book.

1 Introduction

Summary

This is a Primer on auctions. It is intended, in part, to serve as a guide to auctions for the practitioner. This chapter explains what distinguishes auctions from other market transactions. It provides a brief summary of what is essential or helpful for the practitioner.

1.1 Goals of This Primer

Auctions are an increasingly common share of all market transactions. They take a great many forms. Often they greatly improve market liquidity, but they can totally fail to work. Perhaps more than half of all auctions result in some or all of what is being put for auction remaining unsold, and even when this does not occur, the transaction prices can be absurdly low or high, and far from equilibrium levels. This book explores reasons why auctions work or fail. It explains how auction design affects bidding decisions and how bidding decisions affect the outcome, sometimes quite dramatically.

Auctions are highly structured market transactions primarily used in thin markets (markets with few participants and infrequent transactions). In such markets the standard supply-and-demand models of competitive market forces usually cannot be relied on to explain the outcomes, since in most auctions just a few bidders can have a large influence on the outcome. An auction is perhaps better characterized as a bargaining process. Moreover in auctions, unlike most other markets, offers and counteroffers are typically made within a structure defined by a set of rigid and comprehensive rules. This book provides a fairly complete range of auction structures and discusses how they can affect auction outcomes.

This book is intended to serve both as an introductory text on auctions as well as a guide for practitioners, that is, those interested in managing or bidding in auctions. Auctions are highly structured negotiations with a defined set of rules for offers, counteroffers, price determination, and allocations. These rigid structures mean that auctions can be analyzed by mathematical models more accurately and completely than can most other types of market transactions.

This book provides a guide for modeling, analyzing, and predicting the outcomes of auctions. To be useful and effective, such a guide must cover some essential elements of auction theory. In practice, auction theory will rarely provide an exact and prescriptive model that can be applied directly. Nevertheless, auction theory does provide both insights and specific results that are of direct value to the practitioner.

While a primer or practitioner's guide would ideally provide a complete checklist for what to do to design, set up, and manage an auction, or for bidders to decide how to bid, that scope is too broad to be practical. Rather, the focus here is on principles, tools, and examples that can be used to analyze auctions. This first requires some way to categorize different types of auctions. In this categorization, I explain how the auction design and strategy should depend on the type of auction. Second, I explain the main results from game theory and the theory of auctions in order to provide practical frameworks for analyzing auctions and bid strategies. Third, I review auction experience, mainly from actual auctions but also from experiments. This part of the book illustrates how auction theory can be applied to auction design and strategy decisions.

Because of the increased use of auctions in new settings, auctions have become a more common subject of economic research and writing. Because this Primer is intended as a guide for someone interested in designing an auction and managing it or for someone having to bid, its review of research on game theory and auction theory will stress the linkage between theory and application. Some game theory is crucial for the principles of sound auction design and for properly assessing bidding strategy.

This book largely avoids using formulaic and specific task plans or checklists because most auction design and bid strategy problems are too complex to lend themselves to exact formulas. However, one of the main points of this book is to show how to apply game-theoretic modeling tools to practical auction design and strategy decisions. Practical

difficulties are explained in some detail. Reality never exactly fits any theoretical model, but very simple theoretical models can still be very useful.

Explanations of how auction strategy and auction design can matter in one specific case may have only limited applicability to seemingly very similar specific cases. Similarly the main results of auction theory may have only small direct practical value. What is often most useful are simple mathematical models and a few basic principles for developing them, as well as guidelines to help identify limitations of such models. *Simulations*, in which computer algorithms substitute for bidders, and scaled-down *experiments*, in which human subjects play the role of bidders in repeated trials, can also be useful tools for assessing auction design or bid strategy plans. Much of this book's exposition is devoted to explaining how to develop simple and useful models of auctions.

One additional objective in this book is to explain how both auction design and bid strategy can have a significant effect on prices and allocations. Auctions are a form of imperfectly competitive markets, in which both sides have limited information. More specifically, auction design can have a significant effect on revenues in a *forward* auction, that is, an auction to sell one or more lots, and on costs in a *reverse*, or procurement, auction. I provide many examples in which identical items have sold for significantly different prices in the same auction and at the same time. An analysis of how the auction design affects bidder incentives can usually go a long way in explaining such price anomalies. Price differences are even more common when identical objects are auctioned one at a time and in other types of sequential transactions.

I start with a basic exposition of game theory and with a few of the main results from auction theory. To be useful, a model of an auction must be consistent and must quantitatively represent the incentives and decision-making of the bidders. Mostly this assumes rational bidders. However, behavioral and cognitive limitations of bidders are discussed.

This Primer should not be considered as a comprehensive compendium of modern auction theory. Many topics are omitted—such as two-sided auctions or econometric analysis of auctions¹—and there is relatively limited discussion of some more technical topics, such as mechanism design. The intent here is to provide an introduction and a starting point for the reader to either learn something about auctions

or interested in auction design or strategy.² This Primer still should also be of interest to researchers. There are many practical issues in auction design and strategy that have yet to be studied by researchers. Some of the material in this book is new, largely motivated by a need to extend and expand the existing literature in order to address some design issues that have arisen in recent auctions.

1.2 What Are Auctions?

This section briefly describes what auctions are, and the types of auctions studied in this book. I also provide a brief introduction as to why auctions are used in preference to other market mechanisms.

To start, it is essential to say what constitutes an auction, or at least what will be included in this book. What distinguishes an auction from other types of transactions is not always clearly specified. I will use the term primarily to describe all one-to-many bidding processes: forward auctions, with one seller and many buyers, or reverse auctions, with one buyer and many sellers. At times the term *auction* is extended to situations in which there are a few buyers or a few sellers, namely few-to-many transactions. But in these cases the few will be assumed to be acting in concert in organizing the auction. When there are two sellers, acting totally independently, there are really two independent auctions. In contrast, when the two sellers agree on schedules, starting prices, or other elements of the bidding process, then the two sellers are really acting as a single entity in determining the rules of the auction.

The term auction will be restricted here to what are often called *sealed-bid tenders*. More specifically, one-shot bidding processes, both price-only and multi-attribute ones, are referred to here as sealed-bid auctions. Most of the focus is on price-only or other forms of single-dimensional bidding.

More formally, an auction is defined by the following:

1. *Bidding format rules* These rules govern the form of bids. Bids can be price only, multi-attribute, price and quantity, or quantity only.
2. *Bidding process rules* Included are closing rules, which determine the timing of the auction; specification of the information provided to bidders during the auction; rules about the processes for bid improvements or counterbids; and rules specifying the auction-closing conditions.

3. *Price and allocation rules* These rules determine final prices and the amounts won by each winning bidder. Auctions can be for single items or multiple items. Traditionally multi-lot auctions have been conducted as separate auctions, sometimes sequentially and sometimes simultaneously. Modern auction designs have been developed to facilitate bidder arbitrage for similar items and/or to allow bidders to submit offers for different packages of items.

An auction is one type of market mechanism. More specifically, an auction is an allocation mechanism that mostly uses price criteria for determining *allocations*—that is, which parties win which objects or contracts—and the prices paid. Auctions are more centralized than most other market mechanisms. In most markets, other than *exchanges*, buy and/or sell offers are not all made in one place or submitted through an auctioneer. An auction, however, is not the only type of centralized market mechanism.

An auction is also a price discovery mechanism. Auctions provide a means for the entity conducting the auction—the auctioneer, or the auction *originator*, to collect information from bidders so as to determine prices and allocations. The fact that the information bidders provide can affect their prices means that it can also affect their incentives, as is explained in more detail below.

An exchange (e.g., a stock, commodity, or financial exchange) is at times considered to be a form of auction. Exchanges differ from the auctions considered here in three ways. First, exchanges involve two-sided or many-to-many transactions. Second, transactions on an exchange normally run continuously, whereas auctions are periodic or episodic. Third, exchanges usually have an intermediary setting the price who is neither a net buyer nor a net seller on average. The intermediary (or specialist or trading desk) will have a book of buy and sell orders and adjust prices up or down depending on the relative volumes of such orders. In contrast, in auctions the originator will usually have an interest in getting the best price (i.e., the highest price in a forward auction, and the lowest price in a reverse auction). Auctions can, however, have independent entities to manage the bidding process.³

1.2.1 Why Auctions?

Auctions are not used for most transactions. The question arises why auctions are used at all. When the transaction frequency is high, transactions occur at physical or virtual stores, or in exchanges. When it is

low, but the volume or monetary value is high, an auction is most useful. When the transaction frequency is low, it will not be worthwhile to incur the cost of maintaining a trading site, or store. While the organization of an auction may involve some costs, when the trading events are infrequent, it can be cost effective for auctions to be used.

When the volume or the value is also low, bilateral negotiations and the solicitation of offers through requests for offers (RFOs) or requests for proposals (RFPs) are often most efficient. When the transaction frequency is low, the cost of maintaining a store or exchange is high relative to the transaction value. When the transaction value is low, the cost of organizing and managing an auction can be prohibitive. Thus auctions are best for periodic or episodic high-volume and high-value transactions.

1.2.2 Types of Auctions

Auctions have been around for centuries—at least since the time of Herodotus in 500 BCE. Auctions can take many forms and can involve many types of goods and services. Two main ways in which auctions differ are in the auction format, or rules structure, and the frequency and volume of the transactions. Until recently, most auctions were single-object auctions or sequences of such auctions.

Auction Formats Traditionally most price-only auctions used some variant on one of four basic forms:

1. English, ascending price
2. Dutch, descending price
3. First-price sealed bid
4. Second-price sealed bid

Some auctions allow bids to include multiple components or attributes, and not just price. In multi-attribute bidding processes explicit weights or subjective evaluations are used to decide winners.

The most common form of open auction is still the open, oral-outcry *English* auction. In a standard English auction, the auction manager or auctioneer announces prices—an increasing sequence of prices in a forward auction, and a decreasing sequence in procurement auctions—and bidders indicate whether they are willing to accept the announced price. Bidders need not indicate a willingness to accept each announced price in order to remain eligible, and once one bidder indicates acceptance of the most recent price, the auctioneer will go to the next

increment or decrement. Bidders can often, if they desire, shout out (inject) jump bids. The auction ends when no one is willing to improve on the most recently announced price.

The most common form of *online* auction is still the English auction, or simple variants thereof. Instead of an auctioneer calling out prices, though, the host sets a starting price, and bidders can submit bids at any time during an open bidding window. Each new bid is posted, and the auction host will raise the minimum for subsequent bids, similarly to the manner in which a live auctioneer raises price in an English auction.

Descending-price forward auctions have been used since Babylonian times.⁴ The name “Dutch auction” to describe these types of auctions probably arose from their use in selling tulips in the Netherlands, which dates back at least to the seventeenth century.⁵ In a (forward) Dutch auction, the auctioneer starts with a high price, and gradually lowers the price until a bidder is willing to accept that price. Notice that for a reverse auction, that is, an auction in which bidders are sellers and the auctioneer is a buyer, a declining-price auction would elicit bids from all bidders at a high initial price. So, strategically, a declining-price forward (Dutch) auction, that is, an auction in which bidders are buyers, is strategically quite different from a declining-price reverse auction. Here, the term “Dutch” is reserved for declining-price forward auctions. The bid strategy is clearly different in a Dutch and an English auction. In a Dutch auction, a bidder has to guess when to bid. In an English auction, there is no such guessing. A bidder can stop when his or her value is reached.

Both first-price and second-price sealed-bid auctions, often called sealed-bid tender auctions, award to the high bidder the object for sale in a forward auction, and to the low bidder the contract to supply a product or service in a sealed-bid reverse auction. In a first-price auction, the winning bidder pays the bid amount in a forward auction, or is paid the bid amount in a reverse auction. In a second-price auction, the winning bidder pays, or is paid, the amount offered by the best losing bid.

Transaction Frequency and Volume Auctions also differ in how many units are transacted and how often. An auction for a single object is a one-time event, and the strategic considerations are quite different than they are in multi-object auctions and repeated auctions. The theory and experience for one-shot auctions is a useful reference point for the

subsequent analysis of multi-object auctions. Auctions can be one-time, or infrequent, events for a large number of objects. The major spectrum auctions in the United States, Europe, and elsewhere are examples of such auctions. Such auctions have a *matching* role in determining which objects go to which bidders, as well as the usual allocation role in determining winners and losers. Bidders can also interact in the auction to try to more efficiently sort out who wins what, and at what prices. It is important that the auction rules allow efficient arbitrage, as otherwise both the auctioneer and the bidders can lose.

Auctions can also be recurring, such as the monthly capacity auctions conducted by many electricity transmission operators to ensure adequate generation availability. Many commodities—dairy products and wine, for example—are auctioned periodically. Mostly the same bidders compete in each such auction, and for mostly the same types of products. These periodic auctions are commonly multi-unit, and often multi-object. As such, they have many of the same strategic features as one-shot, multi-object auctions. However, the repetition allows bidders to communicate and interact over time across auctions. This can further affect bidding behavior and the outcomes of the auctions.

Auctions can be categorized in other ways. *Forward* auctions—that is, auctions in which there is one seller and several buyers—are perhaps the most common. But *reverse* auctions, in which there is one buyer and several sellers, are also fairly common. The modeling and analysis of forward and reverse auctions tend to be very similar, save for the fact that in reverse auction, the auctioneer needs to ensure post-auction delivery, rather than just a monetary payment in a forward auction.

1.3 A New Age of Auctions

Auctions have changed. They come in a great many new forms. A recent search at the US Patent Office website produced over 2,000 hits for patents issued that include the term “auction.” While not every patent is for a different auction design, the number of innovations is still large. There are two main reasons for these changes, both in some sense owing something to John von Neumann (1903–1957), a mathematician who made pioneering contributions to both computer science and game theory.

One reason is the developments in game theory, a field that barely existed until the 1960s. Game theory is crucial, or should be crucial, to

auction design. Game theory provides mathematical tools and techniques to analyze alternative auction designs. While game theory, and game theorists, are not always directly involved in auction design, their role has increased over the past ten to fifteen years, both in government agencies (e.g., the US Treasury, other national treasuries, and other governmental agencies in a number of sectors including telecommunications, energy, and natural resources) and in the private sector (where firms, e.g., Google, Microsoft, Yahoo, HP, and IBM, have hired leading game theorists to help improve their pricing tools: remember that an auction is a price discovery mechanism).

The other area in which von Neumann made pioneering contributions was in computer science. Without computers and the Internet, the use of auctions might still be where it was centuries ago. To say that auctions are a price discovery mechanism means bids provide the auction originator information about valuations, which are then used to determine prices and allocations. However, bidders will have incentives to withhold information if the information will affect prices paid. Iterative bidding, and other competitive processes, can mitigate bidders' incentives for strategic withholding. These iterative and competitive processes require efficient data exchange and processing facilities—facilities that would not otherwise be available without computers. Moreover the use of the Internet greatly expands the pool of potential auction participants and reduces participation costs. These factors tend to improve auction results, and make auctions feasible for transactions that could never be realized absent computers and electronic communication networks.

1.3.1 New Types of Auctions

Over the past twenty years there has been a proliferation of different auction types. This section does not catalog all of them. Instead, it briefly explains a few of the ways in which auction design has changed, due in part to changes in electronic communication technology, and in part to developments in game theory. The former allows more information to be transmitted and processed more quickly. The latter provides better incentives, and allows auctions to be introduced in areas where they may never have been possible before.

There have been significant advances in multi-object auctions. A large class of simultaneous auctions for buying or selling a number of substitutes or complements at one time includes the *simultaneous multi-round (SMR) auction*, the *simultaneous ascending auction (SAA)*, the

simultaneous ascending and descending clock auctions,⁶ and package bid auctions, the Anglo-Dutch hybrid auction, simultaneous auctions with intra-round bidding, and many other designs. They have been introduced building off developments in game theory for auction design on the one hand, and in computer and communication technology on the other. In addition a few types of package bid auctions have been introduced, including the SMR auction with hierarchical package bidding used for the \$19 billion auction of 700 MHz licenses conducted by the US Federal Communications Commission in 2008.

Even simple auction designs have changed. As noted above, online auctions typically employ a variant of the standard English auction in which a bidding window is open for a fixed duration, and bidders can enter offers as long as their offer exceeds the previous high bid by a minimum amount and the bidding window is open. This type of auction has been referred to as a “Yankee” auction. It has many variants, including one that extends the bidding window any time a new bid is entered. eBay has a “buy-it-now” feature that allows a bidder to enter a bid and close the auction.

Computers make possible package, or combinatorial, bidding. A package bid is an all-or-nothing bid on a combination of items. Package bidding is intended to address the *exposure* problem: a bidder having a value for a combination or package of items, but having a zero or sufficiently low value for the individual items, will be reluctant to submit bids on the individual items and risk winning them, unless that bidder is fairly certain it will win all the items in the package. Package bidding eliminates this exposure risk. However, package bidding adds a great deal of computational complexity because of the large number of possible combinations. When there are N items available, there will be 2^N possible subsets.⁷ This number will exceed 1,000 with only 10 items available.

Many new designs address the form of bids and the way bids are submitted to the originator.

1.3.2 Auctions Replacing Regulation

One area in which auctions have become an increasingly accepted transaction mechanism is in regulation, especially in the energy and telecommunications sectors. New Zealand and the United States were the first countries to introduce auctions for allocating spectrum rights.⁸ Prior to auctions being used for allocating spectrum rights, a variety of allocation rules—including first-come, first-served; lotteries;

and comparative hearings—were used. Auctions have now been introduced for allocating spectrum rights in dozens of countries. Spectrum auctions have perhaps been the largest auctions in history; the three largest spectrum auctions to date alone generated over \$100 billion in revenues—nearly \$50 billion for the German 3G auction, nearly \$35 billion for the UK 3G auction, and almost \$19 billion for the US 700 MHz auction.

Not all spectrum auctions have been successful. The success of the New Zealand auctions is debatable in view of their evidently low revenues and inefficiencies.⁹ The US FCC's auction 5 resulted in most of the largest bidders declaring bankruptcy shortly after the auction. These bidders tried to renegotiate lower prices, and a number of the large winners were successful in doing so. It took almost ten years and a Supreme Court decision to resolve the conflict between bankruptcy law and the auction rules (*FCC v. NextWave Personal Communications, Inc.*, 537 US 293, 302 (2003)).

Despite a few such problems, auctions appear to be a tool gaining increasing acceptance from regulatory agencies. To date the US Federal Communications Commission has conducted over fifty auctions, the UK, Australian, and Canadian telecommunication regulatory authorities have conducted over half a dozen auctions each, and telecommunication regulatory agencies in many other countries have used auctions for selling spectrum rights. Auctions therefore appear to be a permanent part of the regulatory process for managing spectrum in a great many countries. Spectrum auctions will be discussed in more detail in chapter 2.

Auctions are also increasingly common in the energy sector for electricity and gas. Simultaneous descending clock auctions are or have been used for energy procurement in a number of US states, starting with New Jersey and now including Montana, Illinois, Ohio, and California, and also in Spain and Italy. Other simultaneous auction formats are or have been used for selling entitlements for electricity in Belgium, France, Germany, Alberta (Canada), and Texas. Sealed-bid auctions are being used for energy procurement in Maryland, Delaware, Virginia, and the District of Columbia. Auctions are also being used for selling transmission or interconnector rights in the United States and Europe, and for capacity transactions in much of the United States. In the gas sector, various versions of the simultaneous multi-round clock auction have been conducted in Austria, Germany, and Hungary for capacity rights.

Not all the experience has been positive. California has had particularly troubling experiences—in 1993, and then during the summer and fall of 2000.¹⁰ The failure of California Power Exchange, a set of daily and longer term auctions, is a well-documented example of how not to run an auction. The California QF auctions described below resulted in *negative* energy prices. And some Texas capacity entitlement auctions resulted in difficult-to-explain 50 percent or greater price differences for identical products in the same auction. Both of these auctions are described in more detail in chapters 9 and 10.

In both the energy and telecommunication sectors, participation of qualified bidders is essential for a successful auction. Bidders don't always show up in sufficient numbers to make auctions competitive. Some examples in telecommunications include the first US auctions for 700 MHz spectrum and wireless communications services in 2000 and in 1997. In the latter case the forecast revenues were about 100 times the actual auction revenues. In particular, the Congressional Budget Office had forecast a value of \$1.8 billion, but the auction raised only \$13.6 million.¹¹ Similarly the Ohio First Energy procurement auctions have failed over several years to attract adequate competition to allow First Energy to purchase any of its default service resources through the auction.

1.3.3 Auctions in the Private Sector

There may be more auction activity in the private sector than in the public sector, but much less information is publicly available about private sector auctions. Information about wholesale or business-to-business (B2B) transactions tends to be confidential. There has been an increasing reliance on auctions for both buying and selling in B2B dealings. Auctions are being used for everything from selling agricultural commodities to procuring professional services. A significant number of firms have arisen in the B2B sector, and many have folded or merged. Among the notable firms still around as of 2013 were Perfect Commerce (formerly Commerce One and Perfect), Ariba-Procuri (which includes what were formerly called Ariba, Procuri and Trading Dynamics), and Dovebid. Many of these online B2B auction firms have migrated into supply chain management, and many others have focused on one or two specific supply chains or “verticals,” such as Hambricht for IPOs, ChemConnect for chemicals, and Nexant for energy.

For retail auctions, consumer-to-consumer (C2C) and business-to-consumer (B2C) eBay and Yahoo have the largest market shares. What is important for consumer auctions is participation, and so there has been some tendency toward increasing concentration on the consumer side. However, specialized sites such as those for travel and entertainment have been able to maintain some market presence. Consumer auctions tend to be less rigidly structured, be less promoted, and have lower participation than do B2B or government-run auctions. For this reason, less of what follows applies to these consumer auctions. What has also been notable is that many of the online auction operations have encountered difficulties. There are a few exceptions, notably the online auction site eBay; and for ads, Google and Yahoo, which rely heavily on an auction pricing mechanism to price position on their search pages.

1.4 Why Auction Design (and Management) Matters

What seems to be clear is that no one auction approach works all the time. Paul Klemperer states that auction design is a matter of the “horse for the courses” and not “one size fit all.”¹² There are several reasons auction design matters.

First, not all auctions work equally well. There are many things that can go wrong. In some cases the possibility of misallocations caused by an auction design that puts bidders in a position where they are prone to make miscalculations and bad guesses tends not to matter as much when there are many competitors.

Second, in some cases, notably in common value or affiliated value auctions, vigorous competition can result in overbidding and ex post performance problems.¹³

Third, in multi-lot auctions, misallocations can occur for several reasons. A great deal of auction design work has been devoted to multi-product and multi-unit auctions.¹⁴ One of the main concerns in multi-object auctions is the exposure problem, described above, where a bidder may have a low value for the individual items in a package on which it places a high value. Package bidding and simultaneous auction designs, including the SMR and the SMR with both hierarchical and nonhierarchical package bidding, were designed in part to address this concern. Auctions without package bidding are more likely to leave bidders unable to obtain efficient combinations of lots.

Package bidding raised other concerns, one of which is the *threshold problem*. When two (or more) bidders are seeking individual items for which a third bidder has placed a package bid, then the two bidders may need to find a way to coordinate their bids.

For example, suppose that there are three bidders, B1, B2, and B3, and two items, I and II. Suppose that B1 wants item I, B2 wants II, and B3 wants both. If there is not much competition for I or II alone, neither B1 nor B2 might have to offer very much to be the high bidder on those items. Now suppose that B3 has placed a bid of 8, which is posted, for both items together, and that B1 puts a value of 6 on item I and B2 puts a value of 6 on item II. If, absent B3, these two items would sell for 2 each, then B2 and B3 would each need to at least double their bids to win. Neither may want to do so without knowing how much the other might want to offer. This can result in B3 winning, when B1 and B2 should. The reverse can happen when B3 has the high combined value but cannot enter a package bid. This possibility has been a topic of active research,¹⁵ and recently the UK government has introduced versions of *core-selecting* package auctions to address it. Chapters 9 and 10 presents some of the theory of SMR and package auctions. Chapters 9 and 10 also present experience with multi-object auctions in the telecommunications and energy sectors. Package bidding also can add significant complexity. The number of possible combinations can get unmanageably large very fast.

One additional reason auction design matters is that some auctions are better at solving coordination problems for bidders.

On the flip side, there are instances in which it is possible for bidders to coordinate so as to divide the market and limit competition.¹⁶ As a response, regulatory authorities responsible for setting auction rules have limited the information reported to bidders during an auction. The better bidders are able to communicate, the easier is coordination, and at times such coordination can serve primarily to reduce competition in an auction. So the form of bids and the information reported to bidders can affect competition in an auction.

As much as design matters, how well an auction does also depends a great deal on implementation. It is important to emphasize that a primary requisite for any auction is well-prepared bidders. The rules, procedures, and management must facilitate bidder participation and not deter it. Too often the reverse is true, often because the originator wants to guarantee the best outcome from his or her perspective. By optimizing the bidding rules for the originator, participation

is discouraged, which paradoxically results in bad outcomes for the originator. And a competitive auction with less than ideal rules can often result in more efficient outcomes than will a good auction design with low participation. What is required is to enhance prospects for participation. What measures need to be taken so that participation does not sacrifice post-auction performance is discussed in chapter 11.

However, auctions with multiple units of the same object do not always result in uniform prices. One of the great successes in auction design is the development of efficient auction designs for multi-unit auctions that are easy to run. Clock auctions are quite efficient, and fast, for multi-unit, single-product auctions. Clock auctions and SMR auctions also work quite well when the products are substitutes and there are no large bidders. As has been noted elsewhere,¹⁷ however, clock and SMR auctions can be vulnerable to collusive or coordinated bidding. Klemperer designed the Anglo-Dutch hybrid to try to strike a balance between the benefits of an open SMR auction and the risks of collusion.

Many new auction designs involve improvements to the bid submission process or to accommodate online bidding. The time restrictions imposed in many online auctions, specifying a narrow bidding window, create incentives for bidders to wait until the last possible instant to bid—an activity called *sniping*. The clock auction eliminates this incentive, as the auction manager, rather than the bidder, raises the price. However, for reasons explained below, clock auctions are less practical for many consumer and B2B transactions. This has spawned a great deal of effort to adjust for sniping.

1.5 Outline of This Primer

This remainder of the Primer is divided into three parts: (1) theory; (2) auction practice—that is, design, organization, and management; and (3) experience.

The theory part starts with a very brief introduction to basic concepts from game theory. The game theory framework is an essential tool for analyzing auctions. I then explain a number of the more significant results from the auction theory literature, including the revenue, or payoff, equivalence theorem, the winner's curse, optimal auction design, and the theory of simultaneous and sequential auctions. The revenue equivalence theorem is worthy of specific mention as a key result that has widespread applicability. The analysis of simultaneous

and sequential auctions is of great practical concern, as an auction originator with multiple objects, or multiple units, to auction will need to decide on whether to run one auction or more, and if more, how many.

The auction practice sections address measures that can be taken to mitigate bidder collusion and to enhance competition, provisions for information disclosure, volume adjustments, and other basic principles of auction management that can enhance prospects for participation and a competitive outcome. Information disclosure provisions need to balance the benefits of information pooling and the risks of collusion. While full disclosure of all bids can invite market division and coordinated bidding, limited disclosure can often achieve the benefits of full disclosure without increasing the risk of anticompetitive bidding behavior. Volume adjustments are another new administrative tool that can be used to encourage more competitive bidding. These issues are discussed in more detail below. This part of the book also contains some discussion of auction experiments and simulations. Experiments, on one hand, are an increasingly popular research area in which subjects are recruited, and compensated, for participating in auctions staged in controlled environments. Simulations, on the other hand, rely on computer bidders and allow the computer to calculate the outcomes. Both forms of tests have benefits as well as limitations.

The third part of this Primer discusses experience. Most of the discussion surrounds energy and telecommunications auctions, as there is enormous experience in these sectors, and they have been the proving ground for many new auction designs. These sectors also include some of the largest and most newsworthy auctions. This part will discuss auctions in other sectors, including natural resources, commodities, and general procurement.

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