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Debt Dynamics and Fiscal Sustainability

Carlo Cottarelli and Julio Escolano

2.1 Introduction

This chapter presents the analytical relationships that describe government debt dynamics and underpin the sustainability of fiscal policies. A fiscal policy plan is understood here as a plan for revenue and expenditure, or more succinctly, for the resulting primary balances. Sustainability of fiscal policy plans underlies the discussion of debt dynamics. In loose terms, a fiscal policy plan is sustainable if it can be implemented without the government becoming unable to roll over its debt and having to resort to extreme measures such as default or hyperinflation.[1] In more technical terms, a fiscal policy plan can be defined as sustainable if it is compatible with an economic equilibrium—where markets clear, given the policy plan and the characteristics of the economy, including preferences of economic agents, available technology, and market institutions.[2]

The discussion here focuses on features of fiscal policy plans and associated debt dynamics that can point to their sustainability or unsustainability on the basis of observed evidence. Section 2.2 lays out the key concepts of debt dynamics used throughout the chapter and discusses the importance of long-term government solvency (the intertemporal budget constraint) and a stable debt ratio under the different scenarios of the differential between interest rates and growth. Section 2.3 discusses, in this light, various practical methodologies to assess the sustainability of fiscal policies. Some of these methodologies are eminently heuristic, while others are set within an explicit probabilistic setting. Finally, section 2.3 presents some extensions to deal with the debt dynamics in special cases.

2.2 An Overview of Debt Dynamics

The basic identity describing the movement of government debt states that debt at the end of the year (or any other conventional period) is given by the debt at the end of the previous year augmented by interest payments and reduced by the primary balance during the period:

$$D_t = (1 + i_t)D_{t-1} - P_t. \tag{2.1}$$

In this identity, D_t is the debt at the end of year t, and P_t is the primary balance in year t—both in national currency units (e.g., current dollars).[3] Most of the discussion here will be conducted in terms of gross debt (i.e., without netting out asset holdings), and under the assumption that there is no indexed or foreign currency-denominated debt. Extensions for the treatment of net debt, or debt denominated in foreign currency or indexed, will be discussed at the end of this chapter. The interest rate i_t in equation (2.1) is the average effective interest rate paid by the government in year t.[4] Given that governments typically have outstanding debt issued at different dates in the past, the average effective interest rate is not the market yield of government debt in year t.[5] Rather, the current market yield is the rate that the government would have to pay on an additional unit of debt (i.e., the marginal interest rate). As the interest rate varies over time, the average and marginal interest rates will not generally be the same. However, since the time variation of i_t plays only a secondary role in the discussion that follows, it will be assumed to be constant ($i_t = i$). As equation (2.1) is an identity, it conveys no information on the sustainability of the public finances. A discussion of sustainability requires additional structure.

As discussed in chapter 1, the concept of sustainability (as defined above) is closely related to the concept of solvency. It is therefore natural to start an analysis of sustainability by looking at the conditions under which a government can be regarded as solvent based on its ability to meet its budget constraint. When evaluating the economic viability of a specific ongoing investment project, it is common to require that the investment returns be sufficient, by the end of the project, to fully service existing debt (principal and interest) as well as any other debt incurred along the way. The economic operations of a government, however, involve ongoing recurrent revenue and expenditure for general purposes and do not have a natural end point or deadline when all debt must be paid off. Thus a similar characterization of the viability of a fiscal policy plan is given by the government's intertemporal budget constraint (IBC), which considers an infinite horizon: it states that the present value of all future primary balances (discounted at the discount rate i) must equal the initial debt

$$\text{IBC (nominal currency units): } D_0 = \sum_{t=1}^{\infty} \frac{P_t}{(1+i)^t}. \tag{2.2}$$

The IBC is not an accounting identity, since it rules out a large set of fiscal policy plans. Specifically, the policies that it rules out are the Ponzi games:[6] policies where debt and interest are systematically paid by issuing new debt. Equivalently these policies are also ruled out explicitly by the no-Ponzi game condition (NPG):

$$\text{NPG (nominal currency units): } \lim_{t\to\infty} \frac{D_t}{(1+i)^t} = 0. \tag{2.3}$$

It can indeed be shown that the IBC is formally equivalent to the NPG—if either holds, so does the other.[7] Loosely speaking, the NPG states that in the long run, debt cannot grow at a rate equal or higher than the interest rate. This implies that debt service cannot be financed exclusively by issuing more debt because in that case the debt would accumulate at least at the rate of interest. In particular, under policy plans that do not meet the IBC/NPG, the snowball effect (i.e., $(1+i)D_{t-1}$, the first term in the right-hand side of equation 2.1) dominates the dynamics of debt in the long term. This is because under these policy plans either the primary balance is not positive (i.e., is not in surplus) or, if it is positive, it does not keep pace with the stock of debt. Given their equivalence, we will henceforth refer just to the IBC.

While the IBC rules out the Ponzi game type of policy plans, it does not imply that debt must be eventually paid in full. For example, a policy plan that sets the primary balance at the level that is just enough to pay the interest bill ($P_t = iD_0$) will keep debt constant at its initial level (which hence will never be paid back), but it will meet the IBC condition.

As noted, the IBC is written in "dollar" terms, not in relation to GDP. Why then is fiscal solvency often assessed by looking at the behavior of the debt to GDP ratio?

Through algebraic manipulation, equation (2.1) can be expressed as a proportion of (nominal) output:

$$d_t = \frac{(1+i_t)}{(1+\gamma_t)} d_{t-1} - p_t, \tag{2.4}$$

where lowercase letters d_t and p_t represent now the corresponding magnitudes as a ratio to GDP in year t, and γ_t is the growth rate of nominal GDP. We also define the interest-rate–growth differential (IRGD) λ_t as follows:

$$\lambda_t = \frac{i_t - \gamma_t}{1+\gamma_t} = \frac{r_t - g_t}{1+g_t}, \tag{2.5}$$

$$1+\lambda_t = \frac{1+i_t}{1+\gamma_t} = \frac{1+r_t}{1+g_t}, \tag{2.6}$$

where r_t and $g_t$are the real interest and growth rates (both adjusted by the GDP deflator). [8] For ease of notation, we will assume from now on that the IRGD is constant over time ($\lambda_t = \lambda$), as well as the underlying growth and interest rates. Under the representation of debt as a ratio to GDP, the identity describing the movement of the debt ratio, the government intertemporal budget constraint (IBC) and the no-Ponzi game (NPG) condition can be expressed respectively as follows:

$$d_t = (1+\lambda)d_{t-1} - p, \tag{2.7}$$

$$\text{IBC (ratios to GDP) } d_0 = \sum_{t=1}^{\infty}(1+\lambda)^{-t} p_t, \tag{2.8}$$

$$\text{NPG (ratios to GDP) } \lim_{N\to\infty}(1+\lambda)^{-N} d_N = 0. \tag{2.9}$$

Notice, however, that these are completely identical to equations (2.1) to (2.3). The dynamics of debt can indeed also be expressed as a ratio to any other magnitude through formally similar equations—such as debt as a ratio to private financial wealth (as an indicator of potential demand for debt), government assets (as an indicator of collateral coverage), and any other magnitude (rainfall in Texas). Critically, changes in γ (including when the latter is the growth rate of GDP) would be irrelevant for meeting the IBC. So, why is the behavior of d (which definitely depends on the growth rate of GDP, other things being the same) regarded as key to solvency?

To answer this question, we have to consider that the "dollar" value of primary balances typically depends on the level of GDP, as the latter is the base for taxation; that is, the base for the resources that can be used to service debt. So a higher growth rate of GDP implies that revenues rise more rapidly.

Let's focus therefore on fiscal plans in which p_t is constant and equal to p (or p_t rises up to a level that is the maximum sustainable for the economy, i.e., the primary balance in relation to GDP is bounded). Assume also, for the moment, that the interest rate is larger than the growth rate of the economy, meaning $\lambda > 0$, a common occurrence in mature economies.[9]

In this case the stability of d ensures that the IBC condition is met:[10] the two conditions are mathematically equivalent (e.g., see Bartolini and Cottarelli 1994). The intuition is simple: if p is such that d is constant, nominal debt is rising at the GDP growth rate and is therefore rising at a rate below the interest rate, which is the essence of the IBC. Conversely, if the interest rate is higher than the GDP growth rate—or is expected to become so over the long term—the IBC condition, jointly with the observation that the primary surplus as percentage of GDP cannot grow indefinitely, imply a bounded debt-to-GDP ratio. And if the maximum amount of resources—as measured by the primary balance—that a government can potentially mobilize to service its debt is roughly proportional to the size of the GDP, then the debt ratio must be stable if it is to meet the IBC condition (Escolano 2010).[11]

Things are, however, different when the interest rate is lower than the growth rate.[12] A GDP growth rate in excess of the interest rate exerts a powerful stabilizing force on the debt ratio. Essentially it allows persistent primary deficits while maintaining a stable or even declining debt ratio. But with primary deficits the ICB condition cannot be met: the stability of the debt ratio will therefore not be sufficient to ensure that the IBC condition is met. Such stability is therefore a milder solvency condition than the IBC.[13] That is, under a nonpositive IRGD, a constant debt ratio is a Ponzi game. It would be, however, what is sometimes called "an honest Ponzi game," in the sense that the growth of debt is still bounded by the growth of what is ultimately its "collateral," namely GDP, the source of government revenues. That is ultimately why, even if the IBC condition is not met, a stable debt ratio is regarded as sustainable.

2.3 Debt Sustainability Indicators

The definition of sustainability above hinges on the IBC condition or, at least, the stability of the debt ratio, and it has given rise to a number of relevant indicators. An even partial coverage of the increasingly abundant literature on this topic is beyond the scope of this chapter.[14] Instead, we will focus on a few methodologies that are widely used, offering some comments in light of the conceptual discussion above. In general terms, these methodologies fall into three broad categories: (1) *gap measures,* which are based on the difference between the actual value of a fiscal magnitude, such as the primary balance, and the notional value that would meet specific criteria, such as hitting a target debt ratio in specified time or satisfying the IBC condition; (2) estimates of fiscal policy reaction functions (FPRF), which allow testing the consistency of these FPRFs with the IBC condition or the absence of Ponzi-game type explosive debt dynamics; and (3) *fiscal vulnerability indicators,* which flag the likelihood of a future fiscal crisis or fiscal stress when pre-specified values are crossed (often based on their past predictive power).

2.3.1 Fiscal Gap Measures

Debt-Stabilizing Primary

Possibly the simplest indicator of debt sustainability is the gap between the primary balance ratio to GDP that would stabilize the current debt ratio and the actual primary balance ratio (Blanchard 1990; Blanchard et al. 1990). Given a value of the IRGD (λ^*, possibly estimated on the basis of a historical average or a projection), the primary balance ratio that stabilizes the debt ratio at a given level (d^*) is $p^* = \lambda^* d^*$. Thus comparing p^* (often calculated at the current debt ratio) with the actual primary balance ratio (possibly cyclically adjusted to capture its underlying level) provides a measure of the adjustment that would be necessary to stabilize the debt ratio.

The *s*1 Indicator

An alternative indicator of sustainability is the gap between the primary balance ratio that would bring the debt ratio to a pre-specified level within a given period and the present actual primary balance ratio (European Commission 2009). This indicator is often called the "*s*1" indicator and it is regularly used in the European Union to assess the sustainability of fiscal policy in member countries—with a target debt ratio of 60 percent and a period consistent with the surveillance horizon under the Stability and Growth Pact. The following formula can be used to calculate the primary ratio (p^*) that must be maintained for N years to reach a target debt ratio of d_N^* if the initial debt ratio is d_0:

$$p^* = \frac{\lambda}{(1+\lambda)^{-N} - 1}\left[(1+\lambda)^{-N} d_N^* - d_0\right].$$

A similar approach has been used in the IMF's *Fiscal Monitor* to calculate adjustment needs benchmarks for advanced and emerging economies.[15] The *s*1 indicator, however, requires a choice, to some extent arbitrary, of the target debt ratio (d_N^*) and of the time allowed for hitting the target (N).

The *s*2 Indicator

The *s*2 indicator aims at benchmarking the long-term sustainability of a fiscal policy plan. Typically, this policy plan comprises a short- to medium-term period for which policies are specified in detail and a subsequent long-term projection that keeps the primary balance ratio to GDP constant except for the effects of some variables of interest for which long-term projections are available—such as health and pension expenditure, or fiscal revenue from natural resources subject to depletion. This indicator is regularly used in the European Union to measure long-term adjustment needs of member economies taking into account long-term trends in age-related expenditure (European Commission 2009, 2011).

Given a path of projected primary balances for all future periods $\left(\{p_t\}_{t=1}^{\infty}\right)$, the sustainability indicator *s*2 is defined as the fixed annual addition at perpetuity (expressed as a ratio to the contemporaneous GDP) to the projected primary balances that would be necessary to meet the IBC condition. Thus, using the IBC equation (2.8), we can define the *s*2 indicator implicitly as follows:

$$d_0 = \sum_{t=1}^{\infty} (1+\lambda)^{-t} (p_t + s2),$$

or by its equivalent explicit formula

$$s2 = \lambda d_0 - \lambda \sum_{t=1}^{\infty} (1+\lambda)^{-t} p_t.$$

Since there is no presumption that the shape of the projected sequence of primary balances is optimal or that a fixed annual addition is the best policy approach, the indicator should be considered a benchmark and not necessarily a policy recommendation or a measure of the adjustment needed in any particular year.

2.3.2 Sustainability Tests Based on the Fiscal Policy Reaction Function (FPRF)

This strain of sustainability tests aims at estimating whether the fiscal policy reaction function, as revealed by the past behavior of fiscal variables, is consistent with debt sustainability—provided that the same policy reaction function is maintained in the future. A FPRF specifies the behavior of the key fiscal policy instruments (typically, the primary balance ratio) as a function of the debt ratio and other relevant variables—such as the output gap. Sustainability tests are useful to reveal the need for change in the way fiscal policies are conducted (i.e., a change in the FPRF) before the debt dynamics becomes explosive. Also sustainability tests have been used to assess the risk of adverse market reactions in the near future—the size of the "fiscal space" available. Thus, if the estimated policy reaction function leads to explosive debt dynamics, the distance between the current debt ratio and the point at which its dynamics become explosive can be used as a measure of the available fiscal space.

Many of the proposed tests aim at detecting a co-integrating relationship linking the primary balance—or primary expenditure and revenue separately—and the outstanding stock of debt. Under some conditions these co-integrating relationships are sufficient for the IBC to hold. Specifically, most sustainability tests follow different variations of the methodology proposed by Bohn (1998, 2005, 2007). This methodology produces an explicit estimate of the FPRF—which can be itself of independent interest. It posits the existence of a FPRF as follows:

$$p_t = \alpha d_{t-1} + \beta X_t + \upsilon_t, \tag{2.10}$$

where p_t and d_t are, as before, the primary and debt ratios respectively; X_t is a vector of exogenous variables influencing the primary balance (e.g., the output gap, trade openness, terms of trade, fixed effects); υ_t is a random shock; and α and β are parameters to be estimated. The coefficient α measures the forcefulness with which government policies react to an increase in the debt ratio: other things equal, an increase in the debt ratio of Δd will prompt an increase in the primary balance ratio of $\alpha\Delta d$. It is commonly assumed that the stochastic process $\beta X_t + \upsilon_t$ is bounded and that the IRGD is positive ($\lambda > 0$).[16] Using the FPRF given by equation (2.10) and the identity describing the movement of the debt ratio (equation 2.7), the dynamics of the debt ratio is given by the following:

$$d_t = (1 + (\lambda - \alpha))d_{t-1} - \beta X_t - \upsilon_t. \tag{2.11}$$

In this setting, whether the debt path meets the IBC condition hinges on the value of α, which can be statistically estimated and tested. It is clear, from equation (2.11) that the asymptotic rate of growth of the debt ratio is $\lambda - \alpha$, since the last two right-hand-side terms are bounded. Thus, when $\alpha > 0$ (and therefore $\lambda - \alpha < \lambda$), the debt ratio will meet the NPG (equation 2.9), and a fortiori the IBC (Bohn 1998).

Taking into account the existence of a maximum primary surplus ratio, Ostry, Ghosh, Kim, and Qureshi (2010) and Ghosh, Kim, Mendoza, Ostry, and Qureshi (2011) estimate a nonlinear policy reaction function—with a nonlinear $\alpha(d_{t-1})$ replacing αd_{t-1} in equation (2.10). They find cross-country evidence that the primary balance reacts increasingly less to higher values of the debt ratio and even declines after some point.[17] Based on the estimated nonlinear policy reaction function, they calculate a "point of no return" for the debt ratio. Beyond that point, the debt ratio sets on an explosive dynamics that the increasingly weaker reaction of the primary balance (because the nonlinearity) is impotent to stop. They propose that the difference between this point of no return and the actual debt ratio can be seen as a measure of the fiscal space available for a country.[18]

2.3.3 Sustainability Indicators

This approach to measuring the sustainability of fiscal policy aims a systematizing the use of a broad variety of fiscal and macroeconomic indicators to flag the risk of a fiscal crisis. It builds on the literature on early warning system models—until recently, covering mainly external and financial crises.[19] We will focus here on the methodology proposed by Baldacci, McHugh, and Petrova (2011), Baldacci et al. (2011), and IMF (2011b).[20] Rather than testing the IBC condition or estimating the deviation with respect to that condition, this methodology starts with a large set of indicators, and identifies those that have shown a past capacity to predict subsequent episodes of fiscal stress. Then, these indicators are aggregated into a single index which can be viewed as a summary indicator of the likelihood of a fiscal distress episode. Both the definition of fiscal stress episodes and the choice of indicators are based on the conceptual framework of Cottarelli (2011), which is discussed in the opening chapter of this book.[21]

The first step in this methodology is choosing the features that define episodes of fiscal stress, characterized as periods of extreme government funding difficulties, such as debt default, or extremely high inflation or sovereign yields. In the second step, a threshold value is chosen for each indicator which, when crossed, signals that the indicator predicts a crisis. For each indicator, the threshold is chosen to minimize a combination of false positives (type I errors) and false negatives (type II errors).[22] Then, associated to each indicator, a dummy 0–1 variable is constructed according to whether the indicator sends a crisis signal (1) or a non-crisis signal (0). These variables, weighted by their predictive power (one minus the total misclassification error ratio), are aggregated into an index. In this way, variables are assigned a higher weight the higher their forecasting accuracy.

This methodology is valuable in building early warning systems, aiming to flag the potential for fiscal crises in the short term. Also it has the virtue of letting data speak for themselves, imposing little a priori structure as to the causal nexus between specific policies and eventual fiscal outcomes. It is ill-suited, however, to identify an unsustainable current fiscal stance that could result in a fiscal crisis only in the medium or long term or to single out the specific fiscal policy features that need to be changed.

2.4 Some Special Issues

2.4.1 Foreign Currency-Denominated and Inflation-Indexed Debt

The discussion of debt dynamics in this chapter, as in most of the literature, assumes that all debt is denominated in domestic currency and nonindexed. In practice, however, many countries issue sovereign debt denominated in foreign currencies or indexed to inflation. We develop here the expressions of the debt ratio dynamics focusing on the case of foreign currency-denominated debt, and then we point out how these expressions can be adapted to inflation-indexed debt.

The law of movement of the debt ratio (equation 2.7) when there is foreign currency-denominated debt is formally identical to the case when all debt is denominated in domestic currency. The only difference is that the IRGD (λ) must be replaced by an adjusted IRGD ($\hat{\lambda}$) that takes into account the change in the stock of debt and interest payments due to the variation of the exchange rate. To this end, the interest rate (i) must be replaced by an adjusted return rate (ρ) that incorporates these exchange rate effects:

$$\hat{\lambda}_t = \frac{\rho_t - \gamma_t}{1 + \gamma_t}, \tag{2.12}$$

$$d_t = \left(1 + \frac{\rho_t - \gamma_t}{1 + \gamma_t}\right) d_{t-1} - p_t = \left(1 + \hat{\lambda}_t\right) d_{t-1} - p_t. \tag{2.13}$$

In these expressions, d_t represents the stock of debt at the end of year t as a ratio to GDP (with foreign currency-denominated debt valued in domestic currency at the end-year exchange rate). As before, γ_t is the nominal growth of GDP and p_t is the primary balance ratio to GDP, both in year t.

Let α_t be the proportion of total debt denominated in foreign currency (measured in domestic currency equivalent) at the end of year t and let ε_t be the depreciation of the exchange rate (measured as units of domestic currency per unit of foreign currency) in year t. Also let i^d and i^f be, respectively, the interest rates on domestic currency-denominated and foreign currency-denominated instruments. Then the expressions for the adjusted return on debt (ρ) and associated adjusted interest rate ($\hat{i}$) are as follows:

$$\rho_t = \hat{i}_t + \alpha_{t-1}\varepsilon_t, \tag{2.14}$$

$$\hat{i}_t = (1-\alpha_{t-1})i_t^d + \alpha_{t-1}(1+\varepsilon_t)i_t^f = \frac{\text{Total budget interest bill in } t}{\text{Total debt at end of } t-1}. \quad (2.15)$$

Notice that as in the domestic currency debt case, $\hat{i}_t$ can be easily calculated as the interest bill reported in the budget divided by the stock of debt at the end of the previous year.[23]

This treatment is easily generalized to debt in several foreign currencies by extending the second term in equations (2.14) and (2.15) with additional terms, one for each foreign currency, weighted by the relative proportion of total debt denominated in that currency (replacing the weighting α_{t-1} above). Since the law of movement of the debt ratio with and without foreign currency-denominated debt (equations 2.13 and 2.7) are formally the same, other debt dynamics expressions and conclusions also carry through with the appropriate replacement of the adjusted IRGD ($\hat{\lambda}$) for the domestic-currency IRGD (λ) and of the adjusted return on debt (ρ) for the domestic interest rate (i).

When some debt is denominated in domestic currency but indexed to inflation, equations (2.12) through (2.15) still apply with appropriate re-interpretations and modifications of the variables. This is because inflation-indexed debt is formally similar to debt indexed to a notional "foreign currency" that maintains its real value.[24] Specifically, α_t represents the proportion of indexed debt (measured at its current inflation-indexed value) relative to total debt; i^f represents the (real) interest rate paid on indexed debt; and ε_t represents the inflation rate according to the appropriate price index. The adjusted return on debt (ρ) then corresponds to the interest payments reported in the budget divided by debt outstanding at the end of the previous year—if budget accounts report interest actually paid as well as accrued though increase of the principal due to indexation, in accordance with international standards of budget accounting and reporting.[25] If the budget reports only interest due for payment during the year, but not the uplift of the principal (cash accounting), the budget interest bill divided by debt outstanding at the end of the previous year corresponds to the adjusted interest rate ($\hat{i}_t$). Finally, the factor $(1+\varepsilon_t)$ in equation (2.15) should be dropped in most cases, depending on the contractual arrangements for the payment of interest.[26]

2.4.2 Gross and Net Debt

Debt dynamics and sustainability analyses are usually conducted on the basis of gross debt—as is done in the rest of this chapter. However, a more comprehensive government solvency analysis requires the consideration of assets as well as liabilities.[27] Also full consistency between flow variables (e.g., budget balance) and stocks (e.g., government's financial net worth) can only be achieved by taking into account the asset side of the government's balance sheet. Incorporating the asset side in debt dynamics analyses is particularly important when governments own large financial assets,[28] but also in more common situations such as when the debt management office pre-finances future expenditure needs well in advance to lock in favorable market conditions.

Factors that hinder the consideration of assets as part of a more comprehensive net debt analysis include the paucity of governments' balance sheet data and the heterogeneous nature of assets in a government's balance sheet—in contrast with liabilities, which are often relatively homogeneous. Specifically, assets have different levels of liquidity and marketability, and they are available to meet government financial obligations in different degrees (e.g., because their sale may require legal reforms or overcoming severe political constraints). As in other financial analyses, the type of assets that is appropriate to net out against debt obligations depends on the purpose of the analysis. Thus countries differ in the assets they consider in their calculations of net debt: ranging from none to liquid financial assets, all financial assets, all financial and nonfinancial assets, and so on. Moreover government assets are often hard to value (e.g., if they are not traded).

The discussion that follows presents the basic relationships that underlie the dynamics of net debt. Let a_t denote the value of assets at the end of year t as a ratio to GDP. Also the superscript a (respectively, d) will indicate that a variable (e.g., interest, interest-rate–growth differential) corresponds to assets (respectively, debt). Thus, for example, i^a (i^d) represents the nominal rate of return earned on assets (rate of interest paid on debt). Finally, p^g and p^n represent the gross and net primary balance ratios to GDP.[29] The gross primary balance is revenue less expenditure before interest paid on debt, while the net primary balance is revenue less expenditure before both receipts of returns on assets and interest paid on debt.[30] All other symbols retain their meanings from previous sections. Specifically, the following reflects the relationship between net and gross primary balance ratios:

$$p_t^g = p_t^n + \frac{i_t^a}{1+\gamma_t} a_{t-1}. \tag{2.16}$$

The laws of movement of asset and debt ratios in terms of the net and gross primary ratios are given by the following equations (the counterparts of equation 2.4):

$$d_t - a_t = (d_{t-1} - a_{t-1}) + \lambda_t^d d_{t-1} - \lambda_t^a a_{t-1} - p_t^n, \tag{2.17}$$

$$d_t - a_t = (d_{t-1} - a_{t-1}) + \lambda_t^d d_{t-1} + \frac{\gamma_t}{1+\gamma_t} a_{t-1} - p_t^g, \tag{2.18}$$

where the IRGD is now different for assets and debt,

$$\lambda_t^a = \frac{i_t^a - \gamma_t}{1+\gamma_t} = \frac{r_t^a - g_t}{1+g_t}, \quad \lambda_t^d = \frac{i_t^d - \gamma_t}{1+\gamma_t} = \frac{r_t^d - g_t}{1+g_t}. \tag{2.19}$$

When assets are brought into the light, it is clear that in analyses based on gross debt, it is often implicitly assumed that nominal assets remain unchanged[31]—and thus that asset ratios decline with the growth of GDP. This follows from implicitly assuming that primary surpluses are used to reduce debt (and not to build up assets) and primary deficits are financed by debt (and not by asset sales). This can be seen by rearranging equation (2.18):

$$d_t = \left(1+\lambda_t^d\right) d_{t-1} - p_t^g + \left(a_t - \frac{1}{1+\gamma_t} a_{t-1}\right). \tag{2.20}$$

This equation differs from equation (2.7), where assets were not considered, by the addition of the last term, which equals the increase in nominal assets as a ratio to GDP. Thus both equations are equivalent (and equation 2.7 holds) if and only if the last term is zero ($a_t = a_{t-1}/(1+\gamma_t)$): that is, the nominal value of assets remains unchanged and the asset ratio to GDP declines at the rate $1/(1+\gamma)$, reflecting the growth in GDP.

For the same reason, the (net or gross) primary balance that stabilizes both the asset and debt ratios at given levels (e.g., a^* and d^*) exceeds the (net or gross) primary balance that stabilizes only the debt ratio—by the asset buildup necessary for the asset ratio to keep pace with the growing GDP. Thus a country that targets a stable net debt (or financial net worth) needs to maintain a tighter fiscal stance than would be necessary to pay just the growth-adjusted gross interest bill ($p^g = \lambda^d d^*$). In particular, the net and gross primary balances that stabilize both the asset and debt ratios are the following (assuming constant interest rates):

$$p^{n^*} = \lambda^d d^* - \lambda^a a^*,$$

$$p^{g^*} = \lambda^d d^* + \frac{\gamma}{1+\gamma} a^* = p^{n^*} + \frac{i^a}{1+\gamma} a^*.$$

Notes

1. While a policy plan may be unviable for reasons other than the debt dynamics it generates (e.g., because it introduces large economic distortions and impedes economic growth), it will most often lead eventually to a loss of government access to credit markets and to difficulties in financing its operations.

2. Of course, the practical interests of markets and policy makers are in assessing the sustainability of policy plans in which the key parameters are not changed in the too distant future. In principle, any policy plan for a finite (and hence observable) period can become, in theory, a sustainable plan by adding to it, in an arbitrarily distant future, the necessary remedial policy actions. But this is uninteresting. Indeed, very often the interest lies in assessing the sustainability of the current policy setting.

3. The variable P_t is interpreted here as the fiscal primary balance (revenue less expenditure before interest payments), but it can be defined, if pertinent, to include also debt-changing

below-the-line operations. The value of these debt-changing below-the-line operations is often computed as a residual (the difference between the change in debt and the overall balance) and it is referred as the "stock-flow adjustment." The stock-flow adjustment may not be zero, inter alia, owing to privatization receipts used to redeem debt, other net acquisition or disposal of financial assets (e.g., financial sector support operations) financed with debt, or accounting adjustments. If the effective interest rate is adjusted for exchange rate-driven valuation changes in the foreign currency-denominated debt (as discussed below), the stock-flow adjustment should not include these valuation changes. The stock-flow adjustment is occasionally a result of deliberate attempts to obfuscate the true deficit (see IMF 2011b; Irwin 2012; Weber 2012). In the analysis that follows we abstract from debt monetization and seigniorage.

4. The average effective interest rate in year t is typically computed as the quotient between actual budgetary interest payments in t and the debt outstanding at the end of year $t-1$.

5. Moreover outstanding debt consists of instruments with various maturities that typically carry different interest rates.

6. Named after Charles Ponzi (1882–1949), a colorful although eventually unsuccessful swindler.

7. See proposition 1 in Escolano (2010).

8. Based on the identities $1+\gamma_t=(1+g_t)(1+\pi_t)$ and $1+i_t=(1+r_t)(1+\pi_t)$, where π_t is the growth of the GDP deflator in year t.

9. This condition, when postulated for a balanced growth path, is often called the "modified golden rule." Growth theory suggests that if the economy uses resources efficiently and private economic agents prefer current to future consumption, then the modified golden rule must hold, at least asymptotically over the long term for an economy that approaches a balanced growth path (see Blanchard and Fischer 1989). This is because, otherwise, private agents could always choose a strictly preferred alternative consumption-saving plan by reducing their holdings of assets (e.g., government debt) or increasing their liabilities. Essentially, in an equilibrium balanced growth path, the modified golden rule must hold, since otherwise economic agents would need to reinvest yearly at perpetuity all the return they obtain on their assets or more (since in a balanced growth path assets grow at the same rate as GDP; a rate that would be equal or higher than the interest rate). Individual price-taking agents would prefer, for example, to consume all their assets initially and avoid investment altogether. Even though this would violate aggregate feasibility, it shows that $\lambda \leq 0$ cannot be a balanced growth competitive equilibrium.

10. That is, the debt ratio may rise and fall, but it must remain within a stable range. It cannot grow unboundedly forever.

11. If p is not constant and can increase without bound, then, in principle, the IBC condition, under a positive IRGD, represents a weaker sustainability concept than the boundedness of the debt ratio. Under the IBC condition, the debt cannot grow at a rate equal or higher than the interest rate—that is, the government must service a proportion of the debt through primary surpluses. However, the debt can still grow faster than GDP, since the interest rate exceeds the GDP growth rate. For example, debt could grow at a rate intermediate between the GDP growth rate and the interest rate. So, in principle, an increasing debt ratio is compatible with the IBC condition. However, this would require an unbounded rise of p_t, which is unreasonable. The existence of a maximum level of the primary surplus-to-GDP

ratio is an eminently reasonable condition, and in practice, the feasible maximum primary surplus is likely to be only a few percentage points of GDP—owing either to Laffer curve effects or, probably before that, to political constraints. As pointed out in Keynes (1924) "when the State's contractual liabilities, fixed in terms of money, have reached an excessive proportion of the national income . . .. The active and working elements in no community, ancient or modern, will consent to hand over to the *rentier* or bond-holding class more than a certain proportion of the fruits of their work" (p. 64).

12. As discussed in a subsequent chapter, this happens in most developing economies; see also Escolano, Shabunina, and Woo (2011).

13. This is because a constant debt ratio will imply that debt grows at the same rate as GDP and thus that its rate of increase will exceed or be equal to the interest rate, implying a violation of the IBC condition.

14. European Commission (2011) contains a recent stocktaking of different methodologies. See also Chalk and Hemming (2000), Trehan and Walsh (1991), Quintos (1995), and Bohn (2005, 2007).

15. See IMF (2012) statistical tables 13a and 13b. The *Fiscal Monitor*, however, considers cyclically adjusted primary balances and a gradual adjustment for about ten years followed by ten years of constant primary balance at the required (high) level to hit the debt target. It also considers varying IRGD for advanced economies, taking into account the endogenous effects of high debt on interest rates and growth (Poghosyan 2012; Kumar and Woo 2010).

16. The assumption that the stochastic process $\beta X_t + \upsilon_t$ is bounded is not particularly restrictive if the control variables in X_t are the output gap, the trade balance as a ratio to GDP, dummy variables representing institutional features, annual percentage changes in the terms of trade, and so on, all of which are naturally bounded. The main results, however, also hold under weaker conditions: "almost surely" (i.e., for all realizations of the stochastic process except possibly for a set of probability zero) if $\beta X_t + \upsilon_t$ is almost surely bounded (with the bound depending on the realization); or in an expected value sense if $\beta X_t + \upsilon_t$ is stationary.

17. The empirical evidence from the literature, however, is still mixed on whether the fiscal response may vary when debt ratios are unusually high (European Commission 2011), on whether emerging and advanced economies (or indeed any two different economies) follow broadly similar FPRFs, or on the stability of FPRFs over time.

18. Note, however, that the point of no return for individual countries is based on panel estimates that involve the same reaction function for all countries, and that empirically only few observations exist describing the reaction at very high debt levels, as these have not been experienced by most countries.

19. See Baldacci, McHugh, and Petrova (2011), Baldacci et al. (2011), and European Commission (2011), and references therein.

20. See the discussion on fiscal sustainability risks in IMF (2011a, app. 3)), in particular, on deriving a fiscal sustainability risk map.

21. See also Schaechter et al. (2012) for a set of indicators for advanced economies compiled based on this conceptual framework.

22. This combination is typically the total misclassified error ratio, defined as the proportion of non-crisis years that were misclassified as crisis years plus the ratio of crisis years that were misclassified as non-crisis years.

23. This is because budget reporting is typically done in domestic currency units, with interest payments on foreign currency-denominated debt converted into domestic currency at the exchange rate of the date when the payment was made. This also corrects for the fact that interest is not always due at the end of the year (as assumed in the formulas) but at different dates throughout the year. Notice, however, that under standard fiscal accounting, the variation in the value of outstanding debt due to exchange rate changes is not reported as interest payments (it is a "below-the-line" holding gain or loss). Hence it is useful to distinguish ρ, which includes the exchange rate-induced increase in principal owed, from $\hat{i}$, which does not include it. Thus $\hat{i}$ closely corresponds to the fiscal accounting concept of interest and can be calculated based on the budget-reported interest bill.

24. Inflation-indexed debt, however, is different from foreign currency-denominated debt in other important respects not covered in this discussion. For example, a country that issues its own currency can always redeem inflation-index debt by issuing domestic currency (i.e., monetizing the debt). This expedient may not be feasible if the debt must contractually be redeemed in foreign currency and the country lacks the necessary foreign reserves and has no access to international credit markets.

25. See IMF (2001, para. 6.42–6.49).

26. The mentioned factor should be dropped if interest in year t is paid on the indexed value of the debt at the end of the previous year. It should be kept if interest is paid on the value of the debt after increasing it by the inflation in year t.

27. See IMF (2011b, app. 3) on the importance of monitoring gross and net debt.

28. For example, in the cases of Norway or Japan.

29. For the purposes of this discussion and the use of these formulas, any holding gains (losses) on assets can be added to (subtracted from) the primary balance to calculate p_t (net or gross). As before, holding gains (losses) on debt could also be added (subtracted) to the primary balance.

30. The gross primary balance has been denoted simply as "primary" balance in the rest of this chapter, where "debt" referred to gross debt.

31. Except for holding gains and losses. It is typically also assumed that the primary balance includes interest receipts (i.e., that it is the gross primary balance).

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