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On the celestial body absorption by 6D black holes

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Abstract. It is shown that a drastic modifications of BH properties in the extra dimension presence makes the method of extra-dimensional BH search by of possible celestial body absorption much more effective and realistic. First, the decrease of Hawking radiation intensity allows to survive till present the extra-dimensional BHs with masses many orders less that the 4D Hawking mass. Second, the strong extra-dimensional gravity allows these BHs to decelerate fast and become captured by white dwarfs or other celestial bodies. And third, the same strong gravity makes possible complete white dwarf absorption by these light BHs for the cosmological time. The possibility of white dwarf detection through either the accompanying neutrino burst or a forming Solar-mass BH allows to set up a constraint on the 6D BH mass fraction at the level of 10^{-16}. This way the 6D gravity in the region of Planck masses from 1 TeV to 1 PeV can be searched for.

Keywords: braneworld cosmology, extra-dimensions, primordial black holes
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INTRODUCTION

One of the most spectacular manifestation of nature realizing extra-dimensional scenarios could be the surprising properties of microscopic black holes (BHs) [1]. First of all their production both at large accelerators and by cosmic ray particles becomes possible. Also the larger BH gravitational field accelerates the matter accretion onto the microscopic BHs at the extra-dimension presence making possible their efficient deceleration and capture by celestial bodies followed by the fast absorption of the latter by the captured BHs. Only the faster evaporation caused by Hawking radiation prevents a complete absorption of the Earth, white dwarfs and other celestial bodies by microscopic BHs produced in particle collisions.

Actually the absorption of celestial bodies by primordial BHs has been considered as a method of microscopic primordial BH search for a long time [2, 3, 4, 5, 6, 7, 8, 9, 10]. Initially Hawking radiation has been extensively used for the search of primordial BHs with initial masses $M \sim M_{4Haw}$, where $M_{4Haw} = 5 \times 10^{11}$ kg is the 4D Hawking mass experiencing most efficient evaporation at present. However Hawking radiation is inefficient for heavier BHs search. Fortunately it was shown in [6] that BHs with masses only slightly exceeding M_{4Haw} can be completely absorbed by white dwarfs (WDs) for the time of their existence. Both the neutrino burst accompanying the absorption process [7] and the observation of BHs with "unnatural" masses $M \lesssim M_{Sun}$ can be used either for detection of such events or establishing constraints on BH abundance.

However the celestial body "absorption method" encounter severe limitations in 4D gravity. First of all, relative weakness of the latter can not provide efficient BH capture by WDs and other celestial bodies. In addition, the relatively large M_{4Haw} value considerably limits the constrains on the mass fraction of BHs with $M > M_{4Haw}$. The existence of the extra dimensions can fortunately change the situation completely.

In this paper we show that the modification of both gravitational field and Hawking radiation process in the presence of extra dimensions makes the absorption method much more efficient and realistic. First of all, the increase of BH gravity at small distances greatly accelerates the accretion process making possible both efficient BH capture and complete absorption at much lower BH masses. In addition, the less intensive Hawking radiation allows to "survive" the BHs with masses much lower than M_{4Haw}. The arising possibility of the complete absorption of celestial bodies of solar masses by BHs with $M \sim M_{DHaw} \leq 10^{-2} M_{4Haw}$ allows to obtain much tighter and more reliable constraints on the primordial BH density fraction in the Universe that in the 4D gravity.

TEV-SCALE GRAVITY AND BLACK HOLE PROPERTIES

We will consider as an example the simplest large extra dimension version of TeV-scale gravity [11, 12] in which D-dimensional Planck mass $M_D > 1 TeV$ determines the scale

$$R_D = \frac{1}{M_D}\left(\frac{M_4}{M_D}\right)^{\frac{2}{D-4}} \qquad (1)$$

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of extra dimensions assumed to be equal for all $n = D-4$ of them. The Planck system of units with four dimensional Planck mass determined [1] like

$$M_4 = 1/\sqrt{8\pi G} = 2.4 \times 10^{18} GeV, \tag{2}$$

where G is the Newtonian gravity constant, is used throughout the paper. Eq. (1) yields the value $R_5 \sim 10^{14}$ cm for one extra dimension case which thus should immediately be abandoned, at least for such low M_5 values as 1 TeV. Fortunately, the value $R_6 = 0.48$ mm is comparable with the scale of modern Newtonian gravity tests and allows to consider the D=6 case as viable, especially if the values $M_6 \gg 1TeV$ are considered (see below). However the 7D value $R_7 = 3.6 \times 10^{-9}$ m and even smaller ones for D > 7, though evidently do not contradict any thinkable gravity test, considerably limit the effect of extra-dimensional accretion, as will be shown below.

The D-dimensional BHs are described by the D-dimensional Schwarzschild solution allowing to obtain both D-dimensional Schwarzschild radius and gravity potential which can be found in [1]. To consider the BH deceleration and mass growth we will only need Bondi radius

$$R_B(M) = \left[\frac{(D-3)}{4c_s^2}\frac{k_D M}{M_D^{D-2}}\right]^{\frac{1}{D-3}}, \tag{3}$$

where c_s is the sound speed in the celestial body unperturbed by the BH presence and $k_D \sim 1$ is numerical coefficient [1]. Bondi radius (3), playing the role of effective accretion radius, determines the accretion BH mass growth rate

$$\frac{dM}{dt} \simeq 4\pi R_B^2(M)\rho c_s, \tag{4}$$

where ρ is unperturbed central celestial body density. There are Eqs. (3) and (4) which describe the increase of matter accretion rate onto microscopic BHs which makes possible the efficient search of the latter by the celestial body absorption method.

To make our consideration realistic we should consider the primordial BHs which survived the Hawking evaporation during the Hubble time $t_{Hub} \simeq 13.7 \times 10^9 yr$. Masses of such BHs has to exceed the "D dimensional Hawking "mass [13]

$$M_{DHaw} \simeq M_D \left(\frac{M_D}{M_4}\frac{t_{Hub}}{t_{Pl}}\right)^{\frac{D-3}{D-1}}, \tag{5}$$

where $t_{Pl} = 2.7 \times 10^{-43}$ s is the Planck time in the same units as M_4 [1].

Fig. 1 demonstrates that the value (5) which determines the minimal masses of BHs surviving the Hubble

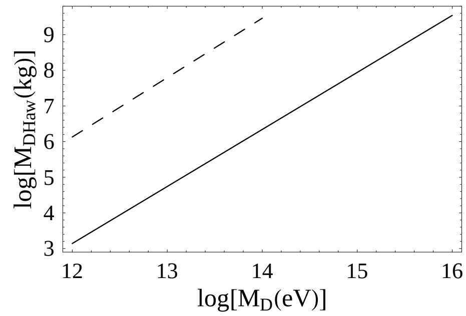

FIGURE 1. Hawking mass vs Planck mass for D=6 (solid) and D=7 (dashed).

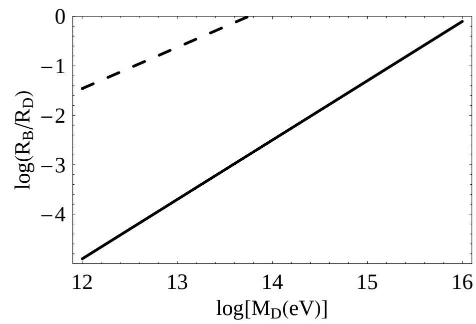

FIGURE 2. Ratio of Bondi radius to extra dimension size for D=6 (solid) and D=7 (dashed).

time are indeed considerably lower than the 4D Hawking mass $M_{4Haw} = 5 \times 10^{11}$ kg.

To estimate the possibility of the mass growth of such light BHs inside celestial objects one first should determine the role of "extra dimensional accretion" which manifests itself until $R_B < R_D$. Fig. 2 demonstrates that the latter condition is fulfilled for $1TeV < M_6 < 10^4 TeV$ and for much narrow interval for the 7D Planck mass M_7. Unfortunately, the latter does not provide considerable advantage for 7D BH search. The small maximum 7D BH mass predetermined by the small $R_7 = 3.6 \times 10^{-9}$ m value is a problem.

A really fast accretional growth of the D dimensional BH mass M proceeds according to Eq. (4) until the Bondi radius (3) which, naturally, grows with M reaches the R_D value. Thus, substituting $R_B = R_D$ into Eq. (3) one can resolve the latter to obtain the final mass M_f reached after the completion of the extra-dimensional accretion. Both the mass growth and Hawking evaporation will proceed at $M > M_f$ in the 4D regime with no considerable influence of extra dimensions. To avoid fast Hawking evaporation and to accelerate considerably the future BH mass growth the M_f value, at least, has to exceed M_{4Haw}. However Fig. 3 demonstrates that the final 7D BH mass reaches the latter only in the narrow region near M_7 not providing wider possibilities of 7D BH search than the

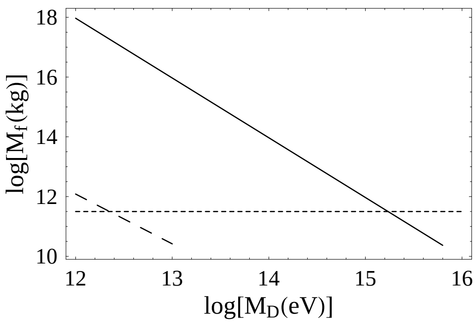

FIGURE 3. Final mass of extra-dimensional accretion vs Planck mass for D=6 (solid) and D=7 (dashed). Short dash horizontal line shows 4D Hawking mass.

LHC. At the same time Fig. 3 demonstrates that the "absorption method" allows to search for D=6 gravity in the 6D Planck mass interval $1TeV < M_6 < 1PeV$ in which $M > M_{4Haw}$ and the method of WD absorption can readily be applied.

EXTRA DIMENSIONAL BH CAPTURE BY WHITE DWARFS

As was already mentioned, WD represent itself the most suitable object for the search of 4D BHs with masses not much exceeding M_{4Haw}. The time of absorption of a WD with central density ρ by a 4D BH with mass M equals [7]

$$T_{abs} = \frac{27 \times 10^9 yr}{(M/10^{12}kg)\ \left(x + \sqrt{1+x^2}\right)^{3/2}}, \qquad (6)$$

where $x = 0.80\sqrt[3]{\rho/(10^9 kg/m^3)}$ is the relativistic parameter of the degenerate electron Fermi gas at central WD density ρ unperturbed by the BH presence. Eq.(6) shows that the densest relativistic WDs with $\rho = 10^{11} \div 10^{12} kg/m^3$ can be absorbed for the Hubble time by any BH with $M > M_{4Haw}$, while the most widespread WDs with $\rho = 10^8 \div 10^9 kg/m^3$ by BHs several times heavier. Thus one may assume that nearly each WD can be completely absorbed by each 6D BH which has passed the stage of 6D accretion and acquired the mass shown in Fig. 3. The last problem we should consider is the possibility of BH capture by WDs. To treat the latter we will follow the first version of Ref. [9] which was too optimistically applied to the problem of capture of 4D BHs by stars.

We will assume that primordial BHs in the Milky Way have Gaussian velocity distribution

$$f(v, \sigma_{BH}) = \frac{1}{(2\pi\sigma_{BH})^{3/2}} exp\left(-\frac{v^2}{2\sigma_{BH}^2}\right), \qquad (7)$$

where $\sigma_{BH} = 1.6 \times 10^5 m/s$ is the one-dimensional dispersion, and number density distribution

$$n_{PBH}(R,z) \simeq 1.8 \times 10^{-24} \left(\frac{10^3 kg}{M_{PBH}}\right) \times \left(\frac{a^2 + R_0^2}{a^2 + R^2 + z^2}\right) \Omega_{PBH}\, m^{-3}, \qquad (8)$$

where Ω_{PBH} is the primordial BH mass fraction, (R,z) are cylindrical Galactocentric coordinates, $a \simeq 6.4kpc$ is the Galactic core radius and R_0 = 8.5 kpc is the Sun Galactocentric distance.

We will also assume that WDs have Gaussian velocity distribution with $\sigma_{WD} \simeq 2.2 \times 10^5/\sqrt{3}\ m/s$ and number density distribution

$$n_{WD} = 6.2 \times 10^{-61} N_{WD} \times \left(e^{-|z|/z_{thin}} + 0.02 e^{-|z|/z_{thick}}\right) e^{-R/R_d} m^{-3}, \qquad (9)$$

where $N_{WD} \sim 10^{10}$ is the total WD number, $R_d \simeq 3.5kpc$ is the disc scale length, $z_{thin} \simeq 3.3 \times 10^2 pc$ and $z_{thick} \simeq 1.4kpc$ are the scale heights of thin and thick discs, respectively.

A possibility of effective deceleration and capture of extra-dimensional BHs by cosmic objects represents the principal point of this paper. Let us consider the 6D BH interaction with a typical WD with density $\rho_{WD} = 10^9 kg/m^3$ and radius $R_{WD} = 10^7 m$ and estimate the rate

$$\Gamma(v_{WD}) = \int d^3\vec{v}\ f(v, \sigma_{BH})\, \pi b^2 |\vec{v} - \vec{v}_{WD}| \qquad (10)$$

of BH caprute by a WD with velocity $\vec{v}_{WD}$, where b is the impact parameter corresponding to BH passage inside the WD cross-section πR_{WD}^2 and $\vec{v} - \vec{v}_{WD}$ is the relative BH-WD velocity at the interstellar distance of about 1pc. Introducing the relative BH-WD velocity $(\vec{v} - \vec{v}_{WD})_{R_{WD}}$ on the WD surface the conservation laws of angular momentum and energy can be written in the form

$$b|\vec{v} - \vec{v}_{WD}| = R_{WD}|\vec{v} - \vec{v}_{WD}|_{R_{WD}} \qquad (11)$$

and

$$|\vec{v} - \vec{v}_{WD}|^2 = |\vec{v} - \vec{v}_{WD}|^2_{R_{WD}} - \frac{2G(M_{WD} + M)}{R_{WD}}, \qquad (12)$$

respectively. We neglected the BH potential energy at interstellar distances which is about 10^{-7} of the kinetic one. Eqs. (11), (12) allow to find that

$$b^2 = R_{WD}^2 + \frac{2G\,(M_{WD}+M)\,R_{WD}}{|\vec{v}-\vec{v}_{WD}|^2} \simeq \frac{2GM_{WD}R_{WD}}{|\vec{v}-\vec{v}_{WD}|^2}. \qquad (13)$$

To evaluate the rate (10) one should substitute (13) and integrate over the BH velocities obeying the condition

$$\frac{M\,(\vec{v}-\vec{v}_{WD})^2}{2} < |\Delta E|, \qquad (14)$$

where ΔE is the energy which a BH loses passing through a WD. Usually one can assume that $\Delta E \ll E_{av}$, where

$$E_{av} = 3\frac{M\,(\sigma_{BH}^2+\sigma_{WD}^2)}{2} \qquad (15)$$

is the average relative kinetic energy at large distances, and obtain from (10)

$$\Gamma\,(v_{WD}) = 6\pi GM_{WD}R_{WD}\,exp\left(-\frac{v_{WD}^2}{2\sigma_{BH}^2}\right) \times \frac{(\sigma_{BH}^2+\sigma_{WD}^2)}{\sqrt{2\pi}\sigma^3}\,\frac{|\Delta E|}{E_{av}} \qquad (16)$$

and

$$\langle\Gamma\rangle = 3\sqrt{2\pi}GM_{WD}R_{WD}\,\frac{1}{\sqrt{\sigma_{BH}^2+\sigma_{WD}^2}}\,\frac{|\Delta E|}{E_{av}} \qquad (17)$$

after the averaging over the WD Gaussian velocity distribution.

The low ΔE value in the 4D gravity allows only BHs with masses $M > 10^{26}kg$[10] to be captured by WDs and other celestial bodies. We will show now that the presence of extra dimensions improves the situation dramatically. Remind that a BH decelerates both due to direct energy transfer to surrounding matter and to mass increase in the process of accretion. The rate of the kinetic BH energy decrease with length caused by the BH mass increase

$$\frac{dE}{dz} = -\frac{E}{v}\,\frac{dM}{dt} \qquad (18)$$

is directly connected with the mass accretion rate (4). It should be noted that the Bondi radius (3) of 6D BHs with masses $M > M_{6Haw} \simeq 10^3 - 10^8 kg$ exceed the interparticle distance $10^{-12} - 10^{-13}m$ in the WD matter allowing to apply the continuous matter approximation in which Eq. (4) was obtained.

Note that Eq. (18) should be modified if the BH speed is comparable with that of sound in the accreting matter [3, 14]. One can estimate the BH and sound speeds in the case of typical WD, respectively, as

$$|\vec{v}-\vec{v}_{WD}|_{R_{WD}} \geq \sqrt{2GM_{WD}/R_{WD}} \simeq 0.02c \qquad (19)$$

and

$$v_s = xc\sqrt{\frac{m_e}{3m'\sqrt{1+x^2}}} < 0.01c, \qquad (20)$$

where c is the speed of light, m_e is electron mass and $m' \simeq 2\times 1.6610^{-27}kg$ is WD mass which falls on one electron. To take the BH motion influence on the accretion rate we will use a substitution

$$v_s^2 \;\rightarrow\; v_s^2 + (\vec{v}-\vec{v}_{WD})_{R_{WD}}^2. \qquad (21)$$

Note that the latter is simply analogous to the substitution used in 4D case [3, 14] and needs additional substantiation for D > 4.

The BH deceleration due to the direct energy transfer to surrounding matter is more difficult to treat at D > 4 than at D=4. The point is that the specific radial dependence of the Newton force in the latter case creates the situation when energy transfer to surrounding matter at various distances gives comparable contribution to the BH energy loss. The logarithmic approximation can be applied to evaluate the value of the latter [3, 10]. Being proportional to the logarithm of the very large ratio of the largest possible impact parameters to the smallest one, the contribution of the energy transfer to surrounding matter to the BH deceleration more that an order of value exceeds that of matter accretion in the D=4 case. Remarkable that to reach acceptable precision in logarithmic approximation one does not need a detail evaluation of local energy transfer at any, at that number, the most difficult for consideration smallest distances. In addition, a much simplifying pulse approximation can be used in this situation. All this allows to obtain the simple and reliable estimate of energy losses at D=4 [3, 14]. Remind that this type of losses fully manifests itself when the BH velocity exceeds the sound speed, as in the case considered.

A steeper gravitational force radial dependence complicates the situation considerably at D > 4. The main contribution to the BH energy losses comes from smaller distances, only slightly exceeding the “distance of matter absorption” b_{abs}. First of all, the amplifying logarithmic factor does not appear in this case, making to contributions of both mechanisms of BH deceleration comparable. In addition, both considerable displacement and varying pressure of the surrounding matter during BH passage should be taken into consideration to evaluate

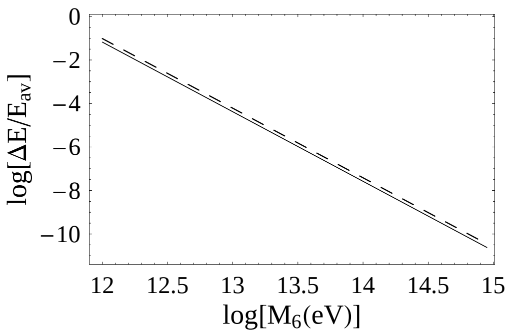

FIGURE 4. Relative energy 6D BH loss in typical WD due to its mass increase (solid) and energy transfer to surrounding matter (short dash line) vs 6D Planck mass.

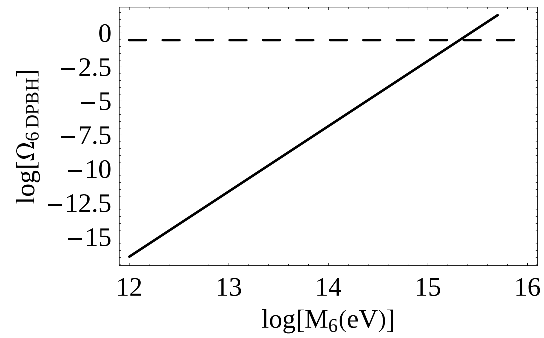

FIGURE 5. Observable 6D primordial BH mass fraction vs 6D Planck mass. Dash horizontal line shows DM density $\Omega_{DM} = 0.3$.

the energy transfer in this situation instead of the simple pulse approximation use in D=4 case. At present we are able to suggest only a simple estimate dE_{tr}/dz of the energy losses by evaluating the local velocity acquired by the surrounding matter in the pulse approximation and integrating corresponding matter kinetic energy density over the region $R > b_{abs}$. The b_{abs} value was obtain from the equality of the acquired local velocity to that of the BH. Fig. 4 indeed demonstrates that both deceleration mechanisms give comparable contributions to the BH deceleration at the length $\Delta z \sim R_{WD}$ of BH passage through a typical WD.

Substituting the typical BH one-pass energy loss value

$$\Delta E \simeq \left(\frac{dE_{abs}}{dz} + \frac{dE_{tr}}{dz} \right) R_{WD} \qquad (22)$$

to the averaged capture rate (17) one can finally evaluate the probability

$$P_{capt} = 2\pi \langle \Gamma \rangle \int_{-\infty}^{\infty} dz \int_{-\infty}^{\infty} dR R\, n_{BH}(R,z)\, n_{WD}(R,z)$$

$$\simeq 4 \times 10^{16} \Omega_{PBH} \left(\frac{10^3 kg}{M} \right) \frac{|\Delta E|}{E_{av}} yr^{-1} \qquad (23)$$

of BH capture by all the galactic WDs. Remind that a WD absorption by a BH can be observed by an accompanying neutrino burst [7]. We will proceed from the assumption that one-megaton neutrino water detectors will soon make it possible to reliably detect one such burst in ten years throughout the Galaxy. The dependence of the detectable mass fraction of 6D primordial BHs versus the 6D Planck mass is given in Fig. 5, where the BH mass was assumed to be equal to 6D Hawking mass M_{6Haw} (5). First of all, the process of WD absorption allows to set up constraints on the 6D primordial BH fraction in all the interval of 6D Hawking masses $1TeV < M_{6Haw} < 1PeV$ in which the 6D accretion "assists" the 4D one sufficiently to absorb WDs completely. However the lowest mass fraction of $\Omega_{PBH} \sim 10^{-16}$ can be detected at minimal possible 6D Planck mass $M_6 = 1TeV$ and BH mass $M \sim 1ton$.

Note also that the constraints on the abundance of the BHs with "unnatural" masses $M \lesssim M_{Sun}$ can also be interpreted in terms of that on the 6D BH mass fraction.

Many approximations have been made to establish the constraints illustrated in Fig. 5. Nevertheless, a more precise evaluation of 6D BH specific energy losses, of one-pass energy loss dependence on both BH impact parameter and WD matter distribution, of WD total number and mass distribution will hardly decrease more than an order of value the efficiency of obtained constraints follows mostly from both efficiency of 6D BH deceleration and WD matter accretion onto them as well as from the large allowable number of small mass 6D BHs.

CONCLUSIONS

Thus it was shown that the drastic modification of BH properties in the extra dimension presence makes possible extra-dimensional BH search by the method of celestial body absorption. First, the decrease of Hawking radiation intensity allows to avoid a complete evaporation of the extra dimensional BHs with masses many orders of value lower than the 4D Hawking mass. The lightest surviving are 6D BHs with masses from one to 10^5 ton at 6D Planck mass of from 1 TeV to 1 PeV.

The light survived extra dimensional BHs can dramatically accelerate their mass growth in the regime of extra-dimensional accretion proceeding until the Bondi accretion radius reaches the size of extra dimensions. However only the masses of 6D BHs can exceed the 4D Hawking mass and both avoid 4D Hawking evaporation and absorb WD in the regime of 4D Bondi accretion in the interval of 6D Planck masses from 1 TeV to 1 PeV. Note that 7D BH mass after extra-dimensional accretion completion slightly exceed the 4D Hawking mass only in the narrow 7D Planck mass interval near 1 TeV.

A key point for observation of WD absorption by microscopic BHs is the possibility of their capture. It was shown that the effective extra-dimensional BH deceleration caused by both the fast mass increase due to extra-dimensional accretion and energy transfer by the large extra-dimensional gravitational BH field to the surrounding WD matter allows 6D BHs to loose up to 20% of their kinetic energy they possess at interstellar distances at one passage through a WD. Such large one-pass energy loss allows considerable BH number to become gravitationally bound by WDs, sink then deep into their matter and absorb the WDs completely. Since the complete WD absorption by BH will has to be accompanied by both the huge neutrino burst and formation of a BH with unusually low mass $M \lesssim M_{Sun}$, the process of WD absorption by extra-dimensional BHs opens up the way of extra-dimension search at 6D Planck masses from 1 TeV to 1 PeV. The possibility of complete absorption a Solar-mass WD by a 6D BH of a ton mass allows to bound the 6D BH mass fraction by down to 10^{-16} of the current DM density.

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REFERENCES

1. S. B. Giddings, and M. L. Mangano, *Phys. Rev.* **D78**, 035009 (2008), arXiv:0806.3381.
2. S. Hawking, *Mon. Not. Roy. Astron. Soc.* **152**, 75 (1971).
3. G. Picchio, *Astron. Astrophys.* **99**, 31–35 (1981).
4. E. V. Derishev, V. V. Kocharovsky, and V. V. Kocharovsky, *JETP Lett.* **70**, 652–658 (1999), (Pisma Zh. Eksp. Teor. Fiz. 70, 642, (1999), in russian).
5. V. V. Tikhomirov, and S. E. Yuralevich, *Vestsi Natsiyanalnai Akademii Navuk Belarusi, Series of Physical-Mathematical Sciences* **4**, 73–78 (2001), (Proceedings of the National Academy of Sciences of Belarus, in russian).
6. V. V. Tikhomirov, and S. E. Yuralevich, *Vestnik BGU* **1**, 76–81 (2002), (Bulletin of Belarus State Uneversity, in russian).
7. V. V. Tikhomirov, and S. E. Siahlo, *Grav. Cosmol.* **11**, 229–234 (2005).
8. V. V. Tikhomirov, and V. V. Malyshchits, *Nonlinear Phenomena in Complex Systems* **10:3**, 303–308 (2007).
9. M. Roncadelli, A. Trves, and R. Turolla (2009), arXiv:0901.1093v1.
10. J. K. Becker, M. A. Abramowicz, and P. L. Biermann (2009), arXiv:0905.3078.
11. N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, *Phys. Lett.* **B429**, 263–272 (1998), hep-ph/9803315.
12. I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, *Phys. Lett.* **B436**, 257–263 (1998), hep-ph/9804398.
13. P. Kanti, *Int. J. Mod. Phys.* **A19**, 4899–4951 (2004), hep-ph/0402168.
14. S. L. Shapiro, and S. A. Teukolsky (1983), *Black holes, white dwarfs, and neutron stars: The physics of compact objects*, New York, USA: Wiley, 645 p.