IV Feedback and Oscillation

A patient comes into a neurological clinic. He is not paralyzed, and he can move his legs when he receives the order. Nevertheless, he suffers under a severe disability. He walks with a peculiar uncertain gait, with eyes downcast on the ground and on his legs. He starts each step with a kick, throwing each leg in succession in front of him. If blindfolded, he cannot stand up, and totters to the ground. What is the matter with him?

Another patient comes in. While he sits at rest in his chair, there seems to be nothing wrong with him. However, offer him a cigarette, and he will swing his hand past it in trying to pick it up. This will be followed by an equally futile swing in the other direction, and this by still a third swing back, until his motion becomes nothing but a futile and violent oscillation. Give him a glass of water, and he will empty it in these swings before he is able to bring it to his mouth. What is the matter with him?

Both of these patients are suffering from one form or another of what is known as *ataxia*. Their muscles are strong and healthy enough, but they are unable to organize their actions. The first patient suffers from *tabes dorsalis*. The part of the spinal cord which ordinarily receives sensations has been damaged or destroyed by the late sequelae of syphilis. The incoming messages are blunted, if they have not totally disappeared. The receptors in the joints and tendons and muscles and the soles of his feet, which ordinarily convey to him the position and state of motion of his legs, send no messages which his central nervous system can pick up and transmit, and for information concerning his posture he is obliged to trust to his eyes and the balancing organs of his inner ear. In the jargon of the physiologist, he has lost an important part of his proprioceptive or kinesthetic sense.

The second patient has lost none of his proprioceptive sense. His injury is elsewhere, in the cerebellum, and he is suffering from what is known as a cerebellar tremor or purpose tremor. It seems likely that the cerebellum has some function of proportioning the muscular response to the proprioceptive input, and if this proportioning is disturbed, a tremor may be one of the results.

We thus see that for effective action on the outer world it is not only essential that we possess good effectors, but that the performance of these effectors be properly monitored back to the central nervous system, and that the readings of these monitors be properly combined with the other information coming in from the sense organs to produce a properly proportioned output to the effectors. Something quite similar is the case in mechanical systems. Let us consider a signal tower on a railroad. The signalman controls a number of levers which turn the semaphore signals on or off and which regulate the setting of the switches. However, it does not do for him to assume blindly that the signals and the switches have followed his orders. It may be that the switches have frozen fast, or that the weight of a load of snow has bent the signal arms, and that what he has supposed to be the actual state of the switches and the signals—his effectors—does not correspond to the orders he has given. To avoid the dangers inherent in this contingency, every effector, switch or signal, is attached to a telltale back in the signal tower, which conveys to the signalman its actual states and performance. This is the mechanical equivalent of the repeating of orders in the navy, according to a code by which every subordinate, upon the reception of an order, must repeat it back to his superior, to show that he has heard and understood it. It is on such repeated orders that the signalman must act.

Notice that in this system there is a human link in the chain of the transmission and return of information: in what we shall from now on call the chain of feedback. It is true that the signalman is not altogether a free agent; that his switches and signals are interlocked, either mechanically or electrically, and that he is not free to choose some of the more disastrous combinations. There are, however, feedback chains in which no human element intervenes. The ordinary thermostat by which we regulate the heating of a house is one of these. There is a setting for the desired room temperature; and if the actual temperature of the house is below this, an apparatus is actuated which opens the damper, or increases the flow of fuel oil, and brings the temperature of the house up to the desired level. If, on the other hand, the temperature of the house exceeds the desired level, the dampers are turned off or the flow of fuel oil is slackened or interrupted. In this way the temperature of the house is kept approximately at a steady level. Note that the constancy of this level depends on the good design of the thermostat, and that a badly designed thermostat may send the temperature of the house into violent oscillations not unlike the motions of the man suffering from cerebellar tremor.

Another example of a purely mechanical feedback system—the one originally treated by Clerk Maxwell—is that of the governor of a steam engine, which serves to regulate its velocity under varying conditions of load. In the original form designed by Watt, it consists of two balls attached to pendulum rods and swinging on opposite sides of a rotating shaft. They are kept down by their own weight or by a spring, and they are swung upward by a centrifugal action dependent on the angular velocity of the shaft. They thus assume a compromise position likewise dependent on the angular velocity. This position is transmitted by other rods to a collar about the shaft, which actuates a member which serves to open the intake valves of the cylinder when the engine slows down and the balls fall, and to close them when the engine speeds up and the balls rise. Notice that the feedback tends to oppose what the system is already doing, and is thus negative.

We have thus examples of negative feedbacks to stabilize temperature and negative feedbacks to stabilize velocity. There are also negative feedbacks to stabilize position, as in the case of the steering engines of a ship, which are actuated by the angular difference between the position of the wheel and the position of the rudder, and always act so as to bring the position of the rudder into accord with that of the wheel. The feedback of voluntary activity is of this nature. We do not will the motions of certain muscles, and indeed we generally do not know which muscles are to be moved to accomplish a given task; we will, say, to pick up a cigarette. Our motion is regulated by some measure of the amount by which it has not yet been accomplished.

The information fed back to the control center tends to oppose the departure of the controlled from the controlling quantity, but it may depend in widely different ways on this departure. The simplest control systems are linear: the output of the effector is a linear expression in the input, and when we add inputs, we also add outputs. The output is read by some apparatus equally linear. This reading is simply subtracted from the input. We wish to give a precise theory of the performance of such a piece of apparatus, and, in particular, of its defective behavior and its breaking into oscillation when it is mishandled or overloaded.

In this book, we have avoided mathematical symbolism and mathematical technique as far as possible, although we have been forced to compromise with them in various places, and in particular in the previous chapter. Here, too, in the rest of the present chapter, we are dealing precisely with those matters for which the symbolism of mathematics is the appropriate language, and we can avoid it only by long periphrases which are scarcely intelligible to the layman, and which are intelligible only to the reader acquainted with mathematical symbolism by virtue of his ability to translate them into this symbolism. The best compromise we can make is to supplement the symbolism by an ample verbal explanation.

Let $f(t)$ be a function of the time t where t runs from $-\infty$ to ∞; that is, let $f(t)$ be a quantity assuming a numerical value for each time t. At any time t, the quantities $f(s)$ are accessible to us when s is less than or equal to t but not when s is greater than t. There are pieces of apparatus, electrical and mechanical, which delay their input by a fixed time, and these yield us, for an input $f(t)$, an output $f(t - \tau)$, where τ is the fixed delay.

We may combine several pieces of apparatus of this kind, yielding us outputs $f(t - \tau_1)$, $f(t - \tau_2)$, ..., $f(t - \tau_n)$. We can multiply each of these outputs by fixed quantities, positive or negative. For example, we may use a potentiometer to multiply a voltage by a fixed positive number less than 1, and it is not too difficult to devise automatic balancing devices and amplifiers to multiply a voltage by quantities which are negative or are greater than 1. It is also not difficult to construct simple wiring diagrams of circuits by which we can add voltages continuously, and with the aid of these we may obtain an output

$$\sum_{1}^{n} a_k f(t-\tau_k) \tag{4.01}$$

By increasing the number of delays τ_k and suitably adjusting the coefficients a_k, we may approximate as closely as we wish to an output of the form

$$\int_0^{\infty} a(\tau) f(t-\tau)\, d\tau \tag{4.02}$$

In this expression, it is important to realize that the fact that we are integrating from 0 to ∞, and not from $-\infty$ to ∞, is essential. Otherwise we could use various practical devices to operate on this result and to obtain $f(t+\sigma)$, where σ is positive. This, however, involves the knowledge of the future of $f(t)$; and $f(t)$ may be a quantity, like the coordinates of a streetcar which may turn off one way or the other at a switch, which is not determined by its past. When a physical process *seems* to yield us an operator which converts $f(t)$ to

$$\int_{-\infty}^{\infty} a(\tau) f(t-\tau)\, d\tau \tag{4.03}$$

where $a(\tau)$ does not effectively vanish for negative values of τ, it means that we have no longer a true operator on $f(t)$, determined uniquely by its past. There are physical cases where this may occur. For example, a dynamical system with no input may go into permanent oscillation, or even oscillation building up to infinity, with an undetermined amplitude. In such a case, the future of the system is not determined by the past, and we may in appearance find a formalism which suggests an operator dependent on the future.

The operation by which we obtain Expression 4.02 from $f(t)$ has two important further properties: (I) it is independent of a shift of the origin of time, and (2) it is linear. The first property is expressed by the statement that if

$$g(t)=\int_0^\infty \alpha(\tau)f(t-\tau)\,d\tau \tag{4.04}$$

then

$$g(t+\sigma)=\int_0^\infty \alpha(\tau)f(t+\sigma-\tau)\,d\tau \tag{4.05}$$

The second property is expressed by the statement that if

$$g(t)=Af_1(t)+Bf_2(t) \tag{4.06}$$

then

$$\begin{aligned}&\int_0^\infty a(\tau)g(t-\tau)\,d\tau\\ &\quad=A\int_0^\infty a(\tau)f_1(t-\tau)\,d\tau+B\int_0^\infty a(\tau)f_2(t-\tau)\,d\tau\end{aligned} \tag{4.07}$$

It may be shown that in an appropriate sense *every operator on the past of $f(t)$ which is linear and is invariant under a shift of the origin of time is either of the form of Expression* 4.02 *or is a limit of a sequence of operators of that form*. For example, $f'(t)$ is the result of an operator with these properties when applied to $f(t)$, and

$$f'(t)=\lim_{\epsilon\to 0}\int_0^\infty \frac{1}{\epsilon^2}a\left(\frac{\tau}{\epsilon}\right)f(t-\tau)\,d\tau \tag{4.08}$$

where

$$a(x)=\begin{cases}1 & 0\leqslant x<1\\ -1 & 1\leqslant x\leqslant 2\\ 0 & 2\leqslant x\end{cases} \tag{4.09}$$

As we have seen before, the functions e^{zt} are a set offunctions $f(t)$ which are particularly important from the point of view of Operator 4.02, since

$$e^{z(t-\tau)} = e^{zt} \cdot e^{-z\tau} \tag{4.10}$$

and the delay operator becomes merely a multiplier dependent on z. Thus Operator 4.02 becomes

$$e^{zt} \int_0^\infty a(\tau) e^{-z\tau} d\tau \tag{4.11}$$

and is also a multiplication operator dependent on z only. The expression

$$\int_0^\infty a(\tau) e^{-z\tau} d\tau = A(z) \tag{4.12}$$

is said to be *the representation of Operator* 4.02 *as a function of frequency*. If z is taken as the complex quantity $x + iy$, where x and y are real, this becomes

$$\int_0^\infty a(\tau) e^{-x\tau} e^{-iu\tau} d\tau \tag{4.13}$$

so that by the well-known Schwarz inequality concerning integrals, if $y > 0$ and

$$\int_0^\infty |a(\tau)|^2 \, d\tau < \infty \tag{4.14}$$

we have

$$\begin{aligned} |A(x+iy)| &\leqslant \left[\int_0^\infty |a(\tau)|^2 \, d\tau \int_0^\infty e^{-2x\tau} d\tau \right]^{1/2} \\ &= \left[\frac{1}{2x} \int_0^\infty |a(\tau)|^2 \, d\tau \right]^{1/2} \end{aligned} \tag{4.15}$$

This means that $A(x + iy)$ is a bounded holomorphic function of a complex variable in every half-plane $x \geqslant \epsilon > 0$, and that the function $A(iy)$ represents in a certain very definite sense the boundary values of such a function.

Let us put

$$u + iv = A(x + iy) \tag{4.16}$$

where u and v are real. The $x + iy$ will be determined as a function (not necessarily single-valued) of $u + iv$. This function will be analytic, though meromorphic, except at the points $u + iv$ corresponding to points $z = x + iy$, where $\partial A(z)/\partial z = 0$. The boundary $x = 0$ will go into the curve with the parametric equation

$$u + iv = A(iy) \quad (y \text{ real}) \tag{4.17}$$

This new curve may intersect itself any number of times. In general, however, it will divide the plane into two regions. Let us consider the curve (Eq. 4.17) traced in the direction in which y goes from $-\infty$ to ∞. Then if we depart from Eq. 4.17 to the right and follow a continuous course not again cutting Eq. 4.17, we may arrive at certain points. The points which are neither in this set nor on Eq. 4.17 we shall call *exterior points*. The part of the curve (Eq. 4.17) which contains limit points of the exterior points we shall call the *effective boundary*. All other points will be termed *interior points*. Thus in the diagram of Fig. 1, with the boundary drawn in the sense of the arrow, the interior points are shaded and the effective boundary is drawn heavily.

The condition that A be bounded in any right half-plane will then tell us that *the point at infinity cannot be an interior point*. It may be a boundary point, although there are certain very definite restrictions on the character of the type of boundary point it may be. These concern the "thickness" of the set of interior points reaching out to infinity.

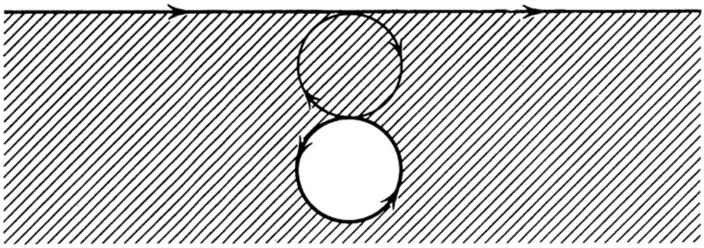

Fig. 1

Now we come to the problem of the mathematical expression of the problem of linear feedback. Let the control flow chart—*not* the wiring diagram—of such a system be as shown in Fig. 2. Here the input of the motor is Y, which is the difference between the original input X and the output of the multiplier, which multiplies the power output AY of the motor by the factor λ. Thus

$$Y = X - \lambda AY \tag{4.18}$$

and

$$Y = \frac{X}{1 + \lambda A} \tag{4.19}$$

so that the motor output is

$$AY = X \frac{A}{1 + \lambda A} \tag{4.20}$$

The operator produced by the whole feedback mechanism is then $A/(1 + \lambda A)$. *This will be infinite when and only when* $A = -1/\lambda$. *The diagram* (Eq. 4.17) *for this new operator will be*

$$u + iv = \frac{A(iy)}{1 + \lambda A(iy)} \tag{4.21}$$

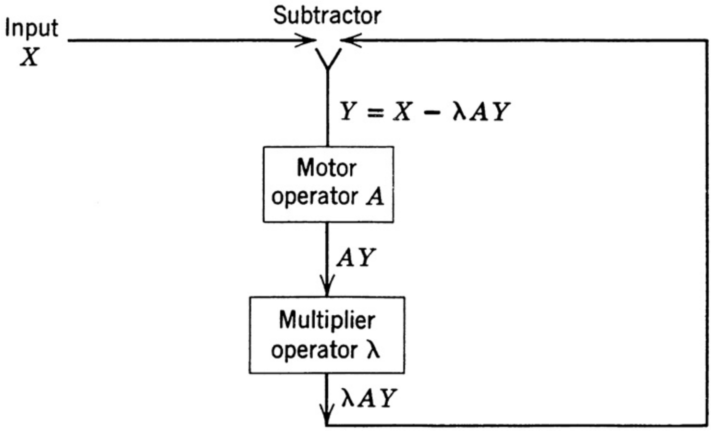

Fig. 2

and ∞ will be an interior point of this when and only when $-1/\lambda$ *is an interior point of Eq.* 4.17.

In this case, a feedback with a multiplier λ will certainly produce something catastrophic, and, as a matter of fact the catastrophe will be that the system will go into unrestrained and increasing oscillation. If, on the other hand, the point $-1/\lambda$ is an exterior point, it may be shown that there will be no difficulty, and the feedback is stable. If $-1/\lambda$ is on the effective boundary, a more elaborate discussion is necessary. Under most circumstances, the system may go into an oscillation of an amplitude which does not increase.

It is perhaps worth considering several operators A and the ranges of feedback which are admissible under them. We shall consider not only the operations of Expression 4.02 but also their limits, assuming that the same argument will apply to these.

If the operator A corresponds to the differential operator, $A(z) = z$, as y goes from $-\infty$ to ∞, $A(y)$ does the same, and the interior points are the points interior to the right, half-plane. The point $-1/\lambda$ is always an exterior point, and any amount of feedback is possible. If

$$A(z) = \frac{1}{1+kz} \tag{4.22}$$

the curve (Eq. 4.17) is

$$u + iv = \frac{1}{1+kiy} \tag{4.23}$$

or

$$u = \frac{1}{1+k^2y^2} \quad v = \frac{-ky}{1+k^2y^2} \tag{4.24}$$

which we may write

$$u^2 + v^2 = u \tag{4.25}$$

This is a circle with radius 1/2, and center at (1/2, 0). It is described in the clockwise sense, and the interior points are those which we should ordinarily consider interior. In this case too, the admissible feedback is unlimited, as $-1/\lambda$ is always outside the circle. The $a(t)$ corresponding to this operator is

$$a(t) = e^{-t/k}/k \tag{4.26}$$

Again, let

$$A(z) = \left(\frac{1}{1+kz}\right)^2 \tag{4.27}$$

Then Eq. 4.17 is

$$u + iv = \left(\frac{1}{1+kiy}\right)^2 = \frac{(1-kiy)^2}{(1+k^2y^2)^2} \tag{4.28}$$

and

$$u = \frac{1 - k^2 y^2}{\left(1 + k^2 y^2\right)^2}, \quad v = \frac{-2ky}{\left(1 + k^2 y^2\right)^2} \tag{4.29}$$

This yields

$$u^2 + v^2 = \frac{1}{\left(1 + k^2 y^2\right)^2} \tag{4.30}$$

or

$$y = \frac{-v}{\left(u^2 + v^2\right) 2k} \tag{4.31}$$

Then

$$u = \left(u^2 + v^2\right)\left[1 - \frac{k^2 v^2}{4k^2 \left(u^2 + v^2\right)^2}\right] = \left(u^2 + v^2\right) - \frac{v^2}{4\left(u^2 + v^2\right)} \tag{4.32}$$

In polar coordinates, if $u = \rho \cos \phi$, $v = \rho \sin \phi$, this becomes

$$\rho \cos \phi = \rho^2 - \frac{\sin^2 \phi}{4} = \rho^2 - \frac{1}{4} + \frac{\cos^2 \phi}{4} \tag{4.33}$$

or

$$\rho - \frac{\cos \phi}{2} = \pm \frac{1}{2} \tag{4.34}$$

That is,

$$\rho^{1/2} = -\sin \frac{\phi}{2}, \quad \rho^{1/2} = \cos \frac{\phi}{2} \tag{4.35}$$

It can be shown that these two equations represent only one curve, a cardioid with vertex at the origin and cusp pointing to the right. The interior of this curve will contain no point of the negative real axis, and, as in the previous case, the admissible amplification is unlimited. Here the operator $a(t)$ is

$$a(t) = \frac{t}{k^2} e^{-t/k} \tag{4.36}$$

Let

$$A(z) = \left(\frac{1}{1+kz} \right)^3 \tag{4.37}$$

Let p and ϕ be defined as in the last case. Then

$$\rho^{1/3} \cos \frac{\phi}{3} + i\rho^{1/3} \sin \frac{\phi}{3} = \frac{1}{1+kiy} \tag{4.38}$$

As in the first case, this will give us

$$\rho^{2/3} \cos^2 \frac{\phi}{3} + \rho^{2/3} \sin^2 \frac{\phi}{3} = \rho^{2/3} \cos \frac{\phi}{3} \tag{4.39}$$

That is,

$$\rho^{1/3} = \cos \frac{\phi}{3} \tag{4.40}$$

which is a curve of the shape of Fig. 3. The shaded region represents the interior points. All feedback with coefficient exceeding 1/8 is impossible. The corresponding $a(t)$ is

$$a(t) = \frac{t^2}{2k^3} e^{-t/k} \tag{4.41}$$

Finally, let our operator corresponding to A be a simple delay of T units of time. Then

$$A(z) = e^{-Tz} \tag{4.42}$$

Then

$$u + iv = e^{-Tiy} = \cos Ty - i \sin Ty \tag{4.43}$$

The curve (Eq. 4.17) will be the unit circle about the origin, described in a clockwise sense about the origin with unit velocity.

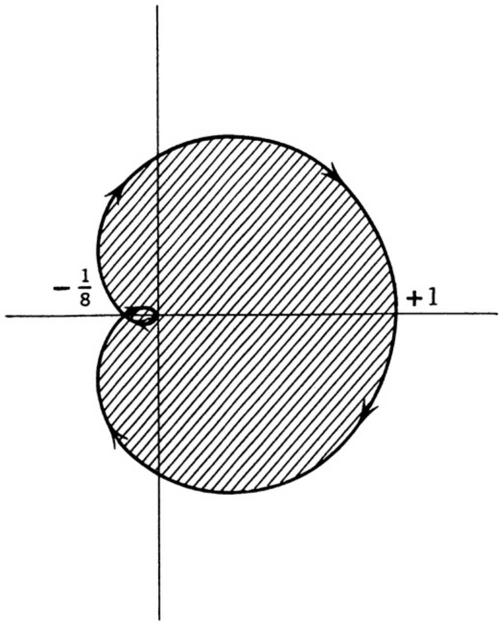

Fig. 3

The inside of this curve will be the inside in the ordinary sense, and the limit of feedback intensity will be 1.

There is one very interesting conclusion to be drawn from this. It is possible to compensate for the operator $1/(1 + kz)$ by an arbitrarily heavy feedback, which will give us an $A/(1 + \lambda)$ as near to 1 as we wish for as large a frequency range as we wish. It is thus possible to compensate for three successive operators of this sort by three—or even two—successive feedbacks. It is not, however, possible to compensate as closely as we wish for an operator $1/(1 + kz)^3$, which is the resultant of the composition of three operators $1/(1 + kz)$ in cascade, by a single feedback, The operator $1/(1 + kz)^3$ may also be written

$$\frac{1}{2k^2}\frac{d^2}{dz^2}\frac{1}{1+kz} \tag{4.44}$$

and may be regarded as the limit of the additive composition of three operators with first-degree denominators, It thus appears that a sum of different operators, each of which may be compensated as well as we wish by a single feedback, cannot itself be so compensated.

In the important book of MacColl, we have an example of a complicated system which can be stabilized by two feedbacks but not by one. It concerns the steering of a ship by a gyrocompass. The angle between the course set by the quartermaster and that shown by the compass expresses itself in the turning of the rudder, which, in view of the headway of the ship, produces a turning moment which serves to change the course of the ship in such a way as to decrease the difference between the set course and the actual course. If this is done by a direct opening of the valves of one steering engine and closing of the valves of the other in such a way that the turning velocity of the rudder is proportional to the deviation of the ship from this course, let us note that the angular position of the rudder is roughly proportional to the turning moment of the ship and thus to its angular acceleration. Hence the amount of turning of the ship is proportional with a negative factor to the third derivative of the deviation from the course, and the operation which we have to stabilize by the feedback from the gyrocompass is kz^3, where k is positive. We thus get for our curve (Eq. 4.17)

$$u + iv = -kiy^3 \tag{4.45}$$

and, as the left half-plane is the interior region, no servomechanism whatever will stabilize the system.

In this account, we have slightly oversimplified the steering problem. Actually there is a certain amount of friction, and the force turning the ship does not determine the acceleration. Instead, if θ is the angular position of the ship and ϕ that of the rudder with respect to the ship, we have

$$\frac{d^2\theta}{dt^2} = c_1\phi - c_2\frac{d\theta}{dt} \tag{4.46}$$

and

$$u + iv = -k_1 i y^3 - k_2 y^2 \tag{4.47}$$

This curve may be written

$$v^2 = -k_3 u^3 \tag{4.48}$$

which still cannot be stabilized by any feedback. As y goes from $-\infty$ to ∞, u goes from ∞ to $-\infty$, and the *inside* of the curve is to the left.

If, on the other hand, the *position* of the rudder is proportional to the deviation of the course, the operator to be stabilized by feedback is $k_1 z^2 + k_2 z$, and Eq. 4.17 becomes

$$u + iv = -k_1 y^2 + k_2 i y \tag{4.49}$$

This curve may be written

$$v^2 = -k_3 u \tag{4.50}$$

but in this case, as y goes from $-\infty$ to ∞, so does v, and the curve is described from $y = -\infty$ to $y = \infty$. In this case, the *outside* of the curve is to the left, and unlimited amount of amplification is possible.

To achieve this we may employ another stage of feedback. If we regulate the position of the valves of the steering engine, not by the discrepancy between the actual and the desired course but by the *difference* between this quantity and the angular position of the rudder, we shall keep the angular position of the rudder as nearly proportional to the ship's deviation from true course as we wish, if we allow a large enough feedback—that is, if we open the valves wide enough. This double feedback system of control is in fact the one usually adopted for the automatic steering of ships by means of the gyrocompass.

In the human body, the motion of a hand or a finger involves a system with a large number of joints. The output is an additive vectorial combination of the outputs of all these joints. We have seen that, in general, a complex additive system like this cannot be stabilized by a single feedback. Correspondingly, the voluntary feedback by which we regulate the performance of a task through the observation of the amount by which it is not yet accomplished needs the backing up of other feedbacks. These we call postural feedbacks, and they are associated with the general maintenance of tone of the muscular system. It is the voluntary feedback which shows a tendency to break down or become deranged in cases of cerebellar injury, for the ensuing tremor does not appear unless the patient tries to perform a voluntary task. This purpose tremor, in which the patient cannot pick up a glass of water without upsetting it, is very different in nature from the tremor of Parkinsonism, or paralysis *agitans*, which appears in its most typical form when the patient is at rest, and indeed often seems to be greatly mitigated when he attempts to perform a specific task. There are surgeons with Parkinsonism who manage to operate quite efficiently. Parkinsonism is known not to have its origin in a diseased condition of the cerebellum, but to be associated with a pathological focus somewhere in the brain stem. It is only one of the diseases of the postural feedbacks, and many of these must have their origin in defects of parts of the nervous system situated very differently. One of the great tasks of physiological cybernetics is to disentangle and isolate loci of the different parts of this complex of voluntary and postural feedbacks. Examples of component reflexes of this sort are the scratch and the walking reflex.

When feedback is possible and stable, its advantage, as we have already said, is to make performance less dependent on the load. Let us consider that the load changes the characteristic A by dA. The fractional change will be dA/A. If the operator after feedback is

$$B = \frac{A}{C + A} \tag{4.51}$$

we shall have

$$\frac{dB}{B} = \frac{-d\left(1 + \frac{C}{A}\right)}{1 + \frac{C}{A}} = \frac{\frac{C}{A^2}\,dA}{1 + \frac{C}{A}} = \frac{dA}{A}\,\frac{C}{A + C} \tag{4.52}$$

Thus feedback serves to diminish the dependence of the system on the characteristic of the motor, and serves to stabilize it, for all frequencies for which

$$\left|\frac{A + C}{C}\right| > 1 \tag{4.53}$$

This is to say that the entire boundary between interior and exterior points must lie inside the circle of radius C about the point $-C$. This will not even be true in the first of the cases we have discussed. The effect of a heavy negative feedback, if it is at all stable, will be to increase the stability of the system for low frequencies, but generally at the expense of its stability for some high frequencies. There are many cases in which even this degree of stabilization is advantageous.

A very important question which arises in connection with oscillations due to an excessive amount of feedback is that of the frequency of incipient oscillation. This is determined by the value of y in the iy corresponding to the point of the boundary of the inside and outside regions of Eq. 4.17 lying furthest from the left on the negative u-axis. The quantity y is of course of the nature of a frequency.

We have now come to the end of an elementary discussion of linear oscillations, studied from the point of view of feedback. A linear oscillating system has certain very special properties which characterize its oscillations. One is that when it oscillates, it always *can* and very generally—in the absence of independent simultaneous oscillations—*does* oscillate in the form

$$A \sin(Bt + C) e^{Dt} \tag{4.54}$$

The existence of a periodic non-sinusoidal oscillation is always a suggestion at least that the variable observed is one in which the system is not linear. In some cases, but in very few, the system may be rendered linear again by a new choice of the independent variable. Another very significant difference between linear and non-linear oscillations is that in the first the amplitude of oscillation is completely independent of the frequency; while in the latter, there is generally only one amplitude, or at most a discrete set of amplitudes, for which the system will oscillate at a given frequency, as well as a discrete set of frequencies for which the system will oscillate. This is well illustrated by the study of what happens in an organ pipe. There are two theories of the organ pipe—a cruder linear theory, and a more precise non-linear theory. In the first, the organ pipe is treated as a conservative system. No question is asked about how the pipe came to oscillate, and the level of oscillation is completely indeterminate. In the second theory, the oscillation of the organ pipe is considered as dissipating energy, and this energy is considered to have its origin in the stream of air across the lip of the pipe. There is indeed a theoretical steady-state flow of air across the lip of the pipe which does not interchange any energy with any of the modes of oscillation of the pipe, but for certain velocities of air flow this steady-state condition is unstable. The slightest chance deviation from it will introduce an energy input into one or more of the natural modes of linear oscillation of the pipe; and up to a certain point, this motion will actually increase the coupling of the proper modes of oscillation of the pipe with the energy input. The rate of energy input and the rate of energy output by thermal dissipation and otherwise have different laws of growth, but, to arrive at a steady state of oscillation, these two quantities must be identical. Thus the level of the non-linear oscillation is determined just as definitely as its frequency.

The case we have examined is an example of what is known as a relaxation oscillation: a case, that is, where a system of equations invariant under a translation in time leads to a solution periodic—or corresponding to some generalized notion of periodicity—in time, and determinate in amplitude and frequency but not in phase. In the case we have discussed, the frequency of oscillation of the system is close to that of some loosely coupled, nearly linear part of the system. B. van der Pol, one of the chief authorities on relaxation oscillations, has pointed out that this is not always the case, and that there are in fact relaxation oscillations where the predominating frequency is not near the frequency of linear oscillation of any part of the system. An example is given by a stream of gas flowing into a chamber open to the air and in which a pilot light is burning: when the concentration of gas in the air reaches a certain critical value, the system is ready to explode under ignition by the pilot light, and the time it takes for this to happen depends only on the rate of flow of the coal gas, the rate at which air seeps in and the products of combustion seep out, and the percentage composition of an explosive mixture of coal gas and air.

In general, non-linear systems of equations are hard to solve. There is, however, a specially tractable case, in which the system differs only slightly from a linear system, and the terms which distinguish it change so slowly that they may be considered substantially constant over a period of oscillation. In this case, we may study the non-linear system as if it were a linear system with slowly varying parameters. Systems which may be studied this way are said to be perturbed secularly, and the theory of secularly perturbed systems plays a most important role in gravitational astronomy.

It is quite possible that some of the physiological tremors may be treated somewhat roughly as secularly perturbed linear systems. We can see quite clearly in such a system why the steady-state amplitude level may be just as determinate as the frequency. Let one element in such a system be an amplifier whose gain decreases as some long-time average of the input of such a system increases. Then as the oscillation of the system builds up, the gain may be reduced until a state of equilibrium is reached.

Non-linear systems of relaxation oscillations have been studied in some cases by methods developed by Hill and Poincaré.[1] The classical cases for the study of such oscillations are those in which the equations of the systems are of a different nature; especially where these differential equations are of low order. There is not, as far as I know, any comparable adequate study of the corresponding integral equations when the system depends for its future behavior on its entire past behavior. However, it is not hard to sketch out the form such a theory should take, especially when we are looking only for periodic solutions. In this case, the slight modification of the constants of the equation should lead to a slight, and therefore nearly linear, modification of the equations of motion. For example, let $Op[f(t)]$ be a function of t which results from a non-linear operation on $f(t)$, and which is affected by a translation. Then the variation of $Op[f(t)]$, $\delta Op[f(t)]$ corresponding to a variational change $\delta f(t)$ in $f(t)$ and a known change in the dynamics of the system, is linear but not homogeneous in $\delta f(t)$, though not linear in $f(t)$. If we now know a solution $f(t)$ of

$$Op[f(t)] = 0 \tag{4.55}$$

and we change the dynamics of the system, we obtain a linear non-homogeneous equation for $\delta f(t)$. If

$$f(t) = \sum_{-\infty}^{\infty} a_n e^{in\lambda t} \tag{4.56}$$

and $f(t) + \delta f(t)$ is also periodic, being of the form

$$f(t) + \delta f(t) = \sum_{-\infty}^{\infty} (a_n + \delta a_n) e^{in(\lambda+\delta\lambda)t} \tag{4.57}$$

then

$$\delta f(t) = \sum_{-\infty}^{\infty} \delta a_n e^{i\lambda nt} + \sum_{-\infty}^{\infty} a_n e^{i\lambda nt} in\delta\lambda t \tag{4.58}$$

The linear equations for $\delta f(t)$ will have all coefficients developable into series in $e^{t\lambda nt}$, since $f(t)$ can itself be developed in this form. We shall thus obtain an infinite system of linear non-homogeneous equations in $\delta a_n + an$, $\delta\lambda$, and λ, and this system of equations may be solvable by the methods of Hill. In this case, it is at least conceivable that by starting with a linear equation (non-homogeneous) and gradually shifting the constraints we may arrive at a solution of a very general type of non-linear problem in relaxation oscillations. This work, however, lies in the future.

To a certain extent, the feedback systems of control discussed in this chapter and the compensation systems discussed in the previous one are competitors. They both serve to bring the complicated input-output relations of an effector into a form approaching a simple proportionality. The feedback system, as we have seen, does more than this, and has a performance relatively independent of the characteristic and changes of characteristic of the effector used. The relative usefulness of the two methods of control thus depends on the constancy of the characteristic of the effector. It is natural to suppose that cases arise in which it is advantageous to combine the two methods. There are various ways of doing this. One of the most simple is that illustrated in the diagram of Fig. 4.

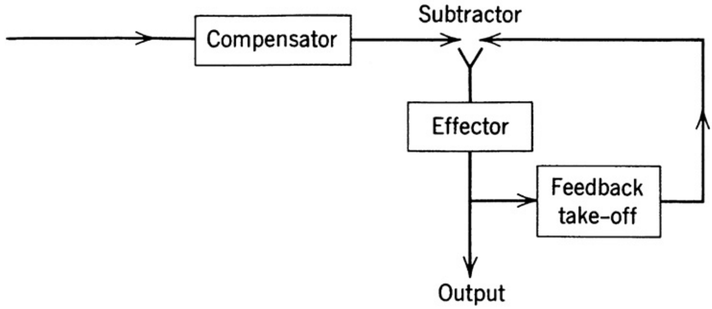

Fig. 4

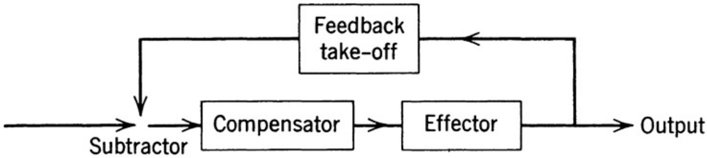

Fig. 5

In this, the entire feedback system may be regarded as a larger effector, and no new point arises, except that the compensator must be arranged to compensate what is in some sense the average characteristic of the feedback system. Another type of arrangement is shown in Fig. 5.

Here the compensator and effector are combined into one larger effector. This change will in general alter the maximum feedback admissible, and it is not easy to see how it can ordinarily be made to increase that level to an important extent. On the other hand, for the same feedback level, it will most definitely improve the performance of the system. If, for example, the effector has an essentially lagging characteristic, the compensator will be an anticipator or predictor, designed for its statistical ensemble of inputs. Our feedback, which we may call an anticipatory feedback, will tend to hurry up the action of the effector mechanism.

Feedbacks of this general type are certainly found in human and animal reflexes. When we go duck shooting, the error which we try to minimize is not that between the position of the gun and the actual position of the target but that between the position of the gun and the anticipated position of the target. Any system of anti-aircraft fire control must meet the same problem. The conditions of stability and effectiveness of anticipatory feedbacks need a more thorough discussion than they have yet received.

Another interesting variant of feedback systems is found in the way in which we steer a car on an icy road. Our entire conduct of driving depends on a knowledge of the slipperiness of the road surface, that is, on a knowledge of the performance characteristics of the system car–road. If we wait to find this out by the ordinary performance of the system, we shall discover ourselves in a skid before we know it. We thus give to the steering wheel a succession of small, fast impulses, not enough to throw the car into a major skid but quite enough to report to our kinesthetic sense whether the car is in danger of skidding, and we regulate our method of steering accordingly.

This method of control, which we may call *control by informative feedback*, is not difficult to schematize into a mechanical form and may well be worthwhile employing in practice. We have a compensator for our effector, and this compensator has a characteristic which may be varied from outside. We superimpose on the incoming message a weak high-frequency input and take off the output of the effector a partial output of the same high frequency, separated from the rest of the output by an appropriate filter. We explore the amplitude-phase relations of the high-frequency output to the input in order to obtain the performance characteristics of the effector. On the basis of this, we modify in the appropriate sense the characteristics of the compensator. The flow chart of the system is much as in the diagram of Fig. 6.

The advantages of this type of feedback are that the compensator may be adjusted to give stability for every type of constant load; and that, if the characteristic of the load changes slowly enough, in what we have called a secular manner, in comparison

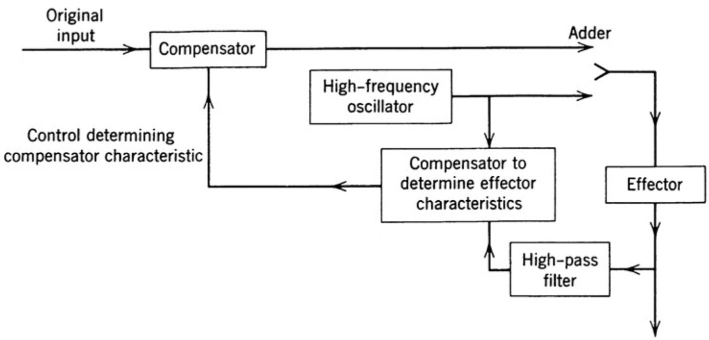

Fig. 6

with the changes of the original input, and if the reading of the load condition is accurate, the system has no tendency to go into oscillation. There are very many cases where the change of load is secular in this manner. For example, the frictional load of a gun turret depends on the stiffness of the grease, and this again on the temperature; but this stiffness will not change appreciably in a few swings of the turret.

Of course, this informative feedback will work well only if the characteristics of the load at high frequencies are the same as, or give a good indication of, its characteristics at low frequencies.

This will often be the case if the character of the load, and hence of the effector, contains a relatively small number of variable parameters.

This informative feedback and the examples we have given of feedback with compensators are only particular cases of what is a very complicated theory, and a theory as yet imperfectly studied. The whole field is undergoing a very rapid development. It deserves much more attention in the near future.

Before we end this chapter, we must not forget another important physiological application of the principle of feedback. A great group of cases in which some sort of feedback is not only exemplified in physiological phenomena but is absolutely essential for the continuation of life is found in what is known as *homeostasis*. The conditions under which life, especially healthy life, can continue in the higher animals are quite narrow. A variation of one-half degree centigrade in the body temperature is generally a sign of illness, and a permanent variation of five degrees is scarcely consistent with life. The osmotic pressure of the blood and its hydrogen-ion concentration must be held within strict limits. The waste products of the body must be excreted before they rise to toxic concentrations. Beside all these, our leucocytes and our chemical defenses against infection must be kept at adequate levels; our heart rate and blood pressure must neither be too high nor too low; our sex cycle must conform to the racial needs of reproduction; our calcium metabolism must be such as neither to soften our bones nor to calcify our tissues; and so on. In short, our inner economy must contain an assembly of thermostats, automatic hydrogen-ion-concentration controls, governors, and the like, which would be adequate for a great chemical plant. These are what we know collectively as our homeostatic mechanism.

Our homeostatic feedbacks have one general difference from our voluntary and our postural feedbacks: they tend to be slower. There are very few changes in physiological homeostasis—not even cerebral anemia—that produce serious or permanent damage in a small fraction of a second. Accordingly, the nerve fibers reserved for the processes of homeostasis—the sympathetic and parasympathetic systems—are often non-myelinated and are known to have a considerably slower rate of transmission than the myelinated fibers. The typical effectors of homeostasis—smooth muscles and glands—are likewise slow in their action compared with striped muscles, the typical effectors of voluntary and postural activity. Many of the messages of the homeostatic system are carried by non-nervous channels—the direct anastomosis of the muscular fibers of the heart, or chemical messengers such as the hormones, the carbon dioxide content of the blood, etc.; and, except in the case of the heart muscle, these too are generally slower modes of transmission than myelinated nerve fibers.

Any complete textbook on cybernetics should contain a thorough detailed discussion of homeostatic processes, many individual cases of which have been discussed in the literature with some detail.[2] However, this book is rather an introduction to the subject than a compendious treatise, and the theory of homeostatic processes involves rather too detailed a knowledge of general physiology to be in place here.

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