Architectural Space in Ancient Greece

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2 Documentation

Unfortunately, there is not a single extant work on architecture—not even a sizable fragment—from ancient Greek times, although it is known that books on the subject were written by the best architects of every period. The only treatise on architecture that has been preserved from antiquity is the *De Architectura* of Vitruvius, the Roman architect and engineer. Although there are several passages in this work that might be cited in support of my hypothesis, they make no specific reference to the system of site planning I have just described, which, at the time when Vitruvius was in practice (about 46–30 B.C.), was no longer in use. Actually, Vitruvius makes no reference at all to methods of planning; he avoids the exposition of general theories and describes shapes and forms only insofar as they relate to building construction.

As writings on architecture were unavailable, I turned to other fields: to the philosophers Plato and Aristotle, the mathematicians Euclid and Proclus, the historian Plutarch, and the traveler Pausanias (although he gives only factual details concerning certain aspects of the sites he visited). Realizing the importance of relating the material achievements of an era to its ideas, I acquainted myself as thoroughly as possible with the literature of the period during which this system of group design prevailed. I paid special attention to philosophic and mathematical concepts that, in my view, seemed closely linked to theories of planning.

The Philosophic Concept of the Universe

A Law Governing the Universe

In the first half of the sixth century B.C. Anaximander introduced into Greek philosophy the idea of a law governing all events in the universe. This idea inspired the legal concept of the polis, the Greek city-state of which every individual was unconditionally a subject.[1] Anaximander's own words on this, from the only fragment of his writings known to us, were recorded by Simplicius: "It is necessary that things should pass away into that from which they were born. For things must pay one another the penalty and compensation for their injustice according to the ordinance of time."[2]

Structure of the Universe

All Greek philosophers assumed that the center of the universe was a corpus; this was usually thought to be the earth, although sometimes, as in the case of the Pythagoreans, it was thought to be fire. We know that Anaximander, for example, held that the universe was a sphere with the earth as its center,[3] and the same concept was formulated by Aristotle: "Therefore all who hold that the world had a beginning say that the earth travelled to the middle."[4]

In connection with this theory most Greek philosophers believed that the universe was spherical, everlasting, and motionless.[5] This view was held by the Pythagoreans and Eleatics and also by Plato, who wrote "The earth lies in the center of a finite, though circular space."[6] The first thesis of Euclid's *Phaenomena* states that the earth lies in the center of the universe.[7]

The Finite or Infinite Nature of the Universe

Plato's allusion to a "finite" space touched on the most important issue in Greek philosophy: whether the universe was finite or infinite. According to Aristotle, "The first [problem] is whether there exists any infinite body, as most of the early philosophers believed, or whether that is an impossibility. This is a point whose settlement one way or the other makes no small difference, in fact, all the difference, to our investigation of the truth. It is this, one might say, which has been, and may be expected to be, the original of all the contradictions between those who make pronouncements in natural science."[8]

The Ionian philosophers maintained that the universe was infinite, and Anaximander, in particular, considered the infinite to be the basic principle of all things.[9] On the other hand, the Pythagoreans, the Eleatics, as well as the Attic philosophers—in short, all schools of philosophy except the Ionian—held that the universe was finite.

Aristotle offers several proofs of the finite nature of space: "Every sensible body has either weight or lightness. . . . Further, every sensible body is in some place, and of place there are six kinds [above and below, before and behind, right and left],[10] but these cannot exist in an infinite body. In general, if an infinite space is impossible, so is an infinite body."[11]

Until the late Hellenistic period Greek philosophy was influenced and sometimes dominated by a mathematical, or geometrical, concept of the universe. The earliest notion of a division of the universe is found in Homer, who divided it into five equal parts.[12] Later, the Pythagoreans asserted the ruling principle of numbers and believed in a geometrically ordered cosmos. The earliest Pythagorean, Petros of Chimera, taught that there were one hundred and eighty-three worlds "arranged in the form of a triangle, each side of the triangle having sixty worlds; of the three left over each is placed at an angle, and those that are next to one another are in contact and revolve gently as in a dance."[13] This theory formed the basis of the concept of the harmony of the spheres enunciated by Aristotle.[14] According to Proclus, the Pythagoreans always considered the equilateral triangle as the basic design of all created matter.[15]

The doctrine concerning the geometrical form of the universe appears also throughout the works of Plato and Aristotle and is reaffirmed later by Plutarch in his *Moralia.*[16] They maintained that the universe was based on five regular polyhedrons: earth was based on the cube, water on the pyramid, fire on the octahedron, air on the dodecahedron, and the heavens (light, or ether) on the icosahedron. Further, they held that these five polyhedrons corresponded to our five senses, resulting in the following relationships:

cube	earth	touch
pyramid	water	taste
octahedron	fire	smell
dodecahedron	air	hearing
icosahedron	light	sight

Euclid taught that the earth formed the mathematical center of the universe, and according to Proclus the universe was built up from the circle and the line.[17]

Geometrical Symbols in Religious Cults

The significance of mathematical symbols in religious rites led to the application of geometrical rules in the organization of space. Use of the symbols that I mention here can be traced for the most part to the Pythagoreans, particularly Philolaos, and in some instances to Plato.

Proclus calls to witness Plato, the Pythagorean authority, and Philolaos (in the *Bacchae*) concerning a theology based on geometrical concepts.[18] Proclus quotes Philolaos as saying that the Pythagoreans identified certain angles with specific gods: the 60° angle with Chronos, Hades, Ares, and Dionysos; the 90° angle with Rea, Demeter, Hera, and Hestia; the 150° angle with Zeus.[19]

Further, according to Archytas in his *Harmony*, the Pythagoreans taught that two stars forming an angle of 30°, 60°, 90°, 120°, 150°, or 180° exercised a powerful effect upon the earth and upon human well-being.[20]

According to Philolaos the Pythagoreans also identified certain geometric forms, such as the circle, triangle, and square, with specific gods, for example, Athena with the triangle, Hermes with the square.[21] A similar observation is found in Plutarch, who records that the Pythagoreans associated certain gods with certain numbers and signs, such as Athena with the equilateral triangle;[22] Hades, Dionysos, and Ares with 60° angle; Rhea, Demeter, Aphrodite, Hestia, and Hera with the 90° angle; and Zeus with the 150° angle.[23] Plutarch also states that the Egyptians identified the right-angled triangle having sides in the proportion 3:4:5 with Osiris and Isis.[24]

The relationships between angles and deities can be summarized as follows:

60° Hades, Ares, Dionysos, Athena, Chronos
90° Rhea, Hera, Demeter, Hestia, Aphrodite, Hermes
150° Zeus

Optics and Perspective

The mathematicians Euclid and Proclus both discuss optics. The contemporary laws of optical perspective given in the first definitions and theories in Euclid's *Optica* show that even at that time the rules governing the inclination of lines in space and the vanishing point of two parallel lines were formulated irrespective of whether they could be applied in an actual drawing. Proclus records that optics were derived from mathematics and were concerned with explaining the causes of false appearances, such as the meeting of two parallel lines in space.[25]

Numbers

Great importance was ascribed to numbers throughout antiquity. In illustration of this I quote from two sources:

Plato: "the great power of geometrical equality amongst both gods and men."[26]

Philolaos: "The nature of Number and Harmony admits of no Falsehood; for this is unrelated to them. Falsehood and Envy belong to the nature of the Non-Limited and the Unintelligent and the Irrational.

"Falsehood can in no way breathe on Number; for Falsehood is inimical and hostile to its nature, whereas Truth is related to and in close natural union with the race of Number."[27]

Certain numbers such as 10 and 12 had a particular significance for the Greeks. The great significance of 10 is referred to by almost all philosophers: Aristotle mentions it in his *Metaphysics*, and it was the subject of a special study by Philolaos in the second half of his book *On the Pythagorean Numbers*. "One must study the activities and the essence of Number" he wrote, "in accordance with the power existing in the Decad [*Ten-ness*]; for it [the Decad] is great, complete, all-achieving, and the origin of divine and human life and its Leader; it shares . . . the power also of the Decad. Without this, all things are unlimited, obscure and indiscernible."[28]

Proportion

The first representation of proportion found in classical writings is that implicit in the arithmetical expressions relating equal differences between two numbers, for example,
3 minus 2 = 2 minus 1
5 minus 3 = 3 minus 1.
Aristotle gives this example:
10 minus 6 = 6 minus 2.

This can be represented as
$a - b = b - c$ (sequential arithmetic progression)
$a - b = c - d$ (nonsequential arithmetic progression)
From the first we arrive at
$a + c = 2b$
and $b^2 - ac = (a - b)^2 = (b - c)^2$. (This proportion is used at Athens, Acropolis I; see pp. 29–30.)

Later, there is reference to geometric proportion. Archytas defined continuous geometric proportion as a sequence of constant ratios between two figures: the first is to the second as the second is to the third.[29]
$a:b = b:c$.
Discontinuous geometric proportion is
$a:b = c:d$.
Still later, harmonic proportion appears:
$(a - b):(b - c) = a:c$.

The first systematic theory of proportions is found in Euclid's *Elements* (Book V), although they were intrinsically known long before his time. To illustrate his theory Euclid used the regular pentagon and, by halving its basic angle, the golden section (although he did not use this name). He states, "A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the less."[30]

From this cursory review of classical sources in philosophy and mathematics, it is clear that they contain several ideas that could give support to my hypothesis, although no specific mention is made of the system of organizing architectural space. The clearest allusion to such a system is found in these words of Philolaos: "And you may see the nature of Number and its power at work not only in supernatural and divine existences but also in all human activities and words everywhere, both throughout all technical production and also in music."[31]

The most likely reasons for the lack of contemporary reference to a system of organizing physical space are, as I have said, that no writings by architects have been preserved from ancient Greek times and that philosophers and mathematicians were less concerned with questions of physical design than with matters more directly related to their own pursuits. The lack of written records, however, does not alter my hypothesis concerning the system I have described.[32]

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1 Werner W. Jaeger, *Paideia: The Ideals of Greek Culture,* trans. Gilbert Highet, New York: Oxford University Press, 1945, p. 110.
2 Ibid., p. 159.
3 Ibid., p. 157.
4 *On the Heavens* 2.13.295a13ff.
5 Empedocles *On Nature,* in Hermann Diels, ed., *Die Fragmente der Vorsokratiker, Griechisch und Deutsch,* 5th ed., Berlin: Weidmann, 1934–1938, I, 31.324–325.
6 Plato *Phaidon* 108CH.
7 Euclides, *Opera Omnia,* ed. I. L. Heiberg and Henricus Menge, Leipzig: Teubner, 1883–1916, vol. 8, *Phaenomena et scripta musica,* ed. Henricus Menge (1916), pp. 10–12.
8 *On the Heavens* 1.5.271b1ff.
9 Jaeger, *Paideia,* p. 158.
10 *Physics* 205b31.
11 *Metaphysics* 11.10.
12 Plutarch *Moralia* 5.13.390C.
13 Plutarch *Moralia* 5.22.422B.
14 *On the Heavens* 2.9.290b12ff.
15 Proclus *Commentary on the first book of elements by Euclid,* Definitions XXIV–XXIX.
16 *Moralia* 5.389.F11.
17 Proclus *Commentary on Euclid,* Definitions XV, XVI.
18 Proclus *Commentary on Euclid,* Prologue, pt. 1.
19 Proclus *Commentary on Euclid,* Definition XXXIV.
20 Archytas *Harmony,* in Diels, *Die Fragmente der Vorsokratiker,* I, 47.B2.436.6–8.
21 In Diels, *Die Fragmente der Vorsokratiker,* I, 402.31.
22 *Moralia* 5.381F.

[23] *Moralia* 5.363A.
[24] *Moralia* 5.379A.
[25] Proclus *Commentary on Euclid,* Prologue, pt. 1.
[26] "ἡ ἰσότης ἡ γεωμετρική καί ἐν Θεοῖς καί ἐν ἀνθρώποις μέγα δύναται." *Gorgias* 508A.
[27] Philolaos *On the Pythagorean Numbers,* quoted by Theo of Smyrna, 106.10, in Diels, *Die Fragmente der Vorsokratiker,* I, 412.9–14. [English translation in Kathleen Freeman, *Ancilla to The Pre-Socratic Philosophers,* Cambridge, Mass.: Harvard University Press, 1952, p. 75.]
[28] *On the Pythagorean Numbers,* in Diels, *Die Fragmente der Vorsokratiker,* I. 411.8–13. [English translation in Freeman, *Ancilla to The Pre-Socratic Philosophers,* p. 75.]
[29] Quoted in Porphyry *Harmonica of Ptolemy,* in Diels, *Die Fragmente der Vorsokratiker,* I, 47B2.436.6–8.
[30] Euclid *Elements* (trans. Heath) 6.188, theorem 3.
[31] *On the Pythagorean Numbers,* quoted by Theo of Smyrna 106.10, in Diels, *Die Fragmente der Vorsokratiker,* I, 412.4–8. [English translation in Freeman, *Ancilla to The Pre-Socratic Philosophers,* p. 75.]
[32] For instance, it is undeniable that the Doric metopes and triglyphs always had a ratio of 3:2, although this is not recorded in any book that has come down to us.