Hints for Selected Exercises

Chapter 1

Section 1.1

2. Before worrying about the radius, try finding the circumference of the circle.

8. The answer depends on whether or not the ANGLE is divisible by 8. You should be able to guess the answer through experimentation, but you probably won't be able to give a proof until you've looked ahead to subsection 1.4.1.

9. In general, of two regular polygons with the same circumference, the one with more sides has a greater area. (This is not simple to prove. A neat proof without calculus was given by Galileo in his *Dialogues on the Two New Sciences*.) Or do a direct calculation.

11. One interesting effect is that programs always close when they would have closed if run on an infinite plane. Can you prove this?

12. Describe the geometric transformations as reflections in certain lines through the turtle's initial position.

13. One result is that eventually, at a small enough scale, all you ever see are straight lines.

Section 1.2

2. The first three vertices the turtle draws determine a circle. Now use congruent triangles to show that the rest of the vertices also lie on this circle.

6. How does the sum of the interior angles relate to the sum of the exterior angles?

7. Consider the closed turtle path formed by the arc and the line. What is the turtle's total turning while traversing this path?

9. Use the result of exercise 8 to relate the other two arcs of the circle to the two angles formed by the arcs and their respective chords. Now use the relation between A and these two angles.

10. Relate the angle turned by the turtle to the angle between the arc and the chord formed by a side of the POLY. Now use exercise 8.

11. Use the result of exercise 10 (although, since the method of proof suggested in exercise 10 is to use the simple-closed-path theorem, you'll have to find another proof in order to avoid a circular argument).

12. Compare with exercise 3 of section 1.1.

Section 1.3

5. Generalize the analysis for $A = 2$, INCREMENT $= 20 = 360/18$ given in subsection 1.3.2. To give a formula, use the fact that the sum of the first $n - 1$ integers is equal to $n(n-1)/2$.

6. In keeping track of the total turning, divide the basic loop into two parts: one where the angle increases and one where the angle decreases. Then add the corresponding angles in pairs. Alternatively, use the following formula, which generalizes the one given in the hint above: If S and T are integers with $T > S$, then the sum of the integers from S through T (inclusive) is equal to $(S+T)(T-S+1)/2$.

8. This can be done with vectors. See section 3.1, exercise 3. We'll leave it to you to find another method of proof.

Section 1.4

2. Use the following fact: If we can draw an n-pointed POLY using an integer angle, then we can also find some integer angle that draws an n-pointed POLY with rotation number 1.

3. The $180 - A$ answer depends on the value of A modulo 4. You should be able to guess the answer by experimenting. To prove it is harder. Try using the symmetry formula $n = \text{LCM}(A, 360)/A$ and comparing this with $\text{LCM}(180 - A)/(180 - A)$. The analysis may be easier if you take advantage of the fact (see exercise 16) that for any integers x and y, $\text{LCM}(x, y) \times \text{GCD}(x, y) = xy$. This reduces the problem to finding the relation between $\text{GCD}(A, 360)$ and $\text{GCD}(180 - A, 360)$.

7. What is the product modulo p of R, $2R, \ldots, (p-1)R$?

11. To show that d can be represented as $\langle\text{integer}\rangle R + \langle\text{integer}\rangle n$, use the fact that R/d and n/d are relatively prime and hence that 1 can be represented as $\langle\text{integer}\rangle R/d + \langle\text{integer}\rangle n/d$. Also, show that d must divide any integer which can be represented as $\langle\text{integer}\rangle R + \langle\text{integer}\rangle n$, which implies both that d must be the smallest such integer and that the integers which can be represented this way are precisely the multiples of

d. Reducing the the numbers of the form $\langle\text{integer}\rangle R + \langle\text{integer}\rangle n$ modulo n gives the multiples of R modulo n.

14. If each of two variables is expressible in the form $aR + bn$, the difference is also expressible in that form. Show that this means the variables in the Euclid process are always of this form, including the final step when the two are both equal to d.

18. Apply the same "reduction rule" EUCLID uses to find the GCD: Imagine that we are in the case where $R < n$, so that we are about to reduce the problem of finding $\text{GCD}(n, R)$ to that of finding $\text{GCD}(n - R, R)$. Then suppose we already know how to find p and q such that $p(n - R) + qR = d$. Then, for that pair p and q, we have $pn + (q - p)R = d$. In other words, if $R < n$ and we know $\text{DIO}(n - R, R) = (p, q, d)$, then we should return $\text{DIO}(n, R) = (p, q - p, d)$. This leads to the computer procedure

```
TO DIO N R
   IF N > R THEN
      [P Q D] ← DIO (N-R, R)
      RETURN [P Q-P D]
```

Fill in the rest of the DIO procedure to handle the cases $N < R$ and $N = R$. (The statement "[P Q D] ← DIO (N-R, R)" sets the three variables P, Q, and D to the three respective pieces of DIO(N-R, R). This is an example of "structure-directed assignment," which is a useful although uncommon feature for a programing language. See appendix A for more information.)

19. Rather than reducing the problem for (n, R) to $(n, n - R)$, try reducing it to (R, rem), where rem is the remainder when n is divided by R. Suppose that the quotient when n is divided by R is quo and the remainder is rem. If $p \times R + q \times \text{rem} = d$, show how to express the solution (x, y) to $xn + yR = d$ in terms of p,q, and quo. Use this to construct FASTDIO as a recursive procedure.

23. If you try some examples, you should notice that $\text{GCD}(F(a), F(b))$ is itself a Fibonacci number. Now, which one is it?

24. Try examples and you will find that x and y are themselves Fibonacci numbers. Which ones are they?

Chapter 2

Section 2.1

2. One of the important results of statistics is that the average value for the square of the distance from home after s steps in a random walk is proportional to s (for s large). This means, roughly, that distance from home increases as $\sqrt{s}$.

9. If the turtle does draw a closed path, compute the total turning over the path and apply the simple closed-path theorem.

Section 2.2

9. Look at the situation from the point of view of the chaser. At each step it moves CHASE.SPEED closer to the evader. But also at each step, the evader moves "outward" from the chaser. Calculate the amount of this outward motion. Suppose that EVADE.SPEED is greater than CHASE.SPEED. Show that if the evader is far from the chaser the outward motion is less than the chaser motion, so the two creatures move closer together; and when the chaser is close to the evader the outward motion is greater than the chaser motion, so the two creatures move further apart. At what point do the two motions balance?

11. You can use analytic geometry and (x, y) coordinates. But once you become familiar with the vector methods in the chapter 3, you should try the following vector approach for finding the point on a given line that is closest to a given point: If the line is represented as $\mathbf{s} + \lambda\mathbf{v}$ (that is, the line parallel to a vector $\mathbf{v}$ and passing through the point $\mathbf{s}$) and the point is $\mathbf{p}$, then the point on the line closest to $\mathbf{p}$ is the point $\mathbf{r} = \mathbf{s} + \lambda_0\mathbf{v}$ for which the line from $\mathbf{r}$ to $\mathbf{p}$ is perpendicular to $\mathbf{v}$—that is, the point for which

$$(\mathbf{p} - \mathbf{s} - \lambda_0\mathbf{v}) \cdot \mathbf{v} = 0.$$

Solving this for λ_0 gives

$$\lambda_0 = \frac{(\mathbf{p} - \mathbf{s}) \cdot \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}}.$$

Hence, the required point is $\mathbf{s} + \lambda\mathbf{v}$, where λ has this value.

12. Suppose that the defender has speed k times that of the attacker, and starts out less than k times as far from the target. (Otherwise, the attacker could actually reach the target.) Consider this description of

the optimal point to head for when $k = 1$—it is the closest point to the target, which is equally far from both players.

Section 2.3

4. Let the length of the branch drawn at the highest level be L. What is the sum of the lengths of the two branches drawn at the next level? What is the sum of the lengths of the four branches drawn at the next level (and so on)?

6. To find the total length, consider that each level has three times as many branches as the preceding one. Use the formula for the sum of a geometric series:

$$x^n + x^{n-1} + \ldots + x^2 + x + 1 = \frac{x^{n+1} - 1}{x - 1}.$$

9. Consider only complete spirals, which result from running EQSPI infinitely forward and backward in time. Then there can be no such thing as a size parameter, because just rotating the spiral does the same thing as increasing or decreasing size.

10. Show that each pass through the loop draws a scale model of the previous pass. Now focus on the net state change achieved in each pass in terms of that of the first pass.

Section 2.4

1. Draw half the side of the largest outer POLY, turn and do the process recursively with a smaller side, and complete the outer POLY without interruption. The trick is to find the factor that gives the reduction in size from one POLY to the next. You will have to use trigonometry.

2. For the length: What is the ratio of the length for level n to the length for level $n+1$? For the area: Decompose into a sum of equilateral triangles. What is the ratio of the total area of the level-n triangles to the area of one of the level-$(n+1)$ triangles?

4. The Hilbert program at level n contains three FORWARD instructions plus four recursive calls. If $f(n)$ is the length of the level-n curve, write an equation for $f(n)$ in terms of $f(n-1)$.

8. If you remove the four outer diagonal lines (darkened in the figure), the curve breaks into four pieces. Try giving a recursive description of each of the pieces.

Chapter 3

Section 3.1

3. Let S be the length of the first side. From each side of the POLYSPI, subtract a vector of length S parallel to the side. What is the sum of the pieces subtracted? What is left? (If the figure closes, then the turtle ends up in the initial heading, so the number of steps taken to close is a multiple of the number of steps in a POLY with the same angle.)

5. In terms of the spirograph analysis, suppose one arm moves in increments of ANGLE1 and the other in increments of ANGLE2. When do the two arms line up?

13. There is a different sense of "approximately symmetric," which becomes better as the difference between the displacements VECTOR(A1,S1) followed by VECTOR(A2,S2) as opposed to VECTOR(A2,S2) followed by VECTOR(A1,S1) becomes visually negligible.

18. Consider the distribution of numbers in the slot process.

19. Look at the assignment of the numbers in the slot process. Show that if the number 1 gets assigned to an odd-numbered slot then all odd numbers get assigned to odd-numbered slots and all even numbers get assigned to even-numbered slots; and conversely if 1 is assigned to an even-numbered slot. Now consider the sums of the numbers in the odd and even slots.

25. Show that there exist integers m and n with $2^m = 2^n \pmod q$. Then show that we must also have $2^{m-1} = 2^{n-1} \pmod q$ and so on.

26. Use the formula

$$1 + x + x^2 + \ldots + x^{r-1} = \frac{x^r - 1}{x - 1},$$

which gives the sum of a geometric series.

27. Rotate the sum by A and apply the vector form of the POLY closing theorem, or else rotate by $3A$ and look at the result.

28. Use Fermat's Little Theorem (see exercises for section 1.4) and the fact stated in exercise 17.

Section 3.2

5. Try to solve for a and b. What is the necessary condition for writing down a solution?

7. As a vector equation this condition is

$$\text{Perp}(a\mathbf{v}_1 + b\mathbf{v}_2) = -b\mathbf{v}_1 + a\mathbf{v}_2,$$

which must hold for any a and b.

8. Use the net displacement formula for DUOPOLY, neglecting the constant center vector.

Section 3.3

2. The points are given by the vector equations

$$\text{NOSE} = \mathbf{P} + \text{TURTLE.HEIGHT} \times \mathbf{H},$$

$$\text{LEFT.LEG} = \mathbf{P} + \frac{1}{2}\text{TURTLE.WIDTH} \times \text{Perp}(\mathbf{H}),$$

$$\text{RIGHT.LEG} = \mathbf{P} - \frac{1}{2}\text{TURTLE.WIDTH} \times \text{Perp}(\mathbf{H}).$$

Section 3.5

2. Use dot product.

3. You could work this out in coordinates, but a neater way is to make use of the definition of dot product in terms of projection. Use the facts that the projection of $\mathbf{v}$ onto $\mathbf{w}$ is 0 if $\mathbf{v}$ and $\mathbf{w}$ are perpendicular, and that the projection of $\mathbf{v}$ onto $\mathbf{v}$ is $|\mathbf{v}|$.

4. Relate $|\mathbf{v}|$, $|\mathbf{w}|$, $\mathbf{v} \cdot \mathbf{w}$ and A to $\text{Proj}(\mathbf{v}, \mathbf{w})$.

5. If the vectors form the sides of a triangle, then $\mathbf{t} + \mathbf{v} + \mathbf{w} = 0$. Notice that the angle opposite $\mathbf{t}$ is 180 minus the angle referred to in the preceding exercise.

6. You should discover that these three-dimensional looping programs close only in the degenerate case when the figure lies in a plane. Nonplanar figures lie along a helix, just as POLYs lie along a circle in two dimensions.

7. We gave two proofs of the POLY closing theorem in subsection 1.2.2. The first proof, that the vertices of POLY lie on a circle, clearly uses the fact that the motion is in a plane. But how about the second proof? That one, remember, showed that assuming that the path doesn't close leads to a contradiction. What is this contradiction, exactly? How does it rely on the fact that the motion is in a plane?

9. ROLL until **ex** is in the plane of **ez**, **p**. Now rotate in the **ex**, **ez** plane (YAW) until **ez** is pointing toward **p**. Finally, ROLL opposite to undo your first ROLL and leave the angle between **ey** and the **ez**, **p** plane the same as when you started. The sine and cosine needed for the first ROLL are $\mathbf{p}\cdot\mathbf{ey}/s$ and $\mathbf{p}\cdot\mathbf{ex}/s$, where s is the square root of $(\mathbf{p}\cdot\mathbf{ex})^2+(\mathbf{p}\cdot\mathbf{ey})^2$. The cosines and sines for the second rotation are then $\mathbf{ez}\cdot\mathbf{p}/|\mathbf{p}|$ and $\mathbf{ex}\cdot\mathbf{p}/|\mathbf{p}|$.

13. A four-dimensional cube is a three-dimensional cube at $t=0$, consisting of the points $(0,0,0,0)$, $(0,0,1,0)$, $(0,1,0,0)$, $(0,1,1,0)$, $(1,0,0,0)$, $(1,0,1,0)$, $(1,1,0,0)$, $(1,1,1,0)$ connected in the appropriate way, plus one at $t=1$ with corresponding vertices connected. A turtle procedure to draw this can be generalized from drawing a cube with the following procedure: Draw a square (FORWARD 1, LEFT 90 done four times), FORWARD 1, PITCH 90, and repeat this whole sequence four times. As far as rotating in four dimensions, you can check that rotating x, y coordinates and leaving z and t alone is a rotation; that is, it leaves all distances the same. In general, there will be one plane where the action is (like x, y here) and an invariant plane (the analog to a fixed axis). That means there are six fundamental rotations.

Chapter 4

Section 4.1

2. The program is equivalent to a repeating fixed instruction sequence that will complete one basic loop when LOCAL.PHASE returns to 0 modulo 360. Thus, it will close when both LOCAL.PHASE and the total turning are 0 modulo 360.

14. Consider some of the techniques used in the interpolation program of exercises 6–13. When exactly is a scissors a nondeformation? What exactly are the conditions under which a SHRINKSEG (exercise 5) causes a change in topological type? Are there other kinds of kinking or unkinking? Look at the criteria for pruned programs (exercise 1 of section 1.2), and make sure you rule out changes that unprune in such a way as to change total turning.

17. To find the extra piece of information, consider the case of no crossing points, then one crossing point.

Section 4.3

1. Imagine the turtle walking along the curve. Is the turtle's left foreleg or right foreleg the one that lies in the inside region?

3. Consider figure 4.16 together with the phenomenon illustrated in figure 4.11b.

5. Consider the set of minimum distances between nonadjoining segments.

Section 4.4

1. Use the ideas in subsection 4.4.3.

2. Spirals may or may not have infinite length or infinite total turning. The `EQSPI` spiral of section 2.3 has infinite total turning but finite length (from any point inward). If a turtle can travel at a constant speed and can turn as fast as necessary to follow any curve, an `EQSPI` maze does not spell doom unless it extends infinitely outward as well as inward.

Chapter 5

Section 5.1

1. How does the question of which side of the curve the turtle considers the "inside" relate to the question of which turning the turtle considers to be positive and which turning it considers to be negative? Compare exercise 1 of section 4.3.

4. Focus on the process of walking. If the turtle's right and left legs are doing the same thing, that adds an additional symmetry constraint beyond each leg independently marching off equal-distance steps.

5. The path can depend on size of the turtle, but not if you use smaller and smaller turtles until the turtle is much smaller than any features of the surface. This is the essence of calculus, and it requires surfaces that do not have arbitrarily small features ("smooth" surfaces).

6. Suppose we decide to compute the excess around a closed path as the net change in pointer heading. Then a trip around the sphere's equator and a trip around a circle in the plane both seem to cause no net change in pointer heading. But look more carefully—what is the difference between what a pointer does on the equator as compared to the planar circle where Excess $= 0$?

8. Consider the answer for exercise 7, where a turtle has one side in the middle of one lane and its other side in the middle of the adjacent lane.

Section 5.2

1. What is the region whose curvature the excess of the path is measuring?

4. Given two subdivisions of the surface, find an even finer subdivision that includes them both.

7. Find a subdivision of a football into polygons for which you know the excesses.

11. A small model of a cone is part of a larger one.

13. For global π look at the "real" radius of the circle. (The "real" center of the circle is not in the surface.) Answers to many of these question will be found in chapters 6 and 7.

15. Fill in the holes. Now how much curvature does the surface have? Cut them out. How much curvature did you remove?

16. Think about the finished umbrella. You can tell its total curvature from a turtle path around the edge. If you start cutting away ribbon after ribbon from the edge, does the total curvature ever increase, indicating that you cut away a strip of negative curvature? Now go back and relate this to things you could measure on the piece.

Section 5.3

2. Cut and paste.

5. This is a simple generalization of the "rectangle" argument for the curvature of the tin can given in the text.

6. The crucial part of the proof is on the turtle path common to both areas. Look back at exercise 5.

10. The result is topologically equivalent to one handle.

12. Choose any point p not on the surface, and let q be the farthest point on the surface from p. What must the surface look like near p? Compare exercise 9 of section 5.2.

13. Since the surface lies in three-dimensional space, it must be equivalent to a sphere with a bunch of handles attached. Now, what can

you say about the total curvature? Could it be zero? Use the previous exercise.

14. Since the surface is closed, exercise 12 implies it must have a point of positive curvature, which can only be a vertex. (No curvature is concentrated along any edge.) But all vertices are the same, so all vertices must have postive curvature. By exercise 13, therefore, the surface must be topologically equivalent to a sphere and hence have total curvature 4π. This total curvature is distributed among v vertices, all containing the same amount of curvature, c. Therefore, $vc = 4\pi$. Each vertex, on the other hand, is made up of a vertex from each of f faces coming together there. If the vertex of the regular polygon has interior angle i, then the curvature at the vertex is given by $c = 2\pi - fi$. (If you don't understand this, go over the section on curvature of cones.) In addition, for a regular polygon with s sides, $i = \pi - 2\pi/s$. Finally, it is clear that f, the number of faces meeting at a vertex, must be at least 3. Now, using all of this, show that i must be less than $2\pi/3$ and therefore s must less than 6; that is, there are no Platonic solids made from polygons with six or more sides. Finally, by analyzing the cases for $s = 3, 4, 5$ show that the only possible Platonic solids are the ones listed. What are v, f, i, and s in each case?

15. The definitions of face, vertex, and edge are at issue.

Chapter 7

10. Use the fact that the length of a chord that subtends an arc of measure θ is $2r \sin \frac{\theta}{2}$, where r is the radius of the circle. Apply this to the chord common to the two arcs that make up the path of the fixed point. Show that the radius of one of these is $\sin r$ and the other is $\sqrt{1 - \sin^2 r \cos^2 \frac{\beta}{2}}$.

11. The proof that a fixed point exists follows the same outline as the proof in the text for the special case of perpendicular axes, except that the great circle going through both poles replaces the Greenwich longitude. The lemma in the text showing that a fixed point implies a rotation completes the proof that any two rotations combine to give a net rotation. For any sequence, the above result can combine the first two into a single rotation, and then that can be combined with the next, and so on.

12. Set up coordinates with $\mathbf{z}$ and $\mathbf{x}$ as before and $\mathbf{y}$ perpendicular to them. Now introduce $\mathbf{t}$ perpendicular to $\mathbf{z}$ but in the plane of arc B.

Show that $\mathbf{t} = (\sin\gamma)\,\mathbf{y} - (\cos\gamma)\,\mathbf{x}$. Finally, rotate $\mathbf{z}$ toward $\mathbf{t}$ to produce $\mathbf{b}$ and take the dot product with $\mathbf{a}$.

13. Use the approximations $\sin x \approx x$ and $\cos x \approx 1 - x^2/2$ and drop very small terms (more than degree 2 in α and β).

17. Check that, for each side of measure α of the original triangle, one will find an angle of α in the polar triangle.

Chapter 8

Section 8.1

2. Use an array (or a list), and store the answer to the ith question in the ith entry of the array. One way to number questions "which face and edge connect to edge e of face f?" is $i = 4f + e$.

8. What is the total curvature of the n-holed torus? If you identify edges in the atlas so that there is only one vertex on the surface, how much curvature will that produce?

15. Think about a turtle carrying a flag around a strip enclosing part of the edge. Better yet, use the recipe for concentrated curvature given in exercise 5 of section 5.3.

16. Qualitative features of this kind of gluing can be seen by viewing it on a special "map" of the surface as follows. Consider the map to be a plane with a line (the glue) drawn on it. Everything is as usual except that the scale of the map changes as you cross the line. As a turtle walks at a constant speed on the surface, it appears to speed up or slow down in crossing the glue line on the map. Now model the turtle as a two-wheeled creature (wheels turning equally fast) and take a look at crossing that line at an angle. One of the turtle's wheels will cross before the other, and its speeding up (or slowing) will cause it to gain (or lose) ground on the other wheel—that is, the turtle will turn. This deflection will not happen if the crossing is made perpendicular to the edge. That dependence on angle of intersection causes it to appear that the curvature concentrated along the edge depends on the measuring process and is not well-defined: A square with edges crossing perpendicular to the line will not have the same total turning as a triangle that crosses the line at the same points.

Section 8.2

2. Remember that curvature can only appear at the vertices.

5. Establish an orientation on each face and check that no contradiction can be found.

8. There are some easy answers: two spheres, two tori, ... But how about surfaces that come in one piece? Think about total curvature.

9. Simply "reversing right and left" is not enough—you can do that by standing on your head. You need an effect that cannot be gotten by any local, continuous operation. How about being able to put your right hand in your left glove? Suppose you took a trip (leaving you gloves at home) that allowed that trick when you got back. Would you be able to read a book you took with you? Would a friend who stayed with your gloves be able to read that book?

Section 8.3

2. Compute using a net which is set up so that ends of the handle match with edges of the net.

3. Given nets N_1 and N_2, show how to produce a "combined" net N_{12} that "contains" both N_1 and N_2 by essentially laying one on top of the other and adding vertices as necessary. Be careful to show that N_{12} can be constructed in all cases to satisfy the requirements of being a net. Show that N_1 and N_2 can each be transformed to N_{12} by elementary operations.

5. Show that such a surface can be "closely approximated" by a piecewise flat surface. To get started, consider how to approximate any simple closed curve in the plane by a polygonal path and look at a continuous deformation from one to the other.

10. Assume that the net does not have any concentrated curvature along its arcs. Then justify the assumption later if you can. You can use the net itself to define pieces of surface and paths around which to compute curvature. The trick is to relate angles turned by the turtle to interior corner angles again, and to note that the sum of corner angles at a vertex is always 2π.

11. What does the "local terrain" look like to a turtle standing at the "bad" vertex? Where does the proof of $K = 2\pi\chi$ for piecewise flat surfaces break down for the double cube?

14. A useful parameter for a regular net is $\chi_{\text{local}} = V - E + F$ *per face*. If S = number of sides to a face and T = number of arcs from a vertex, show that $\chi_{\text{local}} = 1 - S/2 + S/T$. Then try $S = 3, 4, 5, \ldots$ in sequence, checking for solutions to $\chi = F\chi_{\text{local}}$.

15. Use the hint to exercise 14.

16. $\chi_{\text{local}} = 0$ is sufficient to ensure a regular net. This gives $S = 3, 4, 6$ (see exercise 15). These are easy to draw. Try drawing $S = 5$, $T = 3$ or $S = 8$, $T = 3$.

17. Consider tiling an ever larger piece of the plane. As an example take a square consisting of n^2 smaller squares. At any stage we have three types of squares: those on the interior of the tesselation, those on the edge, and those in the corner. We also know that the Euler characteristic of the entire net must be equal to 1. If one takes χ_I, χ_E and χ_C to be the local Euler characteristics for each of these types, then $(n-2)^2\chi_I + 4(n-2)\chi_E + 4\chi_C = 1$. In order for this to hold at large n, the coefficient of n^2 (χ_I) must certainly be zero. This argument generalizes to show $\chi_I = 0$ for any infinite-plane tesselation.

Chapter 9

Section 9.1

2. Only (3) and (4) will be affected. Assume that sin and cos now take degrees as inputs, not radians, and you want to convert to degrees before returning `DEMON.TURNING`.

3. $TT(r) = 2\pi - \theta$; $C(r) = (2\pi - \theta)r$ with θ the constant pie angle.

9. Look back to exercise 7 of section 5.1.

10. Straight-line latitudes replace the circular latitudes of the old scheme. Leaping is in the **x** direction.

Section 9.2

8. Looking into the region would be something like looking at a reflection of the world around you in a spherical mirror, except it would be an entirely different, infinite world *inside* the sphere you would be seeing! If you were small and flexible enough to squeeze through the spherical opening (the neck of the slice in the figure), you would find yourself in a

different flat region of space. In science fiction terms, the spherical neck is a "wormhole" connection between two otherwise normal universes.

9. To point out heading, you will need to choose a spatial dimension and use it to represent time. For the parabolic trajectory use the dimension perpendicular to the plane containing it. The stream will be a parabolic arch in spacetime (like a sheet of paper bent so that its cross-section is a parabola), and droplets will be headed diagonally "up and over" within it.

Section 9.3

3. The largeness of the "falling" effect of gravity is due to the large speed of light.

8. Take a vote.

11. The key to the independence of method of sensing ticks is that the message mechanism doesn't change in time.

12. Construct two events that are spacelike with respect to one another, and observe them from a moving frame of reference.

13. Lorentz rotations leave 45° lines *in the plane of rotation* invariant. If a light ray has a heading not in that plane (not going directly toward or away from the sun), it will not project into that plane as a 45° line.

14. Your hand is actually accelerating the apple, which only appears "at rest" in your hand.

Section 9.4

2. Use the formula in the text, `TURNING.PER.TIME` $= m/r^2$. Ignore any other small effects, such as leaping.

3. Very eccentric orbits near the sun give the most precession. Generally speaking, you know that your simulation is getting near perfect accuracy when reducing the step size does not change the result much. You can estimate error by assuming it to be proportional to step size: If you reduce step size from `STEPSIZE` to `F` $\times$ `STEPSIZE`, the error in the result will be about `F` $\times$ $\langle$change in result$\rangle$.

7. The vector projection $(\mathbf{v} \cdot \mathbf{S}) \times \mathbf{S}$ is, in coordinates, just setting the non-S coordinates to zero.

11. Align the gyroscope in the plane of the orbit for maximum visible effect, and take the orbit near the surface of the earth. Use the formulas TURNING.PER.ANGLE $= m/r$ (see subsection 9.4.4), and $m = GM/c^2$ in physical units (see exercise 4).

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Turtle Geometry

The Computer as a Medium for Exploring Mathematics

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