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Discussion

Discussion: "The Resistance of Clamped Sandwich Beams to Shock Loading" (Fleck, N. A., and Deshpande, V. S., 2004, ASME J. Appl. Mech., 71, pp. 386–401)

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Fleck and Deshpande (FD) proposed an analytical model to encapsulate the response of clamped sandwich beams to blast loadings in air and under water. Four pertinent issues, viz., (1) application of Taylor's model [1] to air blast loadings, (2) the limiting impulse transferable to a stationary rigid face in an air blast, (3) inconsistencies in the proposed shock model, and (4) application of the shock theory to certain micro-architectured core materials, require clarification. Detailed comments are as follows:

(1) The net pressure acting on the front face of the sandwich beam [Eq. (15) of FD] was proposed to be the superposition of the incoming and reflected blast wave pulses. This is acceptable for the typically *weak* shock (or sound) waves generated in underwater explosions because water is nearly incompressible. However, by using the results of the *weak* shock analysis to estimate typical impulses delivered to the front face of the sandwich beam by *strong* shock waves in air blasts (pages 389 and 397 of FD), FD imply that their modified Taylor's model [1] is also applicable to *nonlinear*, finite amplitude disturbances propagating in a compressible medium. This needs to be justified or qualified.

(2) For air blast loadings, FD rightly propose that full transmission of the blast impulse to the sandwich beam, assuming a stationary rigid front face, be considered for safe design. However, the limiting impulse transmitted is based on a *weak* shock in water, where the reflected overpressure (peak pressure) is twice the incident overpressure, given by FD to be

$$I = \int_0^\infty 2p_o e^{-t/\theta} dt = 2p_o\theta, \tag{1}$$

where p_o is the overpressure and θ the decay constant. By contrast, the Rankine-Hugoniot relations predict a reflected overpressure of eight times the incident overpressure for a *strong* shock propagating in an ideal gas, giving a limiting impulse of $8p_o\theta$ from momentum considerations, a classical result well known to fluid mechanicists [2–4]. The actual reflected overpressure could reach a factor of 20, or even higher, if real gas effects, such as dissociation and ionization of the air molecules, are taken into consideration [4]. Therefore, it is questionable whether the performance charts constructed based on Eq. (1) provide a safe guide for the design of *air-blast* resistant sandwich beams.

(3) Instead of establishing a local balance of energy across the shock wave[2] front, FD presents a global energy balance of the sandwich beam where the following two inconsistencies arise from the use of the term $\sigma_{nY}\varepsilon_D X$ [Eq. (28) in FD]:

(i) That the energy absorbed per unit volume of core material ($\sigma_{nY}\varepsilon_D$) is independent of the initial front face velocity v_o. For an aluminium alloy foam core, this is not consistent with FD's own recent studies reported in [5], nor with the authors' experimental data [6].

(ii) That the change in the internal energy[3] density of the core material $[\rho_c U]$ ($=\sigma_{nY}\varepsilon_D$ by FD) is *independent* of the particle velocity jump $[v]$ across the shock front, where $[] \equiv ()_d - ()_u$, U is the internal energy per unit mass, ρ_c is the core density and the symbols used by FD also apply here. By contrast, the basic jump conditions predict that [7]

$$[\rho_c U] = \sigma_u[\varepsilon] + \rho_c[v]^2/2. \tag{2}$$

This can also be confirmed with an idealized example using ABAQUS explicit. Figure 1 shows a two-dimensional honeycomb (comprised of 21×74 regular cells of 4 mm edge length and 0.34 mm wall thickness) with a relative density of 0.1, which is fixed at the right end and compressed at a constant velocity from the left. The aluminium alloy cell walls have rate-independent, elastic, perfectly plastic properties identical to those used by Chen et al. [8]. Each cell wall is modeled using four general-purpose shell elements (S4R of ABAQUS) and self-contact simulations have been incorporated. Figure 2 shows the internal energy density (using "ALLIE" in ABAQUS) of the honey-

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[2]Note that the term "shock wave" is used to describe a progressive cell-crushing phenomenon (see Fig. 1). This is an inertial phenomenon often associated with the high velocity compression of cellular materials and structures [9,10]. In a sense, it is not the classical shock wave described by fluid mechanicists and shock physicists. Rather, the deformation response exhibits "shock-type" characteristics and the field variables appear to obey the basic jump conditions [6,7]. Terms such as compaction or consolidation waves could equally be used.

[3]Since all forms of non-mechanical energies and the energy dissipated by viscous effects and by time-dependent deformation are neglected, the contributions to the internal energy of a fixed body of material are the recoverable elastic strain energy and the energy dissipated by plasticity [11].

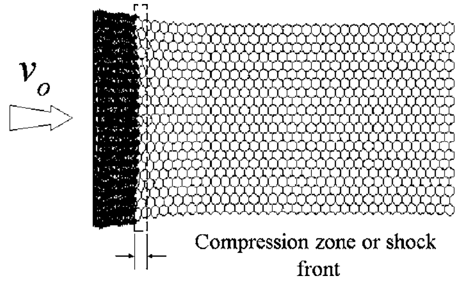

Fig. 1 In-plane compression of a honeycomb at up to $\delta/L \approx 50\%$, where δ is the displacement of the left end measured from its initial configuration and L is the initial length of the structure

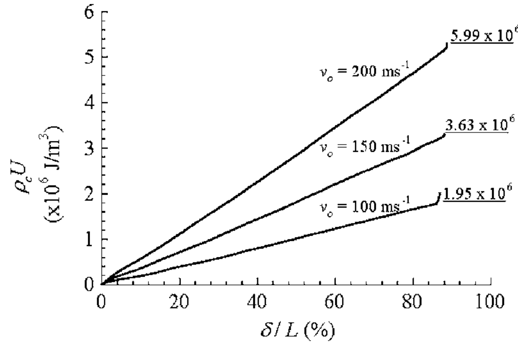

Fig. 2 Internal energy density of the honeycomb vs impact-end displacement at three velocity levels. The overall energy absorbed at full locking predicted by Eq. (2) is given by the underscored values. Parameters used in their calculations were $[\varepsilon]=\varepsilon_D=0.85$, $\sigma_u=0.7$ MPa, and $[v]=v_o$. The stress measured at the fixed end (σ_u) remains almost constant during compression and varies little with impact velocity.

comb at three velocity levels. The upturn at the end of each curve shows an increased absorption of energy by the honeycomb at full locking with impact velocity. In all cases, the energy dissipated by rate-independent plastic deformation occurs mostly within the compression zone highlighted in Fig. 1, and accounts for more than 95% of the internal energy density plotted. Since $[v] \approx v_o$, Fig. 2 shows that $[\rho_c U]$ is *dependent* upon $[v]$ at every stage of the compression process. Therefore, it is questionable whether energy conservation has been achieved by Eq. (28) of FD.

(4) Two of the core topologies depicted in Figs. 2(*d*) and 2(*e*) of FD are likely to exhibit strong "Type-II" inertial effect where the shock analysis is known to be unsatisfactory [9,10]. Previous studies of the dynamic compression of wood along the grain and of the out-of-plane compression of honeycombs have shown that a shock model based upon their respective quasi-static stress-strain curve consistently under-predicts their dynamic strength, and this is particularly evident at the higher impact velocities where a shock is expected to form. The reasons for this have been identified and explained by Reid et al. [9,10].

An important conclusion of FD's analysis is the apparent advantage of sandwich construction over solid plates of the same mass per unit area to blast loadings. However, despite the good agreement between the predictions of the model and the results of FE simulations by Xue and Hutchinson [12], which also assumes that the limiting reflected overpressure is *twice* the incident overpressure for air blast loadings, the efficacy of FD's model should be treated with caution for the above reasons and requires further validation.

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