PREFACE AND ACKNOWLEDGMENTS

Der Mensch spielt nur, wo er in voller Bedeutung des Wortes Mensch ist, und er ist nur da ganz Mensch, wo er spielt.[1]

—Friedrich Schiller

This book contains games that are special in the following sense. When confronted with them for the first time, most people (including us) consider them unsolvable. Surprisingly, this perception changes when the games are translated into mathematics. The mathematical language nicely reveals the problem's underlying structure. Existing mathematical theory then allows the reader to quickly solve the game.

For us, understanding how mathematics unfolds the game's solution were moments of joy, and we wrote this book in the hope that the reader will experience the same. We therefore invite any reader to join us in being astonished about the fact that the game is solvable, in thinking about possible solutions, in having ideas that unfortunately do not work, and in enjoying the moment of understanding.

During the presentation of solutions, we mainly focus on conveying mathematical concepts and ideas. In particular, we explain the mathematical concepts using many examples. We aimed at maintaining mathematical precision, but we also avoided overly complex mathematical notation. This way we hope that

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1. The quote translates roughly as "Humans play only where they can be truly human, and they are fully human only when they play."

readers, ranging from high school students to those dealing with mathematics in their professional lives, can find pleasure when reading the book. Readers who are used to concise mathematical language can find additional details in the appendixes and pointers to the relevant literature.

All of the games can be performed in front of an audience. We have added some sections with practical advice, but interested readers can certainly complement these suggestions with their own ideas on how best to perform the games. The games can be presented in about 60–90 minutes, which makes them suitable for seminar talks or special lectures, such as Christmas lectures.

Most of the games described and their solutions are well known in some form or another. In places, we took the liberty of slightly changing the setup or the presentation of solutions. We did our best to supply information on their origins, as far as this is known to us. We refrain, however, from providing extensive references in the main text (e.g., regarding mathematical concepts and results) so as to not disturb the flow of the text more than necessary.

Each chapter is self-contained in that it contains a complete description of the problem, an introduction to the relevant mathematical theory, some notes on the history of the problem (to the extent that we are aware of it), and in some cases, comments on variations of the game and practical advice for a group performance. It is therefore possible to start reading with any chapter.

The majority of the games include some kind of randomness, and we have implemented some of the games in the programming language R. The code can be downloaded at https://github .com/mathemagicalgames/ and can be used for simulating games and their winning strategies.

Mathematics allows us to express thoughts and arguments in a concise and unambiguous way. Some math expressions are used so frequently that it is worthwhile to introduce notation for

them. The ones that we use throughout this book are collected in appendix A. The list there can be checked when encountering some unknown expressions—we hope that we have explained all relevant terms in that appendix. The experienced reader will realize that the list mainly contains standard notation. Appendix B contains further information on binary numbers, the convergence of sequences and series, and the exponential and logarithmic functions, for example. These concepts are widely used in mathematics, and readers who have not been introduced to them before might benefit from reading these chapters. Appendix C contains further details on the individual chapters. We expect this information to be most valuable for experienced readers who have been exposed to university level mathematics.

We hope this book appeals to people who are already interested in mathematics but also to people who are mainly interested in performing the games (and who might still take a look at the maths during some dark winter evening).

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