Chapter 1

Key concepts of the STR

1.1 INTRODUCTION

This chapter introduces the key concepts of uncertainty analysis which are discussed in this Scientific and Technical Report (STR). Understanding these concepts is a necessary first step in the pursuit of the goal of integrating uncertainty into model-based assessments of water resource recovery facilities (WRRF) design and operations for the purpose of quantifying risk.

The chapter opens with the definition of risk and uncertainty. Uncertainty is a particularly problematic term as it is often used interchangeably with risk, reliability and other similar terms. In addition, these terms may have other meanings in uncertainty evaluations conducted in other disciplines (additional definitions of concepts and terms relevant to the topics covered in this STR can be found in Appendix A). This section also includes how uncertainty has been classified by the scientific community and makes an important distinction between uncertainty and variability.

The chapter closes with a summary of the way uncertainty and variability can be evaluated with the use of models and statistical methods.

1.2 RISK

Risk can be defined as the exposure to events that if realised, result in some sort of loss. Identifying and quantifying – when possible – potential risks is a starting point for risk assessment. Risk, from a traditional engineering perspective, can be quantified as the probability of a specific failure occurring multiplied by the cost of the resulting damage. Therefore, risk quantification has two components: assessment of the probability that the risk will actually manifest, and determination of the associated cost. Certain risks are more amenable to quantification than others. Conversely, other risks can only be assessed qualitatively, for example, by stating that the probability is likely of unlikely, or that the cost of failure is low, medium or high.

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Major types of risk in wastewater treatment projects include (Talebizadeh, 2015):

- Non-compliance;
- Loss of reputation;
- Financial loss;
- Not winning a contract or contract annulment.

Such events are called hazards and their expected frequency of occurrence is usually quantified by a probability. Probabilities describe the expected likelihood of occurrence of an event. It answers the question: is it 'likely' or 'unlikely' that the event will happen? Risk is then calculated as the product of the probability of failure and the cost of failure.

Uncertainty assessment and propagation are the methods with which we quantify probabilities and thus, risk.

BOX 1.1 RISK

Risk = [Probability of failure] * [Cost of failure]

To quantify risk the probability of a hazard must be calculated.

This is achieved by assessing uncertainty.

1.3 UNCERTAINTY

Uncertainty can be defined as the degree of inability to determine or predict the exact behaviour of a system or process both now and in the future.

1.3.1 Classification of uncertainty

Uncertainty arises at many points in engineering projects. Although, this has long been recognised, development of a framework for incorporating uncertainty analysis into model-based decision support in wastewater treatment has lagged.

Researchers have classified uncertainty into categories depending on the methods and tools used to quantify or characterise it, in order to provide a common ground for communication between project participants (Refsgaard *et al.*, 2007; Walker *et al.*, 2003). A widely used approach defines three dimensions/categories of uncertainty: nature, location and level. These dimensions are discussed below in greater detail.

1.3.1.1 Nature of uncertainty

The nature of uncertainty refers to whether the uncertainty can be reduced with measurements or further research (e.g., due to experimental uncertainty in the determination of kinetic parameters) or whether it is due to the inherent variability of a system and cannot be reduced with any further research (e.g., frequency of observed events such as heavy rainfall or toxic spills) (see Section 1.3.2).

1.3.1.2 Level of uncertainty

The level of uncertainty is an expression of the scale of uncertainty associated with an identified risk. Based on Walker *et al.* (2003), the Task Group settled on four levels of uncertainty that define a spectrum ranging from complete determinism to indeterminacy (Figure 1.1):

Quantifiable uncertainty can be quantified and described with statistical methods. The random error in a measurement by a sensor, or in the triplicate analysis of a COD (chemical oxygen demand) sample are two examples of quantifiable uncertainty. Quantifiable uncertainty would also include the error in estimating a population mean from a set of samples.

Scenario uncertainty is uncertainty associated with the use of scenarios to examine possible outcomes that may develop in the future. Scenarios do not forecast what will happen in the future. Instead, they assess what might happen. Realistic assumptions about relationships and/or driving forces within the model can be established. It is not possible, however, to derive the probabilities of the scenarios taking place. Scenario uncertainty can be presented as the range of discrete outcomes from a scenario analysis.

Recognised ignorance is the state where the existence of uncertainty is recognised, but the uncertainty does not lend itself to quantification, nor to study by means of scenario analysis. In this situation, the mechanisms and functional relationships of the phenomena impacted by uncertainty are too poorly understood to enable any useful analysis beyond recognising that there is uncertainty, but it cannot be characterised.

Total ignorance is defined as the state where those involved are not aware of uncertainty. It is unknown what is unknown.

Figure 1.1 depicts these four levels of uncertainty lying between determinism and indeterminism.

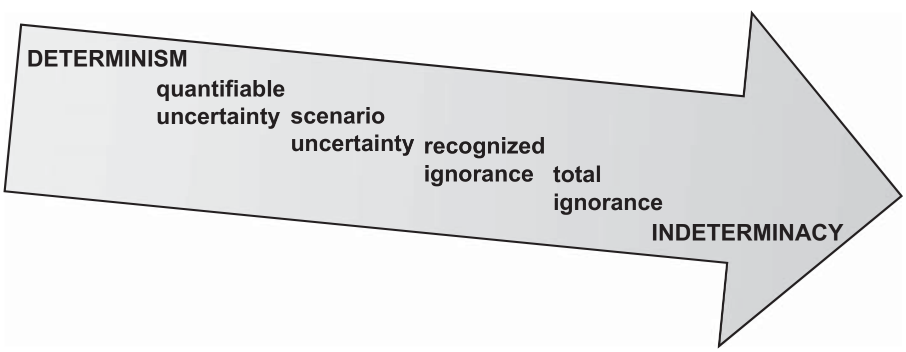

Figure 1.1 Level of uncertainty (after Walker *et al.*, 2003).

1.3.1.3 Location of uncertainty

The location of uncertainty refers to the instance where uncertainty manifests itself in the modelling process. Walker *et al.* (2003) suggested five generic locations: context uncertainty, model uncertainty, input uncertainty, parameter uncertainty and model output uncertainty. Walker's conceptualisation of these uncertainties is elaborated upon below.

- **Context uncertainty**: Context refers to the economic, political, social and technical conditions and circumstances that influence the model boundaries and frame the issues that the model is to address. Context uncertainty also relates to the suitability of a model for its intended purpose.
- **Model uncertainty**: All models involve simplifications of the system under study. Model structure uncertainty refers to uncertainty as to whether the model is an adequate representation of the real system it represents. In addition, errors can arise when implementing a model into a simulator. This is associated with the translation of the model into a program code and its execution on a computer and also includes uncertainty due to software errors.

- **Input uncertainty**: Input uncertainty is comprised of two sub-categories, system data uncertainty and external driving force uncertainty. Data uncertainty include uncertainty in, for example, the flow and concentrations to be input to a model for the purpose of projecting plant behaviour under some future condition. External driving force uncertainty relates to uncertainty associated with changes in conditions that are outside the model boundaries. For example, land-use policies in the catchment to a treatment plant might change and open the catchment to more rapid development. The knock-on effects from this would result in a change in flow and wastewater characteristics for the plant.
- **Parameter uncertainty**: Treatment plant models include many kinetic and stoichiometric parameters. The values of many of these parameters are known only approximately. Parameter uncertainty is associated with the lack of knowledge regarding the true value of these parameters as well as the different techniques used for the selection of parameter values during model calibration.
- **Model output uncertainty**: Model output uncertainty is the accumulated uncertainty caused by all the uncertainties in all the above locations as propagated through the model.

The Task Group chose to modify the Walker *et al.* (2003) framework. Context uncertainty is placed outside of the scope of uncertainties that are to be considered in this STR for model-based decision making. 'Source' of uncertainty is used instead of 'location' of uncertainty. The Task Group has chosen to organise the sources of uncertainty as follows:

- Inputs (includes experimental error);
- Model structure;
- Numerics (software implementation issues);
- Model output.

Section 1.4 provides more details on the classification that the Task Group has chosen to implement. Chapter 5 provides details on how to apply this framework to a wastewater project.

1.3.2 Separating variability and uncertainty

As discussed by Kelly and Campbell (2000), the EPA risk guidance and policy document (US EPA, 1997) and the report by the National Academy of Sciences titled Science and Judgment in Risk Assessment (NRC, 1994) call for separating variability and uncertainty in risk assessments.

There is an important difference between variability and uncertainty (and which quantities should be considered variable, uncertain or both). Although both can be described mathematically in the same way, for example by using density functions (Figure 1.2), they are very different in nature (Table 1.1). The Task Group has selected the definitions included in Box 1.2, in order to clarify the confusion often seen in the literature.

In this STR, uncertainty which is classified as irreducible (also aleatoric uncertainty), is designated as variability whereas uncertainty classified as reducible (also epistemic uncertainty), will be referred to simply as uncertainty.

1.3.3 Sources of variability and uncertainty

The WERF (Water Environment Research Foundation) study: 'Evaluating the Performance of Nutrient Removal Treatment Processes' (Bott & Parker, 2011) highlighted that plant performance variability depends on site-specific conditions: 'Local conditions impact the performance achieved on average and in terms of statistical variability. These factors include process design, climate impacts, wet weather flow influences, attributes of the service area, variation in influent flows and loadings, presence or absence of industrial contributions, whether solids processing is accomplished on the same site, sustained or interrupted supplies of chemicals, construction impacts, mechanical failures, the difficulty in operating the process, the ability to automate the controls of a process, the closeness of operation to design flows and loadings and others'.

The study examined plant data and identified specific factors that can be classified as external and linked to the 'environment' or internal and linked to the 'system'. The environment was detailed as the (past and future) inputs to the treatment plant as well as the responses of the receiving water body to the outputs of the treatment plant. The system was the WRRF.

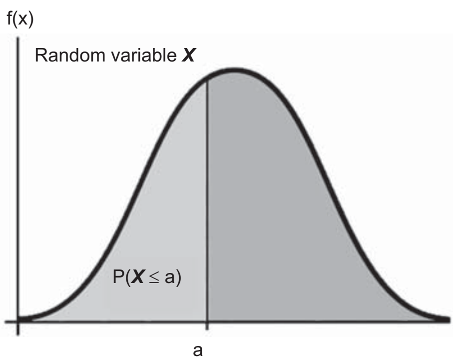

Figure 1.2 A density function can be used to represent either uncertainty or variability.

Table 1.1 Distinction between variability and uncertainty.

	Uncertainty	**Variability**
Origin	Lack of knowledge	'Real spread' of values (in time or space)
Reducibility	Partly reducible by further investigation	Irreducible
Representation	Probability distributions, density function	Time series, frequency distributions, density function
Example	Triplicate analysis of a COD sample, substrate hydrolysis rate	Influent COD (e.g., daily loads of one year)

BOX 1.2 VARIABILITY VS. UNCERTAINTY

Variability: is defined as the 'real spread' of values (in time or space) of a well-specified (statistical) population (Example: observed daily average COD load in the influent of a specific treatment plant over 5 years). The spread of these values is not reducible by further knowledge acquisition. Variability is a property of the population, not of the state of knowledge (Kelly & Campbell, 2000).
Uncertainty: results from a lack of knowledge. Parameter uncertainty is the uncertainty about the appropriate values for model parameters (e.g., half-saturation constants, hydrolysis rates). Model structure uncertainty pertains to the adequacy of the model equations and the model resolution in view of the modelling objective. Unlike variability, uncertainty is partly reducible: for example, further measurements or deeper investigations into the relevant processes might increase knowledge.

Examples of sources of variability and uncertainty originating from the environment are:

- Climate effects, energy use (resulting in operating strategy changes) and collection system/sewer characteristics;
- Wastewater characteristics (flows, loads, temperature, alkalinity, pH, fractionation);
- Growth or loss within the collection area (growth rate, changes in inflow and infiltration, changes in industry);
- Discharge permits.

Examples of sources of variability and uncertainty originating from the system are:

- Biological: microbial growth behaviour, especially at low concentrations;
- Physical: effect of unit operations configurations on removal efficiencies, alpha value (in aeration);
- Physical: non-ideal process behaviour (transport phenomena in aeration systems, non-ideal mixing affecting plant performance, approximation of plug flow hydraulics by using continuous stirred tank reactors (CSTRs) in series);
- Physical−chemical: for example, precipitation stoichiometry and kinetics;
- Biological-colloid chemical: effect of load and composition variations/peaks on sludge composition and floc structure and subsequent effect on sludge sedimentation;
- Unexpected control system behaviour;
- Mechanical failure;
- Operational problems.

1.3.4 Uncertainty analysis approaches

The role that a professional plays within a project influences where she will direct her focus on questions involving variability and uncertainty and the approach to uncertainty analysis that she will prefer. A project manager, who will oversee all aspects of project execution, might want to know how uncertainty affects the critical decisions affecting project development. A risk manager may hone in on uncertainties related to the type of contract and its influence on the risk to the various stakeholders. An engineer may break the project down to its various phases and then move to identify the sources of uncertainty within each phase. A scientist might focus on uncertainties in the data and the mathematical model that will inform the facility design. Tables 1.2−1.4 present three possible (not mutually exclusive) ways of addressing uncertainty (i–iii).

(i) Through project phases (Table 1.2);
(ii) Through modelling project steps (Table 1.3);
(iii) Using a systems analysis framework (Table 1.4).

Table 1.2 Examples of sources of uncertainty through infrastructure project phases (type i).

Phase	Example
Regulatory	Future effluent limits
Planning	Design horizon, design load
Preliminary design	Configuration type, critical growth rate, yield
Detailed design	Number of pumps, aerator layout
Construction	Unexpected geotechnical issues
Commissioning	Stability of processes
Operations	Toxic spills, foaming and bulking, sludge settling

Table 1.3 Examples of sources of uncertainty across the steps of a modelling project (type ii).

Phase	Example
Project definition	System boundary, required prediction accuracy
Data collection	Representativeness of historical data
Plant model-setup	Choice of biological model
Calibration/validation	Model parameter values
Simulation	Choice of scenarios, uncertainty propagation settings

Table 1.4 Examples of sources of uncertainty in a systems analysis framework (type iii).

Phase	Example
Aggregation/sampling error	Point measurements of rainfall
Measurement error	Random, systematic, gross errors
Input uncertainty	Catchment behaviour
Parameter uncertainty	Kinetic, mass-transfer related
Model structure	Monod vs. Haldane kinetics
Numerical	Insufficient numeric resolution

Researchers are typically more familiar with type (iii) structuring of uncertainty within a systems analysis framework whereas practitioners will normally be at ease with type (i) structure. Type (ii) can be interpreted as a combination of (i) and (iii) and reflects the decisions typically taken by the modeller (see also Rieger *et al.*, 2013). It is important to note that a modelling project could be implemented for any of the engineering project phases (see Chapter 5).

As the STR has the objective to facilitate the transfer of methods from research to practice, it is helpful to structure the sources of uncertainty within a framework which practitioners can easily relate to. Therefore, the first two approaches are emphasised.

Chapters 2–6 discuss the use of models for the evaluation of variability, quantifiable (statistically) uncertainty and scenario uncertainty.

1.4 INCORPORATING VARIABILITY AND UNCERTAINTY ANALYSIS IN MODELS

Mathematical models (such as the IWA Activated Sludge Models (Henze *et al.*, 2000)) coupled with statistical methods can assist practitioners in assessing variability and quantifiable uncertainty during plant design and operation. This section focuses specifically on uncertainties associated with the application of process models. Models can be used in any one of the project phases discussed in Table 1.2.

1.4.1 Variability and uncertainty in model steps

Rieger *et al.* (2013) in developing standards for Good Modelling Practice, defined a five-step model development protocol. Uncertainties that arise at each step are shown in Figure 1.3 and discussed in the following sections.

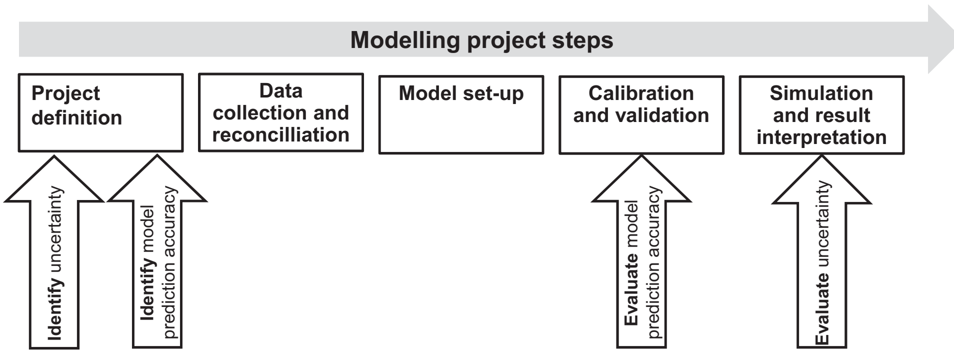

Figure 1.3 Modelling project steps and sources of uncertainty.

Project definition involves identifying the goals of the modelling project and setting the criteria for predictive accuracy. Based on the engineering project phase the pertinent sources of uncertainty will be identified in this stage.

Uncertainties associated with the project definition will differ depending on the project phase for which the modelling exercise is conducted (see Table 1.2 for project phase). For example, greater model accuracy will be demanded if the model is built to support detailed design rather than preliminary design. Also, at later project phases, more information will have been acquired and this will tend to reduce uncertainty.

Data collection and reconciliation is performed to improve data quality by removing noise and other artefacts in the data. This is necessary as raw data collected for the model will normally contain errors, outliers and gaps. Data gaps may be filled in using interpolation or other methods.

Uncertainties associated with data collection and reconciliation include questions as to whether the data are suitable for the intended model application. For example, the data might contain so many gaps that filling those gaps results in an undesirable skew.

Plant model set-up, also known as structure identification, refers to the identification/selection of an appropriate model structure. The definition of model structure includes reactor design (e.g., fixed-bed reactor vs. suspended growth) and reactor configurations (e.g., anoxic and aerobic zones, plug flow vs. CSTR) and biokinetic model selection (e.g., ASM or Monod/inhibition terms).

Uncertainty about the exact reactor and plant configurations (e.g., dimensions), and the true reaction mechanisms, results in a model structure which is subject to error, known as model structure uncertainty.

Calibration or parameter optimisation of a model consists of the adjustment of its numeric parameters, for example, kinetic parameters, to fit observed data. The purpose of calibration is to obtain a model that better reflects observations made under a specific set of conditions (operation).

Validation of a model consists of the evaluation of the model performance on observed data by comparing the simulation results of a calibrated model to an independent set of observations.

Uncertainty in the data (e.g., measurement errors) can result in model parameter values that are different from their true values.

Simulation and result interpretation relates to the application of a model to evaluate process performance (e.g., effluent ammonia concentrations are simulated). Once a model is finalised, it is often used to generate predictions of plant performance (based on currently available information) under some future condition to assess whether design and operation objectives will be met.

All the uncertainties encountered during model development will propagate through the model leading to model output uncertainty. As defined previously, model output uncertainty is the difference between the predicted values and the response of the real plant when operated under the conditions reflected in the model inputs. The more closely the model represents the real plant process and the more representative the input data, the better the model predictions and the less uncertainty.

The steps described above apply to projects where models are used for existing plants, where influent and plant data are available. In green field design situations where data are often not available, the data collection and reconciliation step and the calibration and validation steps are omitted. Data from nearby plants or similar catchments and default model parameter values can be used. However, regardless of the application, the uncertainties described above still exist. Chapter 5 discusses in more detail the uncertainty-related tasks that need to be considered at each stage of a wastewater treatment modelling project.

1.4.2 Sources of variability and uncertainty in models

1.4.2.1 Model input variability

Input variability results from the variable pattern exhibited by an input variable to the model (e.g., variable rainfall in a catchment). This input variability when propagated through the model, will cause variability in predicted plant performance (Figure 1.4).

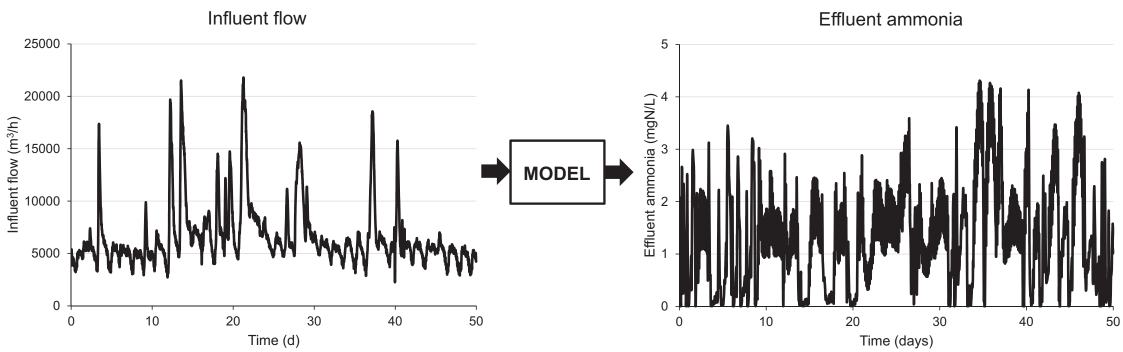

Figure 1.4 Impact of influent variability on effluent ammonia.

1.4.2.2 Model input uncertainty

Input uncertainty is a result of, for example, observation or analytical error and results from the variability of repeated measurements. Another source of input uncertainty is a result of filling in data gaps by interpolation or other methods. Data filling introduces varying errors which propagate through the model and affect the uncertainty of the model output.

1.4.2.3 Model structure uncertainty

Model structure uncertainty can be defined as uncertainty in model predictions originating from assumptions and simplifications made in the structure of the mathematical model. A mathematical model is always a simplified representation of reality. This leads to some degree of uncertainty in the model output predictions, originating from the process detail or rigour that is missing in the model. Uncertainties with the model structure are often associated but not limited to the following model selections:

- Influent fractionation model;
- Biological and chemical model;

- Hydraulic model;
- Aeration system model;
- Clarifier model;
- Models of sensors, actuators and equipment in plant operations;
- Interfaces between models.

Figure 1.5 includes a schematic depiction of the sources of model structure uncertainty for an example activated sludge plant that includes aeration tanks and final clarifiers. The sources have been sub-divided based on their spatial description (hydraulics) and the conversion process description (influent, biological or settling model).

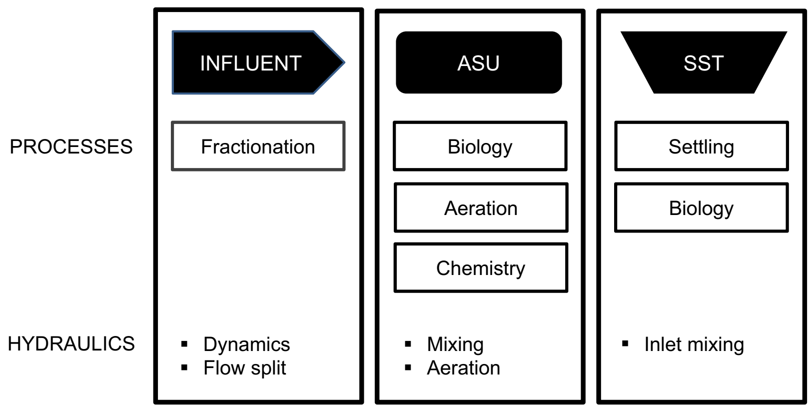

Figure 1.5 Schematic overview of the sources of uncertainty for an example WRRF model. The blocs indicate sources related to the influent model, the activated sludge unit (ASU) model and the secondary sedimentation tank (SST) model.

It should be noted that in whole plant models plant configuration simplifications also introduce model structure uncertainty.

1.4.2.4 Model parameter uncertainty

Treatment plant models contain many parameters that must be assigned values prior to model execution. Lack of knowledge regarding the best parameter value for a given system is an example of model parameter uncertainty. This is especially true for new processes that have not been properly characterised through multiple model development efforts. Examples of parameters without scientific consensus include parameters in models for nitrous oxide production, de-ammonification and the use of alternate substrates for denitrification.

Parameters that introduce uncertainty in biological process models are typically divided into kinetic (rates) and stoichiometric (coefficients). An example of a stoichiometric parameter is yield while, examples of kinetic parameters include bacterial growth, decay and hydrolysis rates as well as their associated half saturation constants. Rate parameters become important when the associated substrate or electron acceptor becomes ‘growth limiting’ and half saturation coefficients are critical for process sizing. Stoichiometric parameters are important for determining how a mass of reactant is distributed amongst one or more products for any given transformation. A very important set of parameters that introduce uncertainty are the parameters and ratios included in the influent fractionation models.

Some parameters are known with greater confidence (lower uncertainty) than others. For example, the yield of ordinary heterotrophic bacteria (OHO) on readily biodegradable (fermentable) substrate (S_B) growing aerobically is well characterised. A value in the range 0.60–0.67 g of OHO COD per gram of S_B consumed has been found to work well in modelling projects. However, uncertainty in the value of yield might be important when alternative substrates such as methanol, ethanol or glycerol are used for denitrification as these have not been as widely researched. As yield relates the consumption of carbon to electron acceptor consumption any uncertainty in the alternative substrate yield estimate could significantly affect the modelled economics of using one of these substrates for denitrification.

Whereas the standard yield is well established, the values of other standard parameters are not well known at all. For example, when modelling hydrolysis, the conversion of particulate biodegradable COD to S_B, plays an important role in the prediction of the process behaviour for systems that have a very short SRT or for processes that largely metabolise particulate substrates (aerobic and anaerobic digestion). Because conditions are never constant developing experiments to accurately estimate the hydrolysis rate under all possible conditions is essentially impossible which can introduce considerable uncertainty in the model predictions when this process is critical to the model output.

This section has illustrated the variability in parameter knowledge using a couple of simple examples, but the reader is reminded that models have hundreds of parameters and each has a different level of uncertainty and each of these parameters has a different impact on the model predictions. This realisation leads to the conclusion that it is important to carefully consider the level of uncertainty in each parameter and ultimately understand the impact that that uncertainty may or may not have on the model output.

1.4.2.5 Numerical uncertainty

Process model simulation is carried out in a number of consecutive steps. First, the real process is described by a mathematical model (a process model using ordinary or partial differential equation (ODE or PDE)). This is then approximated by a simulation model (numerical method) to be implemented in a computer (Bürger *et al.*, 2011). In a third step, the simulation model needs to be implemented in a software platform. The lumped uncertainty coming from approximating the mathematical model by a simulation model and its implementation is called numerical uncertainty. Each of the steps includes errors and approximations resulting in numerical uncertainty. As discussed by Claeys *et al.* (2010), no automatic tool exists to quantify this uncertainty.

The main sources of numerical uncertainty (Figure 1.6) can be classified as either numerical, implementation uncertainty, coding uncertainty, solver suitability, solver coding uncertainty and machine uncertainty (based on Claeys *et al.*, 2010).

The only source of uncertainty a model user can influence directly is the choice of solver and its accuracy settings (solver suitability). Claeys *et al.* (2010) show that this source of numerical uncertainty (caused by

Numerical uncertainty					
Source of uncertainty	Solver coding uncertainty	Machine uncertainty	Numerical implementation uncertainty	Coding uncertainty	Solver suitability
Type of modeller	Software developer	Software developer	Model implemeter	Model implemeter	Model implemeter Model user

Figure 1.6 Overview of the different sources of numerical uncertainty and modeller type that can influence this uncertainty (based on Claeys *et al.*, 2010).

the incorrect application of the solvers) can be more important than parameter uncertainty. As there are no analytical solutions for most (if not all) of the ordinary differential equations (ODEs) and partial differential equations (PDEs) used in wastewater treatment modelling, numerical methods are used to approximate the solution for these models.

The solver coding uncertainty and machine uncertainty can only be prevented by the software developers. Since the model is implemented in a computer and computers have a finite precision floating point arithmetic, any computation involves rounding errors. So, machine uncertainty is caused by the rounding errors and can become very important in iterative processes such as numerical integration.

Numerical implementation uncertainty and coding uncertainty are sources of numerical uncertainty that are introduced when a mathematical model is implemented in a software platform (Hauduc *et al.*, 2010). This implementation is executed in two steps, the translation of the model into a simulation model and the actual coding work. In both steps, conceptual and technical errors, leading to uncertainty, can be introduced.

1.4.2.6 Model output uncertainty

As discussed earlier, model output uncertainty is the accumulated uncertainty caused by all the uncertainties in all the above locations as propagated through the model. Model output uncertainty can be defined as epistemic (reducible) uncertainty and it relates to the differences between the true values of the output quantities and the values predicted by the model. Quantification of model output uncertainty serves as qualification and acceptance of the models used; whose outputs inform a model-based decision-making process.

1.4.3 Evaluation methods

Monte Carlo simulation, expert knowledge with fuzzy logic and optimisation are among the most widely used methods for exploring the combined effects of how the various sources of uncertainty propagate through the model and affect model output. The project phases where these methods can be used are listed in Table 1.5. The methods are critically reviewed in Chapter 4 and additional technical details have been included in Appendix B.

Table 1.5 Methods used for model uncertainty analysis.

Phase	The Most Used Methods	Main Applications
Planning	Monte Carlo Expert knowledge and fuzzy logic Scenario analysis	Output requirements Technology selection Scenario analysis
Preliminary design	Monte Carlo and mixed optimisation methods	Performance evaluation Plant dimensioning Control system selection
Detailed design	Monte Carlo combined with: (i) CFD, (ii) optimisation	Exact dimensions Control system design
Operations	Monte Carlo	Process analysis Optimisation Control

Monte Carlo analysis is the dominant method used to evaluate quantifiable (epistemic) uncertainty and can be applied across all stages of design. In the early stages of design, when there are many degrees of freedom and more uncertainty, Monte Carlo and scenario analysis are the preferred techniques. As the design process progresses (and the degrees of freedom are reduced), Monte Carlo is still the dominant technique, but it is sometimes coupled with mixed optimisation-based techniques (GA, NLP, Pareto frontiers).

Expert knowledge and fuzzy logic have also been used, but mostly for technology selection and scenario analysis.

Optimisation methods are typically used in preliminary and detailed design to develop exact dimensions or during operational assessment to develop control settings and process optimisation.

Chapter 4 reviews the uncertainty analysis methods described in the literature in the field of wastewater. Appendix D discusses pertinent methods from other fields.

1.5 SUMMARY

This chapter presented the important concepts necessary to acquire baseline understanding of the classification of uncertainty and how it relates to process modelling. The key points from this chapter can be summarised as follows:

Uncertainty mainly stems from imperfect or unknown information (lack of knowledge) and can be reduced by more research. Uncertainty can be classified as quantifiable or unquantifiable based on whether statistical methods and models can be used to evaluate it.

In this STR, uncertainty which is classified as irreducible (also aleatoric uncertainty), is designated as variability. Variability is the 'real spread' of values (in time or space) of a measurable quantity.

In this STR, uncertainty classified as reducible (also epistemic uncertainty), will be referred to simply as uncertainty.

Quantifiable (statistical) uncertainty in reference to the value of a parameter or quantity can be described with a probability density function.

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