This PDF includes a chapter from the following book:

Type-Logical Syntax

9 English Modal Auxiliaries

By this point, the reader should have noticed one property that distinguishes our approach from earlier or competing proposals in other lexicalist theories of syntax such as HPSG. As compared to theories such as HPSG, Hybrid TLCG (and TLCG in general) assumes a more abstract interface between surface syntax and semantic interpretation. The (seemingly) abstract and complex lexical entries for scopal operators involving prosodic lambda abstraction that we have introduced in the analyses of various phenomena in the preceding chapters all embody this (from a certain perspective) somewhat untransparent perspective on the architecture of the syntax-semantics interface. We have shown above that this approach can successfully handle a number of complex empirical phenomena that have proven refractory for previous lexicalist/phrase structure–based theories, but one question naturally arises at this point: To what extent is our approach compatible with previous analyses of major syntactic phenomena in the lexicalist tradition?

Addressing this issue properly would call for another monograph-length discussion, one clearly beyond the scope of the present work. Instead, in this chapter we attempt to achieve a more modest goal of clarifying a hidden connection between our approach and previous lexicalist approaches by focusing on a specific empirical domain, namely, the analysis of English modal auxiliaries (in particular, as they interact with the scope of negation). The point we would like to make in this case study is that the somewhat abstract analyses of (semantic) operators and scope we have proposed in the present work is not so distant from the lexicalist analyses familiar in the literature as it may initially appear since the two are indeed closely related to one another from a certain perspective.

The grammar of English modal auxiliaries is a particularly suitable topic for this purpose for the following reasons. First, the treatment of English auxiliary verbs has been one of the central topics in the literature of lexicalist syntactic theories from the early days of GPSG (e.g., Gazdar et al. 1982) up to the most recent version of HPSG (see Sag et al. 2020). Second, in our analysis of the scope anomaly in Gapping in

chapter 3, we assumed an analysis of modal auxiliaries as higher-order scopal operators that departed from the traditional VP/VP analysis in lexicalist theories of syntax. While this assumption is crucial for giving a systematic account of the scoping patterns of auxiliaries in Gapping sentences, one may naturally wonder whether an analysis along these lines can capture the properties of modal auxiliaries that the traditional VP/VP analysis is designed to account for. In any event, while we have shown that the more traditional VP/VP analysis is derivable as an entailment in our type logic from the somewhat novel higher-order operator treatment proposed in chapter 3, the relationship between the two is not yet entirely clear, and a more systematic comparison is called for at this point. The discussion in the present chapter aims to shed light on this question by formulating an explicit analysis of the scopal interactions between modal auxiliaries and negation, an empirical domain which, so far as we are aware, has not received attention in the previous literature on TLCG.

9.1 Modals and Negation: The Empirical Landscape

A review of the basic data illustrating scopal relationship between modals and negation reveals little in the way of systematic semantic conditions which allow one to predict this relationship for any given modal, despite certain semantic aspects of modal operators which appear to be relevant. No single semantic dimension seems to be sufficient to account for the differences in behavior of the modals with respect to negation, let alone explain why the operators introduced by some modals must outscope negation, others must scope under negation, and still others scope freely. The modals differ from each other in terms of quantificational force—in effect, the sets of worlds they quantify over—and modal base, that is, whether they have deontic, epistemic, or bouletic interpretations. But no combination of these semantic properties accounts for the particular scope relation which any given modal auxiliary has with respect to negation. Thus in (442), *should/must* and *need* denote (respectively different flavors of) universal quantification over the relevant possible worlds but have opposite scoping vis-à-vis negation. And this scoping holds regardless of the type of modal base:

(442) a. John should/must not criticize Mary. $\Box\neg\mathbf{criticize}(\mathbf{m})(\mathbf{j})$, deontic
        b. Mary must not have arrived yet. $\Box\neg\mathbf{yet}(\mathbf{arrive}(\mathbf{m}))$, epistemic
        c. That rook should not leave the sixth rank.

                                                           $\Box\neg\mathbf{leave}(\mathbf{6th})(\iota(\mathbf{rook}))$, bouletic

(443) a. John need not criticize Mary. $\neg\Box\mathbf{criticize}(\mathbf{m})(\mathbf{j})$, deontic
        b. Mary need not have arrived yet. $\neg\Box\mathbf{yet}(\mathbf{arrive}(\mathbf{m}))$, epistemic
        c. That rook need not leave the sixth rank. $\neg\Box\mathbf{leave}(\mathbf{6th})(\iota(\mathbf{rook}))$, bouletic

Must invariably scopes over negation, regardless of which of three possible interpretations it conveys, whereas exactly the opposite holds for *need*. Thus, despite their shared quantificational force, all that they have in common is that each permits only one scopal ordering of $\square$ and $\neg$.

This in itself might be taken to support at least a semantically based distinction between the (universal) modals displaying a fixed order with respect to negation and the (existential) modals which, as shown by *can*, *could*, and *may*, can scope either above or below negation, across all available interpretations:

(444) John may/can/could not criticize Mary.

$$\lozenge\neg\mathbf{criticize}(\mathbf{m})(\mathbf{j}), \neg\lozenge\mathbf{criticize}(\mathbf{m})(\mathbf{j}) \text{ (epistemic/deontic)}$$

But this generalization does not extend to *might*, whose quantificational force is also existential but which only has a single scopal ordering with respect to negation:

(445) You might not criticize Mary quite so much.

$$\lozenge\neg\mathbf{criticize}(\mathbf{m})(\mathbf{j}), \#\neg\lozenge\mathbf{criticize}(\mathbf{m})(\mathbf{j}) \text{ (epistemic/deontic)}$$

Might, it is true, does not always display a prominent deontic interpretation. But in (445), there is arguably an interpretation of *might* which suggests that the speaker is advocating for the hearer's forbearance in the latter's interactions with Mary, rather than merely noting the possibility of such forbearance in one or another set of circumstances. Yet even with its existential status and dual deontic/existential versions, *might* can only scope wider than negation, never narrower.

These considerations reflect the difficulty in identifying a consistent set of semantic factors which jointly covary with—let alone determine—the relative order of modal scope with respect to negation in the case of any given auxiliary. The following table lists the relevant patterns for the major familiar modal auxiliaries:

(446)

Modal	Scopal pattern
will	$\mathbf{F} > \neg$
would	$\mathfrak{W} > \neg$
shall	$\mathbf{F} > \neg$
should	$\square > \neg$
ought	$\square > \neg$
might	$\lozenge > \neg$
must	$\square > \neg$
may	$\lozenge < > \neg$
can	$\lozenge < > \neg$
could	$\lozenge < > \neg$
need	$\neg > \square$

Iatridou and Zeijlstra (2013) argue that a principled account of the patterns in (446) can be given directly in terms of the sensitivity displayed by the individual modals to the scope of negation. *Need* is a known negative polarity item (NPI; see Levine [2013] for discussion of the somewhat unusual behavior of this NPI), and hence, when it appears with a local negator such as *not* or *never*, it always scopes under negation. Iatridou and Zeijlstra propose that the invariably wide scope of *must*, *should*, *ought*, and so on, with respect to local negation reflects their status as *positive* polarity items (PPIs). On their account, the different scopal relations between different types of modals and negation is a consequence of the "reconstruction" possibilities of modals depending on their polarity statuses—NPI, PPI, or neutral modals—as summarized in the following table:[1]

(447)

	PPI modals	Neutral modals	NPI modals
Universal	*must*, *should*, *ought to*, *to be to*	*have to*, *need to*	*need*
Existential	—	*can*, *may*	—

On Iatridou and Zeijlstra's account, the auxiliaries are raised to the head of TP, and hence above negP. In the case of a sentence such as *John need not worry*, *need* cannot be licensed unless it is reconstructed back under negP, due to its NPI status. By contrast, PPI modals such as *must*, *should*, and *ought* are prohibited from reconstruction, again due to their lexical property as PPIs. Neutral modals such as *can* and *may* optionally reconstruct to their original sites, giving rise to scope ambiguity with negation.

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1. One might wonder about the classification of *must* and *should* as PPIs, given that they can appear un-problematically in the scope of negation in sentences such as *I don't think that John should be even one little bit nice to anyone in that room*, where the NPIs *even*, *anyone*, and *one little bit* appear with no hint of ill-formedness. But here it is crucial to bear in mind that polarity items as a broad class are known to be sensitive to not only semantic scope effects but syntactic contexts as well; see Richter and Soehn (2006) for a survey of syntactic conditions on a range of NPIs in German. Iatridou and Zeijlstra (2013) argue that the same syntactic sensitivity holds for PPIs and note that

> PPIs . . . are fine in the scope of negation or any other context that is known to ban PPIs if this context is clause-external (Szabolcsi 2004, 24–27), as illustrated in (i)–(iv):

> (i) I don't think that John called someone.   not > [$_\text{CP/IP}$ some
> (ii) No one thinks/says that John called someone.   no one > [$_\text{CP/IP}$ some
> (iii) I regret that John called someone.   regret > [$_\text{CP/IP}$ some
> (iv) Every boy who called someone got help.   every [$_\text{CP/IP}$ some

What seems to hold for the PPI modals, then, is that they cannot be in the scope of negation that originates in syntactically *local* operators.

Thus, Iatridou and Zeijlstra's approach accounts for the scopal relations between modals and negation in terms of the structural relationships between them at LF, induced by the polarity properties of different types of modals. The TLCG account we propose below follows Iatridou and Zeijlstra in taking this correlation between polarity and relative scope of semantic operators to be the key property underlying the superficially observed scope relations between modals and negation. But the specific assumptions about the syntax-semantics interface is different, mainly due to the different assumptions about the status of covert structural representations underlying semantic interpretation.

9.2 Capturing the Modal/Negation Scope Interaction

9.2.1 NPI and PPI Modals

In order to capture the polarity sensitivity of different types of modal auxiliaries in English, we posit a syntactic feature *pol* for category S that takes one of the three values $+$, $-$, and $\varnothing$. The treatment of polarity here follows the general approach to polarity marking in the CG literature by Dowty (1994), Bernardi (2002), and Steedman (2012) but differs from them in some specific details. Intuitively, S_{pol+} and S_{pol-} are positively and negatively marked clauses respectively, and $S_{pol\varnothing}$ is a "smaller" clause that isn't yet assigned polarity marking. To avoid cluttering the notation, we suppress the feature name *pol* in what follows and write S_{pol+}, S_{pol-}, and $S_{pol\varnothing}$ simply as S_+, S_-, and $S_\varnothing$, respectively. Positive-polarity modals are then lexically specified to obligatorily take scope at the level of S_+. Negative-polarity modals, on the other hand, are lexically specified to take scope at the level of $S_\varnothing$, before negation turns an "unmarked" clause to a negatively marked clause. We assume further that complete sentences in English are marked either *pol+* or *pol−*; thus, inhabitants of the type $S_\varnothing$ do not appear as stand-alone sentences.

The analysis of PPI and NPI modals outlined above can be technically implemented by positing the following lexical entries for the modals and the negation morpheme (where $\alpha, \beta \in \{\varnothing, -\}$ and $\gamma \in \{b(se), f(in)\}$):

(448)   a. $\lambda\sigma.\sigma(\mathsf{should}); \lambda\mathscr{G}.\Box\mathscr{G}(\mathrm{id}_{et}); S_{f,+} \upharpoonright (S_{f,\beta} \upharpoonright (VP_{f,\alpha}/VP_{b,\alpha}))$

       b. $\lambda\sigma.\sigma(\mathsf{need}); \lambda\mathscr{G}.\Box\mathscr{G}(\mathrm{id}_{et}); S_{f,\varnothing} \upharpoonright (S_{f,\varnothing} \upharpoonright (VP_{f,\varnothing}/VP_{b,\varnothing}))$

       c. $\lambda\sigma.\sigma(\mathsf{not}); \lambda\mathscr{G}.\neg\mathscr{G}(\mathrm{id}_{et}); S_{\gamma,-} \upharpoonright (S_{\gamma,\varnothing} \upharpoonright (VP_{b,\varnothing}/VP_{b,\varnothing}))$

We assume that different modals are assigned the following syntactic categories, depending on their polarity sensitivity:

(449)

PPI	**NPI**
$\mathbf{S}_{f,+} \restriction (\mathbf{S}_{f,\beta} \restriction (\mathbf{VP}_{f,\alpha}/\mathbf{VP}_{b,\alpha}))$	$\mathbf{S}_{f,\varnothing} \restriction (\mathbf{S}_{f,\varnothing} \restriction (\mathbf{VP}_{f,\varnothing}/\mathbf{VP}_{b,\varnothing}))$
should	*need*
must	*dare*
ought	
might	
can	*can*
could	*could*
may	*may*
will	*will*
would	*would*

We now illustrate the working of this fragment with the analyses for (450a) (which involves a PPI modal) and (450b) (which involves an NPI modal).

(450)   a.   John should not come.
        b.   John need not come.

The derivation for (450a) goes as follows:

(451)

$$\dfrac{\text{john;}\ \mathbf{j};\text{NP} \quad \dfrac{\begin{bmatrix}\varphi_4; \\ h; \text{VP}_{f,\varnothing}/\text{VP}_{b,\varnothing}\end{bmatrix}^4 \quad \dfrac{[\varphi_1; f; \text{VP}_{b,\varnothing}/\text{VP}_{b,\varnothing}]^1 \quad \text{come; } \mathbf{come}; \text{VP}_{b,\varnothing}}{\varphi_1 \circ \text{come}; f(\mathbf{come}); \text{VP}_{b,\varnothing}}}{\varphi_4 \circ \varphi_1 \circ \text{come}; h(f(\mathbf{come})); \text{VP}_{f,\varnothing}}}{\text{john} \circ \varphi_4 \circ \varphi_1 \circ \text{come}; h(f(\mathbf{come}))(\mathbf{j}); \text{S}_{f,\varnothing}}$$

$$\vdots$$

$$\dfrac{\text{john} \circ \varphi_4 \circ \varphi_1 \circ \text{come};\ h(f(\mathbf{come}))(\mathbf{j}); \text{S}_{f,\varnothing}}{\lambda\varphi_1.\text{john} \circ \varphi_4 \circ \varphi_1 \circ \text{come};\ \lambda f.h(f(\mathbf{come}))(\mathbf{j});\ \text{S}_{f,\varnothing} \restriction (\text{VP}_{b,\varnothing}/\text{VP}_{b,\varnothing})}{\restriction} I^1$$

$$\begin{array}{l}\lambda\sigma.\sigma(\text{not}); \\ \lambda\mathscr{G}.\neg\mathscr{G}(\text{id}_{et}); \\ \text{S}_{\gamma,-} \restriction (\text{S}_{\gamma,\varnothing} \restriction (\text{VP}_{b,\varnothing}/\text{VP}_{b,\varnothing}))\end{array}$$

$$\dfrac{\text{john} \circ \varphi_4 \circ \text{not} \circ \text{come};\ \neg h(\mathbf{come})(\mathbf{j});\ \text{S}_{f,-}}{\lambda\varphi_4.\text{john} \circ \varphi_4 \circ \text{not} \circ \text{come};\ \lambda h.\neg h(\mathbf{come})(\mathbf{j}); \text{S}_{f,-} \restriction (\text{VP}_{f,\varnothing}/\text{VP}_{b,\varnothing})} I^4$$

$$\begin{array}{l}\lambda\sigma.\sigma(\text{should}); \\ \lambda\mathscr{G}.\square\mathscr{G}(\text{id}_{et}); \\ \text{S}_{f,+} \restriction (\text{S}_{f,\beta} \restriction (\text{VP}_{f,\alpha}/\text{VP}_{b,\alpha}))\end{array}$$

$$\text{john} \circ \text{should} \circ \text{not} \circ \text{come};\ \square\neg\mathbf{come}(\mathbf{j});\ \text{S}_{f,+}$$

The key point here is that although both *should* and *not* are lexically specified to take scope at the clausal level, their scopal relation is fixed. Specifically, once *should* takes scope, the resultant clause is S_+, which is incompatible with the specification on the argument category for *not*. This means that negation is forced to take scope before the PPI modal does.

Exactly the opposite relation holds between the NPI modal *need* and negation. Here, after negation takes scope, we have S_-, but this specification is incompatible with the argument category for the NPI modal, which requires the clause it scopes over to be $S_\varnothing$. Thus, as in (452), the only possibility is to have *need* take scope before the negation does, which gives us the $\neg > \square$ scopal relation.

(452)

$$\text{john;} \quad \cfrac{\begin{bmatrix} \varphi_4; \\ h; \text{VP}_{f,\varnothing}/\text{VP}_{b,\varnothing} \end{bmatrix}^4 \quad \cfrac{[\varphi_1; f; \text{VP}_{b,\varnothing}/\text{VP}_{b,\varnothing}]^1 \quad \text{come; } \textbf{come; VP}_{b,\varnothing}}{\varphi_1 \circ \text{come; } f(\textbf{come}); \text{VP}_{b,\varnothing}}}{\cfrac{\varphi_4 \circ \varphi_1 \circ \text{come; } h(f(\textbf{come})); \text{VP}_{f,\varnothing}}{\text{john} \circ \varphi_4 \circ \varphi_1 \circ \text{come; } h(f(\textbf{come}))(\textbf{j}); \text{S}_{f,\varnothing}}}$$
$$\textbf{j}; \text{NP}$$

$$\vdots$$

$$\lambda\sigma.\sigma(\text{need}); \qquad \cfrac{\begin{array}{c}\text{john} \circ \varphi_4 \circ \varphi_1 \circ \text{come;} \\ h(f(\textbf{come}))(\textbf{j}); \text{S}_{f,\varnothing}\end{array}}{\begin{array}{c}\lambda\varphi_4.\text{john} \circ \varphi_4 \circ \varphi_1 \circ \text{come;} \\ \lambda h.h(f(\textbf{come}))(\textbf{j}); \\ \text{S}_{f,\varnothing}\upharpoonright(\text{VP}_{f,\varnothing}/\text{VP}_{b,\varnothing})\end{array}} \upharpoonright I^4$$

$$\lambda\mathscr{G}.\square\mathscr{G}(\text{id}_{et});$$
$$\text{S}_{f,\varnothing}\upharpoonright(\text{S}_{f,\varnothing}\upharpoonright(\text{VP}_{f,\varnothing}/\text{VP}_{b,\varnothing}))$$

$$\lambda\sigma.\sigma(\text{not}); \qquad \cfrac{\text{john} \circ \text{need} \circ \varphi_1 \circ \text{come; } \square f(\textbf{come})(\textbf{j}); \text{S}_{f,\varnothing}}{\begin{array}{c}\lambda\varphi_1.\text{john} \circ \text{need} \circ \varphi_1 \circ \text{come;} \\ \lambda f.\square f(\textbf{come})(\textbf{j}); \text{S}_{f,\varnothing}\upharpoonright(\text{VP}_{b,\varnothing}/\text{VP}_{b,\varnothing})\end{array}} \upharpoonright I^1$$

$$\lambda\mathscr{G}.\neg\mathscr{G}(\text{id}_{et});$$
$$\text{S}_{\gamma,-}\upharpoonright(\text{S}_{\gamma,\varnothing}\upharpoonright(\text{VP}_{b,\varnothing}/\text{VP}_{b,\varnothing}))$$

$$\text{john} \circ \text{need} \circ \text{not} \circ \text{come; } \neg\square\textbf{come}(\textbf{j}); \text{S}_{f,-}$$

We assume that modals that give rise to scope ambiguity with negation are simply ambiguous between PPI and NPI variants, as in (449). This accounts for the scope ambiguity of examples such as (444).[2]

Contracted negation presents no special problems, except for one small idiosyncrasy with "neutral" modals that needs to be taken into account. Contracted forms of (un-ambiguously) PPI and NPI modals such as *shouldn't* and *needn't* simply preserve the

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2. Though we have chosen to posit two distinct lexical entries for the "neutral" modals (*can*, *could*, and *may*) for high and low scoping possibilities with respect to negation, corresponding respectively to the scoping properties of the unambiguous modals, it is easy to collapse these two entries for these modals by making the polarity features for the two Ss and two VPs in the complex higher-order category for the modal totally underspecified and unconstrained (except for one constraint $\langle\alpha,\beta\rangle \neq \langle\varnothing,-\rangle$, to exclude the possibility of double negation marking **can not not*), along the following lines:

(i) $\lambda\sigma.\sigma(\text{can}); \lambda\mathscr{G}.\diamondsuit\mathscr{G}(\text{id}_{et}); \text{S}_{f,\alpha}\upharpoonright(\text{S}_{f,\beta}\upharpoonright(\text{VP}_{f,\delta}/\text{VP}_{b,\zeta}))$

By (partially) resolving underspecification, we can derive both the "PPI" and "NPI" variants of the modal lexical entry in (449) from (i), thus capturing scope ambiguity via a single lexical entry. (i) allows for other instantiations of feature specification, but these are either redundant (yielding either high or low scope that is already derivable with the PPI and NPI instantiations in (449)) or useless (i.e., cannot be used in any well-formed syntactic derivation) and hence harmless. Thus, if desired, the lexical ambiguity we have tentatively assumed in the main text can be eliminated by adopting the more general lexical entry along the lines of (i) without the danger of overgeneration.

scope relation between the modal meanings and negation identical to the uncontracted forms (i.e., *should not*, *need not*, and so on.). However, for the "neutral" modals such as *can*, which induce ambiguity in the uncontracted forms, the contracted variants (e.g., *can't*) only have the negation wide-scope meanings ($\neg > \Diamond$). Whatever the source of this idiosyncrasy, assuming that the contracted forms are all stored in the lexicon, the proper scope relation between the modal and negation can be readily captured in the present approach.[3]

9.2.2 Imposing the Clause-Bounded Scope Restriction on Modals and Negation

One issue that needs to be addressed in the higher-order operator analysis of modal auxiliaries and negation we have proposed above is potential overgeneration regarding the syntactic scope of the modal and negation operators. For example, in the following, neither the negation nor the modal auxiliary in the embedded clause can scope over the matrix clause:

(453) John may think Ann should not buy the car.

This clause-boundedness restriction on the scope of modal and negation operators can be captured via the clause-level indexing mechanism we employed in the analysis of extraction pathway marking in chapter 7. To avoid notational clutter, we have not made explicit the relevant assumptions in presenting the lexical entries for the modal and negation operators above, but this can be implemented easily by revising those lexical entries along the following lines:

(454)　a.　$\lambda\sigma.\sigma(\mathsf{should}); \lambda\mathcal{G}.\Box\mathcal{G}(\mathsf{id}_{et}); S_{f,+}^n \!\upharpoonright\! (S_{f,\beta}^n \!\upharpoonright\! (\mathrm{VP}_{f,\alpha}^n/\mathrm{VP}_{b,\alpha}^n))$
　　　　b.　$\lambda\sigma.\sigma(\mathsf{need}); \lambda\mathcal{G}.\Box\mathcal{G}(\mathsf{id}_{et}); S_{f,\varnothing}^n \!\upharpoonright\! (S_{f,\varnothing}^n \!\upharpoonright\! (\mathrm{VP}_{f,\varnothing}^n/\mathrm{VP}_{b,\varnothing}^n))$
　　　　c.　$\lambda\sigma.\sigma(\mathsf{not}); \lambda\mathcal{G}.\neg\mathcal{G}(\mathsf{id}_{et}); S_{\gamma,-}^n \!\upharpoonright\! (S_{\gamma,\varnothing}^n \!\upharpoonright\! (\mathrm{VP}_{b,\varnothing}^n/\mathrm{VP}_{b,\varnothing}^n))$

The explicit indexing on the S and VP/VP categories in these lexical entries ensures that the modal and negation operators take scope directly over the clauses that are "projections" of the VP/VP gaps that they bind. This ensures the clause-boundedness of the scope of these operators.

We illustrate in (455) a failed derivation for (453) in which the embedded negation tries to take scope at the matrix level. The derivation fails due to the mismatch in the index between the VP/VP gap and the outer S for the sign that is given as an argument to the negation operator in (454c) (to make the derivation easier to read, we have used

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3. An alternative would be to assume a syntactic derivation for auxiliary contraction. On this approach, the inability of the PPI variant of *can* to morphologically merge with contracted negation needs to be stipulated in some way or other.

the "slanted" variants of the modals *should* and *may* here (cf. section 9.3.1); this choice is immaterial for the (failed) status of the derivation in (455)).[4]

(455)

$$\frac{\begin{array}{cc} \text{buy} \circ \text{the} \circ \text{car};\\ \mathbf{buy}(\iota(\mathbf{car})); \text{VP}^1_{b,\varnothing} \end{array} \quad \left[\begin{array}{l} \varphi_1;\\ f; \text{VP}^1_{b,\varnothing}/\text{VP}^1_{b,\varnothing} \end{array}\right]^1}{\begin{array}{c} \varphi_1 \circ \text{buy} \circ \text{the} \circ \text{car};\\ f(\mathbf{buy}(\iota(\mathbf{car}))); \text{VP}^1_{b,\varnothing} \end{array}}$$

$$\frac{\vdots}{\begin{array}{c}\text{should};\\ \lambda P \lambda y.\Box P(y);\\ \text{VP}^1_{f,+}/\text{VP}^1_{b,\varnothing}\end{array}}$$

$$\frac{\begin{array}{c}\text{should} \circ \varphi_1 \circ \text{buy} \circ \text{the} \circ \text{car};\\ \lambda y.\Box f(\mathbf{buy}(\iota(\mathbf{car}))); \text{VP}^1_{f,+}\end{array} \quad \begin{array}{c}\text{ann};\\ \mathbf{a};\\ \text{NP}\end{array}}{\text{ann} \circ \text{should} \circ \varphi_1 \circ \text{buy} \circ \text{the} \circ \text{car}; \Box f(\mathbf{buy}(\iota(\mathbf{car})))(\mathbf{a}); \text{S}^1_{f,+}}$$

$$\frac{\begin{array}{c}\text{ann} \circ \text{should} \circ \varphi_1 \circ\\ \text{buy} \circ \text{the} \circ \text{car};\\ \Box f(\mathbf{buy}(\iota(\mathbf{car})))(\mathbf{a});\\ \text{S}^1_{f,+}\end{array} \quad \begin{array}{c}\text{think};\\ \mathbf{think};\\ \text{VP}^{n+1}_{b,\varnothing}/\text{S}^n_{f,+}\end{array}}{\begin{array}{c}\text{think} \circ \text{ann} \circ \text{should} \circ \varphi_1 \circ\\ \text{buy} \circ \text{the} \circ \text{car};\\ \mathbf{think}(\Box f(\mathbf{buy}(\iota(\mathbf{car})))(\mathbf{a})); \text{VP}^2_{b,\varnothing}\end{array} \quad \begin{array}{c}\vdots\\ \text{may};\\ \lambda Q \lambda z.\\ \Diamond Q(z);\\ \text{VP}^2_{f,+}/\text{VP}^2_{b,\varnothing}\end{array}}$$

$$\frac{\begin{array}{c}\text{may} \circ \text{think} \circ \text{ann} \circ \text{should} \circ \varphi_1 \circ \text{buy} \circ \text{the} \circ \text{car};\\ \lambda z.\Diamond\mathbf{think}(\Box f(\mathbf{buy}(\iota(\mathbf{car})))(\mathbf{a}))(z); \text{VP}^2_{f,+}\end{array} \quad \begin{array}{c}\text{john};\\ \mathbf{j};\\ \text{NP}\end{array}}{\begin{array}{c}\text{john} \circ \text{may} \circ \text{think} \circ \text{ann} \circ \text{should} \circ \varphi_1 \circ \text{buy} \circ \text{the} \circ \text{car};\\ \Diamond\mathbf{think}(\Box f(\mathbf{buy}(\iota(\mathbf{car})))(\mathbf{a}))(\mathbf{j}); \text{S}^2_{f,+}\end{array}}$$

$$\frac{}{\begin{array}{c}\lambda\varphi_1.\text{john} \circ \text{may} \circ \text{think} \circ \text{ann} \circ \text{should} \circ \varphi_1 \circ \text{buy} \circ \text{the} \circ \text{car};\\ \lambda f.\Diamond\mathbf{think}(\Box f(\mathbf{buy}(\iota(\mathbf{car})))(\mathbf{a}))(\mathbf{j}); \text{S}^2_{f,+}\!\restriction(\text{VP}^1_{b,\varnothing}/\text{VP}^1_{b,\varnothing})\end{array}}\restriction\text{I}^1 \quad \begin{array}{c}\lambda\sigma.\sigma(\mathbf{not});\\ \lambda\mathscr{G}.\neg\mathscr{G};\\ \text{S}^n_{\gamma,-}\!\restriction\\ (\text{S}^n_{\gamma,\varnothing}\!\restriction(\text{VP}^n_{b,\varnothing}/\text{VP}^n_{b,\varnothing}))\end{array}$$

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To avoid notational clutter, we omit the clause-level indexing feature altogether in the rest of this chapter (and throughout the whole monograph), but it should be assumed that it is implemented along the lines explained above for the purpose of controlling overgeneration.

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4. The same failure of wide scoping over embedded propositional content is observed in connection with negation inside complement VPs in cases such as (i) involving a control verb, cited in Sag et al. (2020, 41) as supporting the interpretation of narrow-scoping negation operators as adjuncts:

(i) Pat considered not doing the homework assignments.

Such cases are straightforward in our account, on the assumption that the lexical entry for *considered*, like other nonauxiliary verbs which embed propositional arguments, will have the syntactic type $\text{VP}^{n+1}_{f,\alpha}/\text{VP}^n_{ger,\alpha}$, which, combining with its gerundive argument, increments the clause level by one.

9.3   Consequences of the Higher-Order Analysis of Modals and Negation

In this section, we discuss some consequences of the higher-order analysis of modals and negation we have presented above. As will be discussed below, the present analysis extends straightforwardly to more complex examples involving these modal and negation operators in ways that may not be fully obvious initially.

9.3.1   Slanting and the VP/VP Analysis of Auxiliaries

The analysis of modal scope presented above can, in a sense, be thought of as a logical reconceptualization of the configurational account proposed by Iatridou and Zeijlstra (2013). Instead of relying on reconstruction and movement, our analysis simply regulates the relative scope relations between the auxiliary and negation via the three-way distinction of the polarity-marking feature *pol*, but aside from this technical difference, the essential analytic idea is the same: the semantic scope of the modal and negation operators transparently reflects the form of the abstract combinatoric structure that is not directly visible from surface constituency, be it a level of syntactic representation (i.e., LF, as in Iatridou and Zeijlstra's account) or the structure of the proof that yields the pairing of surface string and semantic translation (as in our approach, and more generally, in CG-based theories of natural language syntax-semantics).

One might then wonder whether the two analyses are mere notational variants or if there is any advantage gained by recasting the LF-based analysis in a type-logical setup. We do think that our approach has the advantage of being fully explicit, without relying on the notions of reconstruction and movement, whose exact details remain somewhat elusive. However, rather than dwelling on this point, we would like to point out an interesting consequence that immediately follows from our account and which illuminates the relationship between the "transformational" analysis of auxiliaries (of the sort embodied in our analysis of modal auxiliaries as "VP-modifier quantifiers") and the lexicalist alternatives in the tradition of nontransformational syntax (such as G/HPSG and CG).

To see the relevant point, note first that PPI modals such as *should* can be derived in the lower-order category $\mathrm{VP}_{f,+}/\mathrm{VP}_{b,\delta}$ as follows (where $\alpha, \beta, \delta \in \{\varnothing, -\}$):

(456)

$$\dfrac{\lambda\sigma.\sigma(\text{should});\ \lambda\mathscr{G}.\Box\mathscr{G}(\text{id}_{et});\ S_{f,+}\!\upharpoonright\!(S_{f,\beta}\!\upharpoonright\!(\text{VP}_{f,\alpha}/\text{VP}_{b,\alpha}))\qquad \dfrac{\begin{bmatrix}\varphi_3;\\ x;\text{NP}\end{bmatrix}^3\quad \dfrac{\begin{bmatrix}\varphi_1;\\ f;\text{VP}_{f,\delta}/\text{VP}_{b,\delta}\end{bmatrix}^1\ \begin{bmatrix}\varphi_2;\\ g;\text{VP}_{b,\delta}\end{bmatrix}^2}{\dfrac{\varphi_1\circ\varphi_2;\ f(g);\ \text{VP}_{f,\delta}}{\dfrac{\varphi_3\circ\varphi_1\circ\varphi_2;\ f(g)(x);\ S_{f,\delta}}{\lambda\varphi_1.\varphi_3\circ\varphi_1\circ\varphi_2;\ \lambda f.f(g)(x);\ S_{f,\delta}\!\upharpoonright\!(\text{VP}_{f,\delta}/\text{VP}_{b,\delta})}\ \upharpoonright\!\text{I}^1}}}{\dfrac{\varphi_3\circ\text{should}\circ\varphi_2;\ \Box g(x);\ S_{f,+}}{\dfrac{\text{should}\circ\varphi_2;\ \lambda x.\Box g(x);\ \text{VP}_{f,+}}{\text{should};\ \lambda g\lambda x.\Box g(x);\ \text{VP}_{f,+}/\text{VP}_{b,\delta}}\ /\text{I}^2}\ \backslash\text{I}^3}$$

Similarly, the negation morpheme *not* can be slanted to the $\text{VP}_{b,-}/\text{VP}_{b,\varnothing}$ category:

(457)

$$\dfrac{\lambda\sigma.\sigma(\text{not});\ \lambda\mathscr{G}.\neg\mathscr{G}(\text{id}_{et});\ S_{\gamma,-}\!\upharpoonright\!(S_{\gamma,\varnothing}\!\upharpoonright\!(\text{VP}_{b,\varnothing}/\text{VP}_{b,\varnothing}))\qquad \dfrac{\begin{bmatrix}\varphi_3;\\ x;\text{NP}\end{bmatrix}^3\quad \dfrac{\begin{bmatrix}\varphi_1;\\ f;\text{VP}_{b,\varnothing}/\text{VP}_{b,\varnothing}\end{bmatrix}^1\ \begin{bmatrix}\varphi_2;\\ g;\text{VP}_{b,\varnothing}\end{bmatrix}^2}{\dfrac{\varphi_1\circ\varphi_2;\ f(g);\ \text{VP}_{b,\varnothing}}{\dfrac{\varphi_3\circ\varphi_1\circ\varphi_2;\ f(g)(x);\ S_{b,\varnothing}}{\lambda\varphi_1.\varphi_3\circ\varphi_1\circ\varphi_2;\ \lambda f.f(g)(x);\ S_{b,\varnothing}\!\upharpoonright\!(\text{VP}_{b,\varnothing}/\text{VP}_{b,\varnothing})}\ \upharpoonright\!\text{I}^1}}}{\dfrac{\varphi_3\circ\text{not}\circ\varphi_2;\ \neg g(x);\ S_{b,-}}{\dfrac{\text{not}\circ\varphi_2;\ \lambda x.\neg g(x);\ \text{VP}_{b,-}}{\text{not};\ \lambda g\lambda x.\neg g(x);\ \text{VP}_{b,-}/\text{VP}_{b,\varnothing}}\ /\text{I}^2}\ \backslash\text{I}^3}$$

These two lowered categories can be combined to produce the following sign:

(458)

$$\dfrac{\text{should};\ \lambda g\lambda x.\Box g(x);\ \text{VP}_{f,+}/\text{VP}_{b,\delta}\qquad \dfrac{\text{not};\ \lambda g\lambda x.\neg g(x);\ \text{VP}_{b,-}/\text{VP}_{b,\varnothing}\quad [\varphi_1;g;\text{VP}_{b,\varnothing}]^1}{\text{not}\circ\varphi_1;\ \lambda x.\neg g(x);\ \text{VP}_{b,-}}}{\dfrac{\text{should}\circ\text{not}\circ\varphi_1;\ \lambda x.\Box\neg g(x);\ \text{VP}_{f,+}}{\text{should}\circ\text{not};\ \lambda g\lambda x.\Box\neg g(x);\ \text{VP}_{f,+}/\text{VP}_{b,\varnothing}}\ /\text{I}^1}$$

Slanting the NPI modal *need*, on the other hand, yields the following result:

(459)

$$\dfrac{\lambda\sigma.\sigma(\text{need});\ \lambda\mathscr{G}.\Box\mathscr{G}(\text{id}_{et});\ S_{f,\varnothing}\!\upharpoonright\!(S_{f,\varnothing}\!\upharpoonright\!(\text{VP}_{f,\varnothing}/\text{VP}_{b,\varnothing}))\qquad \dfrac{\begin{bmatrix}\varphi_3;\\ x;\text{NP}\end{bmatrix}^3\quad \dfrac{\begin{bmatrix}\varphi_1;\\ f;\text{VP}_{f,\varnothing}/\text{VP}_{b,\varnothing}\end{bmatrix}^1\ \begin{bmatrix}\varphi_2;\\ g;\text{VP}_{b,\varnothing}\end{bmatrix}^2}{\dfrac{\varphi_1\circ\varphi_2;\ f(g);\ \text{VP}_{f,\varnothing}}{\dfrac{\varphi_3\circ\varphi_1\circ\varphi_2;\ f(g)(x);\ S_{f,\varnothing}}{\lambda\varphi_1.\varphi_3\circ\varphi_1\circ\varphi_2;\ \lambda f.f(g)(x);\ S_{f,\varnothing}\!\upharpoonright\!(\text{VP}_{f,\varnothing}/\text{VP}_{b,\varnothing})}\ \upharpoonright\!\text{I}^1}}}{\dfrac{\varphi_3\circ\text{need}\circ\varphi_2;\ \Box g(x);\ S_{f,\varnothing}}{\dfrac{\text{need}\circ\varphi_2;\ \lambda x.\Box g(x);\ \text{VP}_{f,\varnothing}}{\text{need};\ \lambda g\lambda x.\Box g(x);\ \text{VP}_{f,\varnothing}/\text{VP}_{b,\varnothing}}\ /\text{I}^2}\ \backslash\text{I}^3}$$

Note that this resultant category cannot be combined with the lowered negation category in (457) due to feature mismatch (*need* requires its argument to be $VP_{b,\varnothing}$, but *not* marks the VP as $VP_{b,-}$). Thus, the lowered *need* is correctly prevented from outscoping negation.

It is, however, possible to derive *need not* as a complex auxiliary with the correct negation-outscoping semantics:

(460)

$$
\cfrac{
\begin{bmatrix}\varphi_3; \\ x; \text{NP}\end{bmatrix}^3
\quad
\cfrac{
\begin{bmatrix}\varphi_4; \\ h; \text{VP}_{f,\varnothing}/\text{VP}_{b,\varnothing}\end{bmatrix}^4
\quad
\cfrac{
\begin{bmatrix}\varphi_1; \\ f; \text{VP}_{b,\varnothing}/\text{VP}_{b,\varnothing}\end{bmatrix}^1
\quad
\begin{bmatrix}\varphi_2; \\ g; \text{VP}_{b,\varnothing}\end{bmatrix}^2
}{\varphi_1 \circ \varphi_2; f(g); \text{VP}_{b,\varnothing}}
}{\varphi_4 \circ \varphi_1 \circ \varphi_2; h(f(g)); \text{VP}_{f,\varnothing}}
}{\varphi_3 \circ \varphi_4 \circ \varphi_1 \circ \varphi_2; h(f(g))(x); \text{S}_{f,\varnothing}}
$$

$$
\cfrac{
\lambda\sigma.\sigma(\text{not}); \\
\lambda\mathscr{G}.\neg\mathscr{G}(\text{id}_{et}); \\
\text{S}_{\gamma,-}\!\restriction\!(\text{S}_{\gamma,\varnothing}\!\restriction\!(\text{VP}_{b,\varnothing}/\text{VP}_{b,\varnothing}))
\quad
\cfrac{
\cfrac{
\lambda\sigma.\sigma(\text{need}); \\
\lambda\mathscr{G}.\Box\mathscr{G}(\text{id}_{et}); \\
\text{S}_{f,\varnothing}\!\restriction\!(\text{S}_{f,\varnothing}\!\restriction\!(\text{VP}_{f,\varnothing}/\text{VP}_{b,\varnothing}))
\quad
\cfrac{
\begin{array}{c}\vdots \\ \varphi_3 \circ \varphi_4 \circ \varphi_1 \circ \varphi_2; \\ h(f(g))(x); \text{S}_{f,\varnothing}\end{array}
}{
\begin{array}{c}\lambda\varphi_4.\varphi_3 \circ \varphi_4 \circ \varphi_1 \circ \varphi_2; \\ \lambda h.h(f(g))(x); \\ \text{S}_{f,\varnothing}\!\restriction\!(\text{VP}_{f,\varnothing}/\text{VP}_{b,\varnothing})\end{array}
}\!{\restriction}\text{I}^4
}{
\cfrac{\varphi_3 \circ \text{need} \circ \varphi_1 \circ \varphi_2; \Box f(g)(x); \text{S}_{f,\varnothing}}{
\begin{array}{c}\lambda\varphi_1.\varphi_3 \circ \text{need} \circ \varphi_1 \circ \varphi_2; \\ \lambda f.\Box f(g)(x); \text{S}_{f,\varnothing}\!\restriction\!(\text{VP}_{b,\varnothing}/\text{VP}_{b,\varnothing})\end{array}
}\!{\restriction}\text{I}^1
}
}{
\cfrac{
\cfrac{\varphi_3 \circ \text{need} \circ \text{not} \circ \varphi_2; \neg\Box g(x); \text{S}_{f,-}}{\text{need} \circ \text{not} \circ \varphi_2; \lambda x.\neg\Box g(x); \text{VP}_{f,-}}\backslash\text{I}^3
}{\text{need} \circ \text{not}; \lambda g\lambda x.\neg\Box g(x); \text{VP}_{f,-}/\text{VP}_{b,\varnothing}}/\text{I}^2
}
$$

In short, in our type-logical setup, alternative lexical signs that correspond to the lexical entries for the relevant expressions that are directly specified in the lexicon in lexicalist theories of syntax are all derivable as theorems from the more abstract, higher-order entries we have posited above. This is essentially the consequence of the slanting lemma in the revised system augmented with polarity markings. Significantly, the polarity markings ensure that slanting of the higher-order modals and negation preserves the correct scope relations between these operators.

The formal derivability of the lower-order entry from the higher-order entry is an interesting result, as it potentially illuminates the deeper relationship between the "transformational" and "lexicalist" analyses of auxiliaries in the different traditions of the generative grammar literature. The two approaches have tended to be seen as reflecting fundamentally incompatible assumptions about the basic architecture of grammar, but if a formal connection can be established between the two at an abstract level by making certain (not totally implausible) assumptions, then the two may not be as dif-

ferent from each other as they have appeared to be throughout the whole history of the controversy between the transformational and nontransformational approaches to syntax. In any event, we take our result above to indicate that the logic-based setup of TLCG can be fruitfully employed for the purpose of meta-comparison of different approaches to grammatical phenomena in the syntactic literature.[5]

9.3.2 Some Further Consequences of Slanting

We believe that the above discussion of the derivability relation between the higher-order entries of modals and negation and their lower-order counterparts corresponding to the more familiar VP/VP entries in lexicalist theories of syntax has clarified the relationship between our proposal and previous approaches in the lexicalist tradition considerably. At a suitably abstract level, one can see the former as a proper generalization of the latter, rather than seeing the two as embodying mutually incomparable analytic ideas.

But the role of the slanting lemma in our theory is not merely to relate the more abstract analysis to a more surface-oriented familiar analysis in the lexicalist tradition. In fact, slanting plays a crucial role in analyzing certain types of examples that are otherwise difficult to analyze without duplicating the lexicon. Here, we illustrate two such cases, one involving coordination of the modals themselves (this section) and the other involving VP ellipsis with the higher-order modal interpretation (section 9.3.3).

Consider first the conjunction of modals in (461).

(461) Every physicist can and should learn how to teach quantum mechanics to the undergraduate literature majors.

There is a reading for this sentence in which the two modals outscope the subject universal quantifier in each conjunct ('it is possible that every physicist learns . . . and it is deontically necessary that every physicist learn . . .').

Assuming that *and* is of type $(X \backslash X)/X$, combining only expressions whose prosodies are strings, it may appear impossible to derive (461) on the relevant reading, since the modals in (461) must be higher-order to outscope the subject quantifier and therefore must have functional prosodies. In fact, however, a straightforward derivation is avail-

---

5. Familiar examples such as (i) provide motivation for higher-order versions of raising verbs such as *seem*, *tend*, *happen*, and so on:

(i) A unicorn seems to be approaching.

It is straightforward to show that the higher-order lexical entries for such verbs yield, by exactly the same proof narratives as for modals, the lower-order VP/VP signs that correspond to the argument structure taken as standard in a variety of nontransformational frameworks. On the basis of such higher-order $S{\restriction}(S{\restriction}(VP/VP))$ types, we would expect to find cases of raising predicates scoping wide over conjunctions and disjunctions, but the semantics of these verbs makes it difficult to come up with clear cases that would enable us to test this prediction.

able with no additional assumptions or machinery. Note first that the modal auxiliary can be derived in the $((S/VP)\backslash S)/VP$ type:

(462)
$$\cfrac{[\varphi_3;\mathscr{P};S_{f,\alpha}/VP_{f,\alpha}]^3 \quad \cfrac{\cfrac{[\varphi_2;f;VP_{f,\alpha}/VP_{b,\alpha}]^2 \quad [\varphi_1;P;VP_{b,\alpha}]^1}{\varphi_2\circ\varphi_1;f(P);VP_{f,\alpha}}}{\cfrac{\varphi_3\circ\varphi_2\circ\varphi_1;\mathscr{P}(f(P));S_{f,\alpha}}{\lambda\varphi_2.\varphi_3\circ\varphi_2\circ\varphi_1;\lambda f.\mathscr{P}(P);S_{f,\alpha}\!\restriction\!(VP_{f,\alpha}/VP_{b,\alpha})}\upharpoonright I^2} \quad \begin{array}{l}\lambda\sigma.\sigma(\mathsf{can});\\ \lambda\mathscr{F}.\Diamond\mathscr{F}(\mathsf{id}_{et});\\ S_{f,+}\!\restriction\!(S_{f,\beta}\!\restriction\!(VP_{f,\alpha}/VP_{b,\alpha}))\end{array}}{\cfrac{\cfrac{\varphi_3\circ\mathsf{can}\circ\varphi_1;\Diamond\mathscr{P}(P);S_{f,+}}{\mathsf{can}\circ\varphi_1;\lambda\mathscr{P}.\Diamond\mathscr{P}(P);(S_{f,\alpha}/VP_{f,\alpha})\backslash S_{f,+}}\backslash I^3}{\mathsf{can};\lambda P\lambda\mathscr{P}.\Diamond\mathscr{P}(P);((S_{f,\alpha}/VP_{f,\alpha})\backslash S_{f,+})/VP_{b,\alpha}}/I^1}$$

By conjoining two such modals via generalized conjunction, we obtain

(463) $\mathsf{can}\circ\mathsf{and}\circ\mathsf{should};\lambda R\lambda\mathscr{R}.\Diamond\mathscr{R}(R)\wedge\Box\mathscr{R}(R);((S_{f,\alpha}/VP_{f,\alpha})\backslash S_{f,+})/VP_{b,\alpha}$

We apply this functor first to the sign with VP type derived for *learn how to teach quantum mechanics to the undergraduate literature majors* and then to the slanted version of the quantified subject *every physicist*, derivable as in (464):

(464)
$$\cfrac{\cfrac{\cfrac{[\varphi_1;y;NP]^1 \quad [\varphi_2;P;VP_{f,\alpha}]^2}{\varphi_1\circ\varphi_2;P(y);S_{f,\alpha}}}{\lambda\varphi_1.\varphi_1\circ\varphi_2;\lambda y.P(y);S_{f,\alpha}\!\restriction\!NP}\upharpoonright I^1 \quad \begin{array}{l}\lambda\sigma_1.\sigma_1(\mathsf{every}\circ\mathsf{physicist});\\ \mathbf{V_{phys}};S_{f,\alpha}\!\restriction\!(S_{f,\alpha}\!\restriction\!NP)\end{array}}{\cfrac{\mathsf{every}\circ\mathsf{physicist}\circ\varphi_2;\mathbf{V_{phys}}(\lambda y.P(y));S_{f,\alpha}}{\mathsf{every}\circ\mathsf{physicist};\lambda P.\mathbf{V_{phys}}(\lambda y.P(y));S_{f,\alpha}/VP_{f,\alpha}}/I^2}$$

This yields the following result, with the correct semantic translation for (461):

(465)
$$\cfrac{\begin{array}{cc}\begin{array}{c}\vdots\\ \mathsf{every}\circ\mathsf{physicist};\\ \mathbf{V_{phys}};S_{f,\alpha}/VP_{f,\alpha}\end{array} & \cfrac{\begin{array}{c}\vdots\\ \mathsf{can}\circ\mathsf{and}\circ\mathsf{should};\\ \lambda R\lambda\mathscr{R}.\Diamond\mathscr{R}(R)\wedge\Box\mathscr{R}(R);((S_{f,\alpha}/VP_{f,\alpha})\backslash S_{f,+})/VP_{b,\alpha}\end{array} \quad \begin{array}{c}\vdots\\ \mathsf{learn}\ldots;\\ \mathbf{LHT};VP_{b,\alpha}\end{array}}{\mathsf{can}\circ\mathsf{and}\circ\mathsf{should}\circ\mathsf{learn}\ldots;\\ \lambda\mathscr{R}.\Diamond\mathscr{R}(\mathbf{LHT})\wedge\Box\mathscr{R}(\mathbf{LHT});(S_{f,\alpha}/VP_{f,\alpha})\backslash S_{f,+}}\end{array}}{\mathsf{every}\circ\mathsf{physicist}\circ\mathsf{can}\circ\mathsf{and}\circ\mathsf{should}\circ\mathsf{learn}\ldots;\\ \Diamond\mathbf{V_{phys}}(\mathbf{LHT})\wedge\Box\mathbf{V_{phys}}(\mathbf{LHT});S_{f,+}}$$

9.3.3 Higher-Order Modals and Ellipsis

A natural question that arises at this point is whether our analysis of VP ellipsis and pseudogapping will extend to cases in which the higher-order entry for the modal is involved in the ellipsis site, due to scopal interaction with other elements in the sentence (typically, the subject quantifier scoping lower than the auxiliary).

There are indeed examples of exactly this pattern, such as the following:

(466) a. A mathematician will solve this physics problem, someday, but no physicist ever will.

   b. Maybe John and Bill don't solve math problems, but surely every physicist should physics problems.

For example, (466b) has two possible readings. On one reading (*every > should*), the sentence asserts the obligation held by actual physicists of solving physics problems. On the other reading (*should > every*), the sentence says something about what has to be the case about whoever happen to be physicists in the relevant (deontically necessary) possible worlds, so, on this reading it can be (non-vacuously) true even in situations in which there are no physicists in the actual world. These two readings are thus truth-conditionally distinct. A similar, modal wide-scope reading is available in the VP ellipsis example (466a) as well. To obtain this second type of reading, we need the higher-order *will* rather than the lower-order VP/VP entry to host VP ellipsis. Perhaps surprisingly, the analysis of VP ellipsis and pseudogapping from chapter 6 can license the relevant readings for (466) without introducing any additional machinery. We illustrate this point in what follows.

   We start with the proof that an $S\!\upharpoonright\!(S\!\upharpoonright\!(\mathrm{TV}/\mathrm{TV}))$ expression for the higher-order modal is an entailment of the $S\!\upharpoonright\!(S\!\upharpoonright\!(\mathrm{VP}/\mathrm{VP}))$ entry posited in the lexicon:

(467)

$$
\cfrac{\cfrac{\left[\begin{array}{l}\sigma_1;\\ \mathscr{C};\\ S_{f,\beta}\!\upharpoonright\!(\mathrm{TV}_{f,\alpha}/\mathrm{TV}_{b,\alpha})\end{array}\right]^4 \quad \cfrac{\cfrac{\cfrac{\left[\begin{array}{l}\varphi_1;\\ f;\\ \mathrm{VP}_{f,\alpha}/\mathrm{VP}_{b,\alpha}\end{array}\right]^1 \quad \cfrac{\left[\begin{array}{l}\varphi_2;\\ R;\\ \mathrm{TV}_{b,\alpha}\end{array}\right]^2 \quad \left[\begin{array}{l}\varphi_3;\\ x;\\ \mathrm{NP}\end{array}\right]^3}{\varphi_2\circ\varphi_3;\ R(x);\mathrm{VP}_{b,\alpha}}}{\varphi_1\circ\varphi_2\circ\varphi_3;\ f(R(x));\ \mathrm{VP}_{f,\alpha}}\ /\mathrm{I}^3}{\varphi_1\circ\varphi_2;\ \lambda x.f(R(x));\ \mathrm{TV}_{f,\alpha}}\ /\mathrm{I}^2}{\varphi_1;\ \lambda R\lambda x.f(R(x));\ \mathrm{TV}_{f,\alpha}/\mathrm{TV}_{b,\alpha}}}{\sigma_1(\varphi_1);\ \mathscr{C}(\lambda R\lambda x.f(R(x)));\ S_{f,\beta}}}{\lambda\varphi_1.\sigma_1(\varphi_1);\ \lambda f.\mathscr{C}(\lambda R\lambda x.f(R(x)));\ S_{f,\beta}\!\upharpoonright\!(\mathrm{VP}_{f,\alpha}/\mathrm{VP}_{b,\alpha})}\ {}^{\upharpoonright\mathrm{I}^1}
$$

$$
\cfrac{\lambda\varphi_1.\sigma_1(\varphi_1);\ \lambda f.\mathscr{C}(\lambda R\lambda x.f(R(x)));\ S_{f,\beta}\!\upharpoonright\!(\mathrm{VP}_{f,\alpha}/\mathrm{VP}_{b,\alpha}) \qquad \begin{array}{l}\lambda\sigma_0.\sigma_0(\mathsf{should});\\ \lambda\mathscr{F}.\Box\mathscr{F}(\mathrm{id}_{et});\\ S_{f,+}\!\upharpoonright\!(S_{f,\beta}\!\upharpoonright\!(\mathrm{VP}_{f,\alpha}/\mathrm{VP}_{b,\alpha}))\end{array}}{\cfrac{\sigma_1(\mathsf{should});\ \Box\mathscr{C}(\lambda R\lambda x.R(x));\ S_{f,+}}{\lambda\sigma_1.\sigma_1(\mathsf{should});\ \lambda\mathscr{C}.\Box\mathscr{C}(\mathrm{id}_{et});\ S_{f,+}\!\upharpoonright\!(S_{f,\beta}\!\upharpoonright\!(\mathrm{TV}_{f,\alpha}/\mathrm{TV}_{b,\alpha}))}\ {}^{\upharpoonright\mathrm{I}^4}}
$$

With this derived entry for the auxiliary, the derivation for the ellipsis clause of (466b) is straightforward. The derivation is shown in (468).

(468)

$$
\lambda\sigma_1.\sigma_1(\text{every}\circ \\
\text{physicist}); \\
\mathbf{V_{phys}}; \\
S_{f,\varnothing}\!\upharpoonright\!(S_{f,\varnothing}\!\upharpoonright\!NP)
$$

$$
\cfrac{
\begin{bmatrix}\varphi_1;\\ x;\\ NP\end{bmatrix}^1 \quad
\cfrac{
\cfrac{
\begin{array}{c}\lambda\varphi_0.\varphi_0;\\ \lambda\mathscr{F}.\mathscr{F}(\mathbf{solve});\\ TV_{f,\varnothing}\!\upharpoonright\!(TV_{f,\varnothing}/TV_{b,\varnothing})\end{array}\quad
\begin{bmatrix}\varphi;\\ \mathscr{G};\\ TV_{f,\varnothing}/TV_{b,\varnothing}\end{bmatrix}^0
}{\varphi;\mathscr{G}(\mathbf{solve});TV_{f,\varnothing}}\;/E \qquad
\begin{array}{c}\text{physics}\circ\\ \text{problems};\\ \mathbf{physp};NP\end{array}
}{
\cfrac{
\cfrac{\varphi\circ\text{physics}\circ\text{problems};\mathscr{G}(\mathbf{solve})(\mathbf{physp});VP_{f,\varnothing}}{\varphi_1\circ\varphi\circ\text{physics}\circ\text{problems};\mathscr{G}(\mathbf{solve})(\mathbf{physp})(x);S_{f,\varnothing}}
}{
\begin{array}{c}\lambda\varphi_1.\varphi_1\circ\varphi\circ\text{physics}\circ\text{problems};\\ \lambda x.\mathscr{G}(\mathbf{solve})(\mathbf{physp})(x);S_{f,\varnothing}\!\upharpoonright\!NP\end{array}
}\;\upharpoonright\!I^1
}
}{
\begin{array}{c}\text{every}\circ\text{physicist}\circ\varphi\circ\text{physics}\circ\text{problems};\\ \mathbf{V_{phys}}(\lambda x.\mathscr{G}(\mathbf{solve})(\mathbf{physp})(x));S_{f,\varnothing}\end{array}
}
$$

$$
\cfrac{\cdots}{
\begin{array}{c}\lambda\varphi.\text{every}\circ\text{physicist}\circ\varphi\circ\text{physics}\circ\text{problems};\\ \lambda\mathscr{G}.\mathbf{V_{phys}}(\lambda x.\mathscr{G}(\mathbf{solve})(\mathbf{physp})(x));S_{f,\varnothing}\!\upharpoonright\!(TV_{f,\varnothing}/TV_{b,\varnothing})\end{array}
}\;\upharpoonright\!I^0
$$

$$
\cfrac{
\begin{array}{cc}
\begin{array}{l}\lambda\sigma_1.\sigma_1(\text{should});\\ \lambda\mathscr{C}.\Box\mathscr{C}(\mathrm{id}_{et});\\ S_{f,+}\!\upharpoonright\!(S_{f,\beta}\!\upharpoonright\!(TV_{f,\alpha}/TV_{b,\alpha}))\end{array}
&
\begin{array}{l}\lambda\varphi.\text{every}\circ\text{physicist}\circ\varphi\circ\text{physics}\circ\text{problems};\\ \lambda\mathscr{G}.\mathbf{V_{phys}}(\lambda x.\mathscr{G}(\mathbf{solve})(\mathbf{physp})(x));S_{f,\varnothing}\!\upharpoonright\!(TV_{f,\varnothing}/TV_{b,\varnothing})\end{array}
\end{array}
}{
\begin{array}{c}\text{every}\circ\text{physicist}\circ\text{should}\circ\text{physics}\circ\text{problems};\\ \Box\mathbf{V_{phys}}(\lambda x.\mathbf{solve}(\mathbf{physp})(x));S_{f,+}\end{array}
}
$$

The key point of the derivation here is that a hypothetical TV/TV (which later gets bound by the "Geachified" higher-order modal derived in (467)) feeds the ordinary pseudogapping operator (of type TV⌈(TV/TV)) that supplies the meaning of the missing verb. Since the Geachified higher-order modal enters the derivation after the subject quantifier takes scope, we obtain the desired $\Box > \forall$ reading for the sentence. Note in particular that via the systematic interaction of hypothetical reasoning, no special entry for the ellipsis operator (e.g., one that directly takes the higher-order modal as an argument) needs to be posited to derive the relevant modal wide-scope reading for the sentence.

9.3.4 VP Fronting

Work in phrase structure–theoretic approaches to the syntax-semantics interface has tended to follow the treatment of negation in Kim and Sag (2002), which distinguishes *not* (and possibly *never*) as a complement of auxiliaries from *not* as an adjunct to the auxiliaries' VP complements (see section 9.4 for more on this). This approach is supposedly motivated by the ambiguity of sentences with *could not/never* sequences, where both $\neg > \Diamond$ and $\Diamond > \neg$ readings are available.

There is, in fact, only a very sparse empirical base in English for this phrase structure–based analysis of modal/negation scoping relations, a fact that Kim and Sag (2002)

themselves tacitly acknowledge. One of the few lines of argument that they appeal to is the fact that fronted VPs containing *not* adjuncts are always interpreted with narrowly scoping negation, as illustrated in (469):

(469) . . . and NOT vote, you certainly can __, if the nominees are all second-rate.

Data of this sort are intended to provide empirical support for the putative correlation of phrase structural position with the scope of negation (see section 9.4 for further discussion).

But we can readily capture the pattern in (469) in our approach by requiring that clauses hosting topicalization be subject to polarity conditions which induce the effect of entailing narrow scope for the negation within the fronted VP. We start by presenting the topicalization operator in (470a) (with the polymorphic syntactic type X), illustrating its ordinary operation to produce (470b) (where the semantics is simply an identity function, since we ignore the pragmatic effects of topicalization):

(470) a. $\lambda\varphi\lambda\sigma.\varphi\circ\sigma(\boldsymbol{\varepsilon}); \lambda\mathcal{F}\lambda\mathcal{G}.\mathcal{G}(\mathcal{F}); (S_{f,\beta}\!\restriction\!(S_{f,\beta}\!\restriction\!X))\!\restriction\!X$ where $\beta \in \{+,-\}$
     b. . . . and vote, John can __.
     c. #. . . and not vote, John can __. $(\neg > \Diamond)$

The derivation for (470b) is given in (471).

(471)

$$\frac{\dfrac{\dfrac{\text{can};\ \lambda P\lambda y.\Diamond P(y);\text{VP}_{f,+}/\text{VP}_{b,\alpha} \qquad \left[\begin{matrix}\varphi_1;\\ Q;\text{VP}_{b,\alpha}\end{matrix}\right]^1}{\text{can}\circ\varphi_1;\ \lambda y.\Diamond Q(y);\ \text{VP}_{f,+}} \qquad \begin{matrix}\text{john};\\ \mathbf{j};\text{NP}\end{matrix}}{\dfrac{\text{john}\circ\text{can}\circ\varphi_1;\ \Diamond Q(\mathbf{j});\ S_{f,+}}{\lambda\varphi_1.\text{john}\circ\text{can}\circ\varphi_1;\ \lambda Q.\Diamond Q(\mathbf{j});\ S_{f,+}\!\restriction\!\text{VP}_{b,\alpha}}\ {\scriptstyle\restriction I^1}} \qquad \dfrac{\dfrac{\lambda\varphi\lambda\sigma.\varphi\circ\sigma(\boldsymbol{\varepsilon});\ \lambda P\lambda\mathscr{C}.\mathscr{C}(P);\ (S_{f,\beta}\!\restriction\!(S_{f,\beta}\!\restriction\!X))\!\restriction\!X \qquad \begin{matrix}\text{vote};\\ \mathbf{vote};\\ \text{VP}_{b,\alpha}\end{matrix}}{\lambda\sigma.\text{vote}\circ\sigma(\boldsymbol{\varepsilon});\ \lambda\mathscr{C}.\mathscr{C}(\mathbf{vote});\ S_{f,\beta}\!\restriction\!(S_{f,\beta}\!\restriction\!\text{VP}_{b,\alpha})}}{\text{vote}\circ\text{john}\circ\text{can}\circ\boldsymbol{\varepsilon};\ \Diamond\mathbf{vote}(\mathbf{j});\ S_{f,+}}}$$

The requirement on the topicalization operator in (470a) effectively means that $S_{\varnothing}$ is "too small" to host a topicalized phrase. That is, in order to license topicalization, the clause needs to have already "fixed" the polarity value to either $+$ or $-$. This condition turns out to have the immediate effect of enforcing narrow scope on negation in fronted VPs.

To see how this condition works, let's suppose it did not hold; that is, suppose that β could take any of the three polarity values. Then the following would be one way in which *not* inside a topicalized phrase would outscope the modal.

(472)

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{[\varphi_4;Q;\mathrm{VP}_{b,\varnothing}]^1 \quad [\varphi_5;g;\mathrm{VP}_{b,\varnothing}/\mathrm{VP}_{b,\varnothing}]^2}{\varphi_5 \circ \varphi_4;\, g(Q);\, \mathrm{VP}_{b,\varnothing}} \quad \begin{array}{l}\text{john};\\ \mathbf{j};\mathrm{NP}\end{array}}{\text{john} \circ \varphi_5 \circ \varphi_4;\, g(Q)(\mathbf{j});\, \mathrm{S}_{b,\varnothing}}}{\lambda\varphi_5.\text{john} \circ \varphi_5 \circ \varphi_4;\, \lambda g.g(Q)(\mathbf{j});\, \mathrm{S}_{b,\varnothing}{\upharpoonright}(\mathrm{VP}_{b,\varnothing}/\mathrm{VP}_{b,\varnothing})}\,{\upharpoonright}\mathrm{I}^2 \quad \begin{array}{l}\lambda\sigma_0.\sigma_0(\text{can});\\ \lambda\mathscr{F}.\Diamond\mathscr{F}(\mathrm{id}_{et});\\ \mathrm{S}_{f,\varnothing}{\upharpoonright}(\mathrm{S}_{b,\varnothing}{\upharpoonright}(\mathrm{VP}_{b,\varnothing}/\mathrm{VP}_{b,\varnothing}))\end{array}}{\text{john} \circ \text{can} \circ \varphi_4;\, \Diamond Q(\mathbf{j});\, \mathrm{S}_{f,\varnothing}}}{\lambda\varphi_4.\text{john} \circ \text{can} \circ \varphi_4;\, \Diamond Q(\mathbf{j});\, \mathrm{S}_{f,\varnothing}{\upharpoonright}\mathrm{VP}_{b,\varnothing}}\,{\upharpoonright}\mathrm{I}^1$$

$$\cfrac{\cfrac{\begin{array}{c}\vdots\\ \lambda\varphi_4.\text{john} \circ\\ \text{can} \circ \varphi_4;\\ \Diamond Q(\mathbf{j});\\ \mathrm{S}_{f,\varnothing}{\upharpoonright}\mathrm{VP}_{b,\varnothing}\end{array} \quad \cfrac{\cfrac{\begin{array}{l}\text{vote};\\ \mathbf{vote};\\ \mathrm{VP}_{b,\varnothing}\end{array} \quad \begin{bmatrix}\varphi_1;\\ f;\\ \mathrm{VP}_{b,\varnothing}/\mathrm{VP}_{b,\varnothing}\end{bmatrix}^3 \quad \begin{array}{l}\lambda\varphi_2\lambda\sigma_1.\\ \quad \varphi_2 \circ \sigma_1(\boldsymbol{\varepsilon});\\ \lambda\mathscr{F}\lambda\mathscr{G}.\mathscr{G}(\mathscr{F});\\ (\mathrm{S}_{f,\beta}{\upharpoonright}(\mathrm{S}_{f,\beta}{\upharpoonright}\mathrm{X})){\upharpoonright}\mathrm{X}\end{array}}{\varphi_1 \circ \text{vote};\, f(\mathbf{vote});\, \mathrm{VP}_{b,\varnothing}}}{\boxed{\begin{array}{l}\lambda\sigma_1.\varphi_1 \circ \text{vote} \circ \sigma_1(\boldsymbol{\varepsilon});\\ \lambda\mathscr{C}.\mathscr{C}(f(\mathbf{vote}));\mathrm{S}_{f,\varnothing}{\upharpoonright}(\mathrm{S}_{f,\varnothing}{\upharpoonright}\mathrm{VP}_{b,\varnothing})\end{array}}}}{\cfrac{\varphi_1 \circ \text{vote} \circ \text{john} \circ \text{can} \circ \boldsymbol{\varepsilon};\, \Diamond f(\mathbf{vote})(\mathbf{j});\, \mathrm{S}_{f,\varnothing}}{\begin{array}{l}\lambda\varphi_1.\varphi_1 \circ \text{vote} \circ \text{john} \circ \text{can} \circ \boldsymbol{\varepsilon};\\ \lambda f.\Diamond f(\mathbf{vote})(\mathbf{j});\mathrm{S}_{f,\varnothing}{\upharpoonright}(\mathrm{VP}_{b,\varnothing}/\mathrm{VP}_{b,\varnothing})\end{array}}\,{\upharpoonright}\mathrm{I}^3 \quad \begin{array}{l}\lambda\sigma.\sigma(\text{not});\\ \lambda\mathscr{G}.\neg\mathscr{G}(\mathrm{id}_{et});\\ \mathrm{S}_{f,-}{\upharpoonright}(\mathrm{S}_{f,\varnothing}{\upharpoonright}(\mathrm{VP}_{b,\varnothing}/\mathrm{VP}_{b,\varnothing}))\end{array}}{\text{not} \circ \text{vote} \circ \text{john} \circ \text{can};\, \neg\Diamond\mathbf{vote}(\mathbf{j});\, \mathrm{S}_{f,-}}$$

Here, the derivation uses the NPI version of *can* in order to license the negation wide-scope reading. Since the negation is inside the topicalized phrase rather than the main clause, topicalization needs to be hosted by a clause to which negation hasn't yet combined. But this is precisely the possibility that the restriction $\beta \in \{+,-\}$ excludes. That is, the derivation in (472) actually fails to be licensed in our fragment since β in the topicalization operator cannot be instantiated as $\varnothing$ (as in the grayed-in part of the derivation in (472)).

Using the other version of *can* produces the other scopal relation ($\Diamond > \neg$) for (470c), as in the following derivation:

(473)

$$\cfrac{\cfrac{\begin{array}{l}\text{vote};\\ \mathbf{vote};\mathrm{VP}_{b,\varnothing}\end{array} \quad \begin{array}{l}\text{not};\\ \lambda Q\lambda y.\neg Q(y);\mathrm{VP}_{b,-}/\mathrm{VP}_{b,\varnothing}\end{array}}{\text{not} \circ \text{vote};\, \lambda y.\neg\mathbf{vote}(y);\, \mathrm{VP}_{b,-}} \quad \begin{array}{l}\lambda\varphi_2\lambda\sigma_1.\varphi_2 \circ \sigma_1(\boldsymbol{\varepsilon});\\ \lambda\mathscr{F}\lambda\mathscr{G}.\mathscr{G}(\mathscr{F});(\mathrm{S}_{f,\beta}{\upharpoonright}(\mathrm{S}_{f,\beta}{\upharpoonright}\mathrm{X})){\upharpoonright}\mathrm{X}\end{array}}{\lambda\sigma_1.\text{not} \circ \text{vote} \circ \sigma_1(\boldsymbol{\varepsilon});\, \lambda\mathscr{C}.\mathscr{C}(\lambda y.\neg\mathbf{vote}(y));\, \mathrm{S}_{f,\beta}{\upharpoonright}(\mathrm{S}_{f,\beta}{\upharpoonright}\mathrm{VP}_{b,-})}$$

$$
\begin{array}{c}
\begin{bmatrix} \varphi_1; \\ P; \\ VP_{b,-} \end{bmatrix}^1 \quad \begin{bmatrix} \varphi_3; \\ f; \\ VP_{b,-}/VP_{b,-} \end{bmatrix}^3 \qquad \begin{array}{c} \text{john;} \\ \mathbf{j}; \\ \text{NP} \end{array} \\
\hline
\begin{array}{c} \vdots \\ \lambda\sigma_1.\text{not}\circ\text{vote}\circ\sigma_1(\boldsymbol{\varepsilon}); \\ \lambda\mathscr{C}.\mathscr{C}(\lambda y.\neg\mathbf{vote}(y)); \\ S_{f,\beta}\upharpoonright(S_{f,\beta}\upharpoonright VP_{b,-}) \end{array} \quad \begin{array}{c} \lambda\sigma_0.\sigma_0(\text{can}); \\ \lambda\mathscr{F}.\Diamond\mathscr{F}(\text{id}_{et}); \\ S_{f,+}\upharpoonright(S_{b,-}\upharpoonright(VP_{b,\alpha}/VP_{b,\alpha})) \end{array} \quad
\begin{array}{c}
\varphi_3\circ\varphi_1; f(P); VP_{b,-} \\
\hline
\begin{array}{c} \lambda\varphi_3.\text{john}\circ\varphi_3\circ\varphi_1; \\ \lambda P.f(P)(\mathbf{j}); S_{f,+} \end{array} \\
\hline
\begin{array}{c} \lambda\varphi_3.\text{john}\circ\varphi_3\circ\varphi_1; \\ \lambda P.f(P)(\mathbf{j}); S_{f,+}\upharpoonright(VP_{b,-}/VP_{b,-}) \end{array}
\end{array} \upharpoonright I^3 \\
\hline
\begin{array}{c}
\text{john}\circ\text{can}\circ\varphi_1; \Diamond P(\mathbf{j}); S_{f,+} \\
\hline
\lambda\varphi_1.\text{john}\circ\text{can}\circ\varphi_1; \lambda P.\Diamond P(\mathbf{j}); S_{f,+}\upharpoonright VP_{b,-}
\end{array} \upharpoonright I^1 \\
\hline
\text{not}\circ\text{vote}\circ\text{john}\circ\text{can}\circ\boldsymbol{\varepsilon}; \Diamond\neg\mathbf{vote}(\mathbf{j}); S_{f,+}
\end{array}
$$

The slanted version of *not* combines freely with its VP argument to yield a topicalized VP_−, but the type of the mother—in particular, its polarity specification—is determined by the highest scoping operator, *can*, which yields a positive polarity clause.

Cases of VP fronting involving quantified subjects are also straightforward in the present approach. For example, (474) involves VP fronting, but the scopal relation between the subject quantifier and the negated modal is the same as in examples that don't involve VP fronting. Thus, on one reading, it has the $\neg > \Diamond > \forall$ scopal relation (consider, for example, a [typical] situation of a physics department at which some of the professional physicists are employed as technical staff who don't have rights to vote at the department chair election).

(474) But vote, EVERY physicist could not ___.

We illustrate below how this reading is derived in the present approach.

Given what has been said so far, it will be clear that if the negative operator scopes highest in deriving the continuation clause, that clause will be S_−, in compliance with the conditions on the topicalization operator. A proof along these lines will thus take the following form:

(475)
$$
\begin{array}{c}
\begin{array}{c}
[\varphi_2; f; VP_{b,\varnothing}/VP_{b,\varnothing}]^2 \quad [\varphi_1; P; VP_{b,\varnothing}]^1 \\
\hline
\varphi_2\circ\varphi_1; f(P); VP_{b,\varnothing}
\end{array} \quad [\varphi_4; g; VP_{f,\varnothing}/VP_{b,\varnothing}]^4 \\
\hline
\begin{array}{c}
\varphi_4\circ\varphi_2\circ\varphi_1; g(f(P)); VP_{f,\varnothing} \qquad [\varphi_3; y; NP]^3 \\
\hline
\varphi_3\circ\varphi_4\circ\varphi_2\circ\varphi_1; f(P)(y); S_{f,\varnothing}
\end{array}
\end{array}
$$

$$\vdots$$

$$\frac{\varphi_3 \circ \varphi_4 \circ \varphi_2 \circ \varphi_1; \; f(P)(y); S_{f,\varnothing}}{\lambda\varphi_3.\varphi_3 \circ \varphi_4 \circ \varphi_2 \circ \varphi_1; \; \lambda y.f(P)(y); \; S_{f,\varnothing}\!\restriction\!NP} {\upharpoonright}I^3 \qquad \begin{array}{l}\lambda\sigma_1. \\ \quad \sigma_1(\text{every} \circ \\ \quad \text{physicist}); \\ \mathbf{V_{phys}}; \\ S_{f,\varnothing}\!\restriction\!(S_{f,\varnothing}\!\restriction\!NP)\end{array}$$

$$\frac{\text{every} \circ \text{physicist} \circ \varphi_4 \circ \varphi_2 \circ \varphi_1; \; \mathbf{V_{phys}}(\lambda y.f(P)(y)); S_{f,\varnothing}}{\begin{array}{l}\lambda\varphi_4.\text{every} \circ \text{physicist} \circ \\ \quad \varphi_4 \circ \varphi_2 \circ \varphi_1; \\ \lambda f.\mathbf{V_{phys}}(\lambda y.f(P)(y)); \\ S_{f,\varnothing}\!\restriction\!(VP_{f,\varnothing}/VP_{b,\varnothing})\end{array}} {\upharpoonright}I^4 \qquad \begin{array}{l}\lambda\sigma_2.\sigma_2(\text{could}); \\ \lambda\mathscr{F}.\Diamond\mathscr{F}(\mathrm{id}_{et}); \\ S_{f,\varnothing}\!\restriction\!(S_{f,\varnothing}\!\restriction\!(VP_{f,\varnothing}/VP_{b,\varnothing}))\end{array}$$

$$\frac{\text{every} \circ \text{physicist} \circ \text{could} \circ \varphi_2 \circ \varphi_1; \; \Diamond\mathbf{V_{phys}}(\lambda y.g(P)(y)); S_{f,\varnothing}}{\begin{array}{l}\lambda\varphi_2.\text{every} \circ \text{physicist} \circ \text{could} \circ \varphi_2 \circ \varphi_1; \\ \lambda g.\Diamond\mathbf{V_{phys}}(\lambda y.g(P)(y)); S_{f,\varnothing}\!\restriction\!(VP_{b,\varnothing}/VP_{b,\varnothing})\end{array}} {\upharpoonright}I^2 \qquad \begin{array}{l}\lambda\sigma_3.\sigma_3(\text{not}); \\ \lambda\mathscr{G}.\neg\mathscr{G}(\mathrm{id}_{et}); \\ S_{\gamma,-}\!\restriction\!(S_{\gamma,\varnothing}\!\restriction\!(VP_{b,\varnothing}/VP_{b,\varnothing}))\end{array}$$

$$\frac{\text{every} \circ \text{physicist} \circ \text{could} \circ \text{not} \circ \varphi_1; \; \neg\Diamond\mathbf{V_{phys}}(\lambda y.g(P)(y)); S_{f,-}}{\lambda\varphi_1.\text{every} \circ \text{physicist} \circ \text{could} \circ \text{not} \circ \varphi_1; \; \lambda P.\neg\Diamond\mathbf{V_{phys}}(\lambda y.P(y)); S_{f,-}\!\restriction\!VP_{b,\varnothing}} {\upharpoonright}I^1$$

$$\frac{\begin{array}{l}\vdots \\ \lambda\varphi_1.\text{every} \circ \text{physicist} \circ \text{could} \circ \text{not} \circ \varphi_1; \\ \lambda P.\neg\Diamond\mathbf{V_{phys}}(\lambda y.P(y)); S_{f,-}\!\restriction\!VP_{b,\varnothing}\end{array} \quad \dfrac{\begin{array}{l}\lambda\varphi_2\lambda\sigma_1.\varphi_2 \circ \sigma_1(\boldsymbol{\varepsilon}); \\ \lambda\alpha\lambda P.P(\alpha); (S_{f,\beta}\!\restriction\!(S_{f,\beta}\!\restriction\!X))\!\restriction\!X\end{array} \quad \begin{array}{l}\text{vote;} \\ \mathbf{vote}; VP_{b,\varnothing}\end{array}}{\lambda\sigma_1.\text{vote} \circ \sigma_1(\boldsymbol{\varepsilon}); \; \lambda P.P(\mathbf{vote}); S_{f,\beta}\!\restriction\!(S_{f,\beta}\!\restriction\!VP_{b,\varnothing})}}{\text{vote} \circ \text{every} \circ \text{physicist} \circ \text{could} \circ \text{not} \circ \boldsymbol{\varepsilon}; \; \neg\Diamond\mathbf{V_{phys}}(\lambda y.\mathbf{vote}(y)); S_{f,-}}$$

The interaction of negation with ellipsis, illustrated in (477), is straightforward and, in contrast with the Kim and Sag (2002) account discussed above, predicts (correctly) the possibility of ellipsis following narrow-scoping negation, as shown in (476), whose importance we discuss in more detail in section 9.4:

(476) I know everyone's putting pressure on you to vote in this election, but you could always NOT.

(477)
$$\frac{\dfrac{\left[\begin{array}{l}\varphi_2; \\ f; VP_{b,\varnothing}/VP_{b,\varnothing}\end{array}\right]^2 \; \left[\begin{array}{l}\varphi_1; \\ P; VP_{b,\varnothing}\end{array}\right]^1}{\varphi_2 \circ \varphi_1; \; f(P); \; VP_{b,\varnothing}} \quad \left[\begin{array}{l}\varphi_3; \\ g; \\ VP_{b,\varnothing}/VP_{b,\varnothing}\end{array}\right]^3}{\dfrac{\varphi_3 \circ \varphi_2 \circ \varphi_1; \; g(f(P)); \; VP_{b,\varnothing}}{\varphi_3 \circ \varphi_2; \; \lambda P.g(f(P)); \; VP_{b,\varnothing}/VP_{b,\varnothing}} /I^1 \quad \left[\begin{array}{l}\sigma_0; \\ \mathscr{C}; \\ S_{b,\varnothing}\!\restriction\!(VP_{b,\varnothing}/VP_{b,\varnothing})\end{array}\right]^0}}{\dfrac{\sigma_0(\varphi_3 \circ \varphi_2); \; \mathscr{C}(\lambda P.g(f(P))); \; S_{b,\varnothing}}{\lambda\varphi_3.\sigma_0(\varphi_3 \circ \varphi_2); \; \lambda g.\mathscr{C}(\lambda P.g(f(P))); \; S_{b,\varnothing}\!\restriction\!(VP_{b,\varnothing}/VP_{b,\varnothing})} {\upharpoonright}I^3}$$

$$\vdots$$

$$\frac{\begin{array}{ll}\lambda\varphi_3.\sigma_0(\varphi_3\circ\varphi_2); & \lambda\sigma_2.\sigma_2(\text{not}); \\ \lambda g.\mathscr{C}(\lambda P.g(f(P))); & \lambda\mathscr{G}.\neg\mathscr{G}(\text{id}_{et}); \\ S_{b,\varnothing}\!\upharpoonright\!(VP_{b,\varnothing}/VP_{b,\varnothing}) & S_{\gamma,-}\!\upharpoonright\!(S_{\gamma,\varnothing}\!\upharpoonright\!(VP_{b,\varnothing}/VP_{b,\varnothing}))\end{array}}{\sigma_0(\varphi_3\circ\text{not}); \neg\mathscr{C}(\lambda P.(f(P))); S_{b,-}}$$

$$\frac{\lambda\varphi_2.\sigma_0(\varphi_3\circ\text{not}); \lambda f.\neg\mathscr{C}(\lambda P.(f(P))); S_{b,-}\!\upharpoonright\!(VP_{b,\varnothing}/VP_{b,\varnothing})\,|\!\Gamma^2 \quad\begin{array}{l}\lambda\sigma_1.\sigma_1(\text{could}); \\ \lambda\mathscr{F}.\Diamond\mathscr{F}(\text{id}_{et}); \\ S_{f,+}\!\upharpoonright\!(S_{b,-}\!\upharpoonright\!(VP_{f,\alpha}/VP_{b,\alpha}))\end{array}}{\sigma_0(\text{could}\circ\text{not}); \Diamond\neg\mathscr{C}(\lambda P.P); S_{f,+}}$$

$$\frac{}{\lambda\sigma_0.\sigma_0(\text{could}\circ\text{not}); \lambda\mathscr{C}.\Diamond\neg\mathscr{C}(\text{id}_{et}); S_{f,+}\!\upharpoonright\!(S_{b,-}\!\upharpoonright\!(VP_{b,\varnothing}/VP_{b,\varnothing}))}\,|\!\Gamma^0$$

The syntactic type, prosody, and semantics provable for *could not* is thus exactly parallel in all respects to the basic properties of the lexical entries for the higher-order modals and therefore will serve as input to the ellipsis operator in exactly the same way, yielding an ellipsis version whose relation to the sign just derived will be identical to that holding between, for example, the VP ellipsis version of higher-order *could* and the lexical entry for this sign, from which the former is derived via application of the ellipsis operator and then the higher-order Geachified modal, as in (468).

9.3.5 Inversion and Higher-Order Modals

We take inversion to correspond to an alternative ordering of prosodic elements associated with a polar interrogative interpretation, the latter the effect of applying the operator "**?**" to the proposition denoted by the uninverted version.[6] To take a simple example, the correlation of this linear ordering with the specific semantics of a polar interrogative can be captured for the higher-order modal *should* as in (478):

(478) $\lambda\sigma_1.\text{should}\circ\sigma_1(\boldsymbol{\varepsilon}); \lambda\mathscr{F}.\textbf{?}\Box\mathscr{F}(\text{id}_{et}); S_{f,+}\!\upharpoonright\!(S_{f,\beta}\!\upharpoonright\!(VP_{f,\alpha}/VP_{b,\alpha}))$

This sign makes it possible to license (479) on the wide-scope interpretation of the modal, as in (480).

(479) Should every physicist use LATEX?

(480)
$$\vdots$$

$$\frac{\begin{array}{ll}\lambda\varphi_1.\varphi_1\circ\varphi_2\circ\text{use}\circ\text{latex}; & \lambda\sigma_2.\sigma_2(\text{every}\circ\text{physicist}); \\ \lambda y.f(\textbf{use-ltx})(y); S_{f,\beta}\!\upharpoonright\!NP & \mathbf{V_{phys}}; S_{f,\beta}\!\upharpoonright\!(S_{f,\beta}\!\upharpoonright\!NP)\end{array}}{\begin{array}{l}\text{every}\circ\text{physicist}\circ\varphi_2\circ\text{use}\circ\text{latex}; \\ \mathbf{V_{phys}}(\lambda y.f(\textbf{use-ltx})(y)); S_{f,\beta}\end{array}}$$

with left sign:
$$\begin{array}{l}\lambda\sigma_1.\text{should}\circ\sigma_1(\boldsymbol{\varepsilon}); \\ \lambda\mathscr{F}.\textbf{?}\Box\mathscr{F}(\text{id}_{et}); \\ S_{f,+}\!\upharpoonright\!(S_{f,\beta}\!\upharpoonright\!(VP_{f,\alpha}/VP_{b,\alpha}))\end{array}$$

$$\frac{\lambda\varphi_2.\text{every}\circ\text{physicist}\circ\varphi_2\circ\text{use}\circ\text{latex}; \lambda f.\mathbf{V_{phys}}(\lambda y.f(\textbf{use-ltx})(y)); S_{f,\beta}\!\upharpoonright\!(VP_{f,\alpha}/VP_{b,\alpha})}{\text{should}\circ\text{every}\circ\text{physicist}\circ\boldsymbol{\varepsilon}\circ\text{use}\circ\text{latex}; \textbf{?}\Box\mathbf{V_{phys}}(\lambda y.\textbf{use-ltx}(y)); S_{f,+}}$$

6. While we remain agnostic on the semantics of interrogatives, if we take $\textbf{?} = \lambda p.\{p,\neg p\}$, then the interpretation will correspond to the one advocated in Karttunen (1977).

We note that here, as with the other operators we have introduced for higher-order modals, the higer-order entry in (478) can be slanted down to a sign with syntactic type (S/VP)/NP:

(481)

$$\cfrac{\cfrac{\cfrac{\cfrac{[\varphi_1;g;\mathrm{VP}_{f,\alpha}/\mathrm{VP}_{b,\alpha}]^1 \quad [\varphi_2;P;\mathrm{VP}_{b,\alpha}]^2}{\varphi_1 \circ \varphi_2;\, g(P);\, \mathrm{VP}_{f,\alpha} \qquad [\varphi_3;z;\mathrm{NP}]^3}}{\varphi_3 \circ \varphi_1 \circ \varphi_2;\, g(P);\, \mathrm{S}_{f,\alpha}}}{\lambda\varphi_1.\varphi_3 \circ \varphi_1 \circ \varphi_2;\, g(P)(y);\, \mathrm{S}_{f,\alpha}{\upharpoonright}(\mathrm{VP}_{f,\alpha}/\mathrm{VP}_{b,\alpha})}{\upharpoonright}\mathrm{I}^1 \quad \begin{array}{l}\lambda\sigma_1.\mathrm{should}\circ\sigma_1(\boldsymbol{\varepsilon});\\[2pt]\lambda\mathscr{F}.?\Box\mathscr{F}(\mathrm{id}_{et});\\[2pt]\mathrm{S}_{f,+}{\upharpoonright}(\mathrm{S}_{f,\beta}{\upharpoonright}(\mathrm{VP}_{f,\alpha}/\mathrm{VP}_{b,\alpha}))\end{array}}{\mathrm{should}\circ\varphi_3\circ\boldsymbol{\varepsilon}\circ\varphi_2;\, ?\Box P(y);\, \mathrm{S}_{f,+}}$$

$$\cfrac{\cfrac{\mathrm{should}\circ\varphi_3\circ\boldsymbol{\varepsilon}\circ\varphi_2;\, ?\Box P(y);\, \mathrm{S}_{f,+}}{\mathrm{should}\circ\varphi_3;\, \lambda P.?\Box P(y);\, \mathrm{S}_{f,+}/\mathrm{VP}_{b,\alpha}}/\mathrm{I}^2}{\mathrm{should};\, \lambda y\lambda P.?\Box P(y);\, (\mathrm{S}_{f,+}/\mathrm{VP}_{b,\alpha})/\mathrm{NP}}/\mathrm{I}^3$$

This lower-order interrogative operator will first combine with an NP to its right and subsequently with a VP to give correct readings for examples such as *Should John use LaTeX?*[7]

9.4 Comparison with a Phrase Structure–Theoretic Analysis

An alternative approach to the modal/negation scope interaction data is proposed by Kim and Sag (2002) in HPSG, whose key proposal, as already noted, is a distinction between *not* as an auxiliary complement and *not* as a VP adjunct:

(482)   a.   [VP modal [VP *not* [VP . . .]]]
        b.   [VP modal *not* [VP . . .]]

Kim and Sag essentially argue that *not* supports both ellipsis and extraction, motivating its analysis as a syntactic argument, and observe further that in both kinds of constructions negation scopes widely over the entire clause. On the other hand, negation can also scope narrowly, giving rise to the ambiguity of, for example, *You could not vote*, with interpretations available under both $\neg > \Diamond$ and $\Diamond > \neg$ scopings. Since only the first of these is available in, for example, *. . . but vote, you can not*, Kim and Sag apply their assumption that while dependents of heads can be extracted, head phrases themselves cannot be, in order to mandate an analysis of extracted VPs in negated contexts

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7. We provide no analysis of negative interrogative sentences such as *Should we not tell John about the new job posting?*, which is ambiguous (though the ambiguity is apparently resolved by stress placement). The interpretation of the semantics for such examples is currently an active research issue; see, e.g., Romero and Han (2004) and Goodhue (2019) for opposing analyses of such data. Given the considerable uncertainty about the semantic action of negation in negative polar questions, we leave this particular grammatical pattern open.

such as (482a), taking the data to establish a correlation among syntactic behavior, scopal possibilities, and configurational representation that can be tidily summarized as in (483):

(483)   a.   VP extractability and ellipsability, wide scope for negation, and complement status for *not* are correlated.
        b.   Failure of VP extraction and ellipsis, narrow scope for negation, and adjunct status for *not* are correlated.

But there is reason to believe that this correlation lacks any kind of generality. To begin with, note that with modals such as *must* and *should*, narrow scope is perfectly compatible with VP extraction and ellipsis possibilities:

(484)   a.   If the party wants you to not vote in this election, then vote in the election you must not.
        b.   I know John wants you to vote in this election, but you really should not.

Data of this sort make it clear that if we assume Kim and Sag's configurational test for extraction and ellipsis possibilities, there is no one-to-one correlation between the "height" of *not* in the clause on the one hand and the scope of negation with respect to the clausal proposition on the other. Narrow negation scope does not necessarily entail a configuration in which *not* is an argument of the modal.

   Nor is it clear that ellipsis is restricted to wide-scope negation contexts. As we have just seen, complement *not* on Kim and Sag's account can sometimes correspond to the same narrow negation that adjunct *not* is supposedly restricted to. But in the case of *can/could*, the distinction is supposed to be clear: adjunct status always corresponds to narrow-scope negation in such cases. The problem is that data such as (485), with only a narrow-scope negation reading available, is accepted by all native speakers of English we have consulted:

(485)   I know everyone's putting pressure on you to vote in this election, but you could always NOT.

The presence of *always* with wide scope over the *but* clause proposition effectively privileges the narrow scope interpretation of the negation here—a major contraindication to Kim and Sag's analysis, because the *could/can* ellipsis facts are the main empirical support for the correlation of configuration with scope in English. In view of (484) and (485), it is difficult to see any hard predictions that support the configurational analysis: if narrow-scope *not* in *can/could* examples is an adverb, cases such as the latter example pretty much force the conclusion that the prohibition of adverbial remnants in VP ellipsis is a false generalization, at least as far as *not* is concerned—which then leaves no clear distributional distinction allowing us to distinguish the alleged two configurations that Kim and Sag argue are both needed. Both adverbial and complement

not support ellipsis, and complement *not* can scope low as well as high. The same conclusion of course follows more directly if *not* is not an adverb but a complement everywhere.

It does not appear, then, that there is any robust factual basis for assuming anything other than a single combinatorics for *not*, whose narrow-scope interpretation is derivable as a theorem from its single higher-order entry, interacting with the polarity properties of the associated modal. Accordingly, the analysis we provide directly posits only a single tecto type for *not*, with alternative scoping possibilities corresponding to two different versions of the negation operator—the higher-order lexical entry and the lower-order sign derivable from it by the Hybrid TLCG proof theory.

9.5   Conclusion

It will be useful at this point to take stock of our results. Starting with the empirical motivation for higher-order modals, we have shown that the only lexical entries we need for the modals are the higher-order ones and that the analysis of VP ellipsis and pseudogapping from chapter 6 does not need to be modified in any way to license VP ellipsis and pseudogapping in examples involving higher-order, wide-scoping modals. Furthermore, a higher-order version of the negation operator *not*, of the same type (modulo the polarity features) as the higher-order modals, accounts for the idiosyncratic behavior of the modals vis-à-vis the scope of negation via the polarity-marking mechanism incorporating the analytic ideas of Iatridou and Zeijlstra (2013).

Readers familiar with the phrase structure grammar tradition may find the higher-order entry for the modal auxiliaries we have posited somewhat bewildering and unnecessarily abstract. For such readers, we would like to point out that, though seemingly abstract and complex, our proposal has at least the following three properties which we take to be highly desirable. First, it extends more straightforwardly to the puzzling (apparent) scope anomaly data in Gapping and Stripping. Second, our approach is parsimonious in that it requires only the most general form of the entries for modals and negation to be posited in the lexicon to account for their various complex scopal properties. Third, our approach is in fact closely related to the analyses of auxiliary verbs in the lexicalist tradition in that alternative signs for these auxiliary verbs that correspond to the lexically specified entries in lexicalist theories fall out as *theorems* from the higher-order ones in our setup. We take our approach to belong to the same broad tradition as other lexicalist theories, but if our analysis of auxiliaries (and other phenomena we have presented in this book) still appear somewhat alien, that may ultimately reflect different views about the appropriate level of abstraction from the empirically observable data that a theory of grammar should embody. Whatever one takes to be the right answer to this question, we believe we have at least offered an interesting enough alternative to be worth pursuing seriously.