Determination of the seabed porosity and shear modulus profiles using a gravity wave inversion

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SUMMARY
A previously developed Bottom Shear Modulus Profiler (BSMP) theory (Yamamoto & Torii 1986) allows the seabed shear modulus versus depth profile to be extracted by inverting measurements of seabed motion and water wave-induced pressures at one point on the seafloor. Preliminary BSMP experiments (Trevorrow et al. 1988b; Badiey et al. 1988), compared to existing geologic borehole data at two inner continental shelf sites, showed that this method can accurately predict the magnitude and depth structure of the sediment shear modulus. An improved version of BSMP instrumentation system has been developed and deployed in deeper water at outer continental shelf sites. Gravity water wave-induced bottom pressures are measured in a period band from 7 to 200 s, whereas measurements of gravity water wave-induced ground motion are limited to a period band between 7 and 30 s, due to the limited sensitivity of the seismometers used. It is also found that burial of a BSMP in the seabed improved the seabed seismometer coupling significantly. The BSMP inversions of these data extract the shear modulus versus depth profile of the seabed with a depth resolution of a few meters and penetrating as much as 200 m into the seabed. The shear modulus of a sediment at a given depth of burial is a unique function of the porosity of the sediment. Using this relation, the porosity versus depth profile of the seabed is calculated from the shear modulus profile obtained by the BSMP inversion. Excellent agreements are shown between the porosity profiles obtained from the BSMP inversion and the borehole porosity logs. The pressure data indicate that the potential penetration depth of the BSMP inversion is about 2 km into the continental shelf and 20 km into the deep seafloor.

Key words: seabed, porosity, shear modulus, inversion

1 INTRODUCTION
The determination of the physical properties of the seabed is an important and challenging problem of geophysics. Such information is badly needed for modelling the wave propagation in the ocean. Such information is also important for the design of foundations for offshore structures. Our understanding of geologic processes in various continental shelf provinces also requires such information.

Among all of the elastic moduli, the shear modulus is a particularly sensitive indicator of the skeletal structure of sediments. Sedimentological parameters like grain size, porosity, compaction, cementation, etc., are therefore closely related to shear modulus. Recently, the petroleum industry has actively pursued shear-wave seismology in land exploration because of this reason (Danbom & Domenico 1987). On the other hand, active shear-wave seismology has been limited in the marine environment partly because shear waves are difficult to excite acoustically in the marine sediments (Rauch & Schmalfeldt 1983). Recently, an entirely new passive geophysical inverse method called the Bottom Shear Modulus Profiler (BSMP) has been developed as an alternative to active seismic methods. The BSMP theory (Yamamoto & Torii 1986) enables seabed shear modulus profiles to be extracted by inverting measurements of seabed motion and water wave-induced pressure. Experimental verification of the BSMP theory has recently been made through comparisons between the BSMP inversion of wave-induced seabed motion measured by an OBS and existing borehole data at the inner continental shelf off the New Jersey coast (Trevorrow et al. 1988a,b) and at the Great Bahama Bank (Badiey et al. 1988). Due to the limited sensitivity of the seismometers used in these preliminary experiments, the penetration depth of BSMP inversion was limited to 50 m into the seabed at water depths up to 12 m.

An improved version of BSMP instrumentation system was built and tested on the outer continental shelf off the coast of New Jersey in the summer of 1987 (Turgut et al.
investigated by geotechnical engineers (for a summary of these works see, e.g. Richart, Hall & Edwards 1986) has a limited depth resolution of the order of several hundred meters while the airborne electromagnetic method (Won & Smits 1985) neither has a good depth penetration nor a good depth resolution. The BSMP method reported in this paper seems to have a better depth resolution than these electromagnetic methods. The experiments and the BSMP borehole comparisons are reported in this paper.

2 SHEAR MODULUS OF SEDIMENTS

The effects of depositional conditions of sediments on the dynamic characteristics of sediments have been extensively investigated by geotechnical engineers (for a summary of these works see, e.g. Richart, Hall & Woods 1970; Ishihara 1976) and geologists (e.g. Danbom & Domenico 1987; White 1983) in recent years. Although all of the elastic moduli have important effects on elastic wave propagation, the shear modulus is a particularly sensitive indicator of the shear resistance of sediments. Sediments behave as elastic material only when they are subjected to small strain amplitudes, on the order of 10^{-3} or less. The gravity water wave-induced seabed motion under usual circumstances falls into this category (Trevorrow et al. 1988a,b). In this paper we limit our attention only to the elastic behaviour of sediments.

A wealth of experimental data on the relations between the shear modulus and the depositional state of the un lithified sediments has been accumulated by geotechnical engineers (e.g. Hardin & Richart 1963; Kuribayashi et al. 1974; Hardin & Black 1968; Marcuson & Wahl 1972; Seed & Idriss 1970). For an excellent summary of their works and others, readers are referred to Richart, Hall & Woods (1970), Ishihara (1976) and Bryan & Stoll (1988).

The data indicate that the shear modulus, \( \mu \), of sediments is proportional to the square root of the confining effective stress, \( \sigma_0 \), and that the proportionality constant is a function of the void ratio, \( \varepsilon \), of the sediment. The void ratio, \( \varepsilon \), is related to the porosity, \( \beta \), by:

\[
\varepsilon = \frac{\beta}{1-\beta} \quad \text{or} \quad \beta = \frac{\varepsilon}{\varepsilon + 1}.
\]  

This relation holds for any type of un lithified sediments and may be expressed as

\[
\mu = F(\varepsilon)\sigma_0^{0.5}.
\]  

It is interesting to note that the elastic theory of face-centred spheres predicts \( \mu \) proportional to the cubic root of \( \sigma_0 \) (White 1983).

The amplitude function \( F(\varepsilon) \) has been given in the following form by aforementioned authors,

\[
F(\varepsilon) = a \left( \frac{b - \varepsilon}{1 + \varepsilon} \right)^2
\]

where

\[
a = 219 \times 10^3 (\text{Pa})^{0.5},
\]

\[
b = 2.17 \quad \text{for} \quad 0 < \varepsilon < 0.8 \quad \text{(Hardin & Richart 1963)}
\]

\[
a = 939 \times 10^3 (\text{Pa})^{0.5},
\]

\[
b = 2.97 \quad \text{for} \quad 0.6 < \varepsilon < 1.8 \quad \text{(Hardin & Black 1968)}.
\]

The confining effective stress \( \sigma_0 \) is related to the effective overburden pressure or the effective vertical stress \( \sigma_z \) by definition as

\[
\sigma_0 = \frac{1}{2} (\sigma_1 + \sigma_2) = \frac{1}{2} (1 + 2K_0)\sigma_z,
\]

\[
\sigma_x, \sigma_y \quad \text{and} \quad \sigma_z \quad \text{are the three orthogonal effective normal stresses. Here} \quad K_0 \quad \text{is the coefficient of earth pressure at rest, and is related to the internal friction angle} \ \phi_0 \quad \text{of the sediment strata} \quad (\text{e.g. Ishihara 1976}), \quad K_0 = 1 - \sin\phi_0.
\]

Since \( \phi_0 \) in natural sediment strata is about 30°, \( K_0 \) is approximately 0.5. The average value of the more than several hundred sediment cores collected from the continental shelf of the eastern USA is also about 30° (Hathaway et al. 1979). Therefore, it may be reasonable to assume \( \phi_0 = 30° \) or \( K_0 = 0.5 \).

The effective overburden pressure, \( \sigma_z \), at a given depth of burial, \( z \), in the sediment strata is given as

\[
\sigma_z = \int_0^z (\rho_s - \rho_l) \, dz,
\]

where \( g \) is the acceleration of gravity, \( \rho_s \) is the bulk density of sediment and \( \rho_l \) is the density of pore fluid (for sea water \( \rho_l = 1025 \, \text{kg m}^{-3} \)). \( \rho_s \) is given as

\[
\rho_s = \rho_s (1 - \beta) + \rho_l \beta,
\]

where \( \rho_s \) is the density of grain mineral and is equal to about 2650 kg m^{-3} for most sediments. The effective overburden pressure can then be rewritten as:

\[
\sigma_z = g(\rho_s - \rho_l) \int_0^z (1 - \beta) \, dz.
\]
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If the porosity versus depth profile of the seabed is known, the shear modulus profile of the seabed can be estimated using equations (2), (5) and (10). Conversely, if the shear modulus profile of the seabed is known, one can estimate the porosity profiles of the seabed using equations (2), (5) and (10) as well. In the following, we will demonstrate examples of these forward and inverse processes using the borehole porosity profile logs and the shear modulus profiles obtained by the BSMP method.

3 BSMP INSTRUMENTATION AND EXPERIMENTS

In order to improve the penetration depth and depth resolution of the BSMP inversion, an improved BSMP instrumentation system called the High Resolution (HR-) BSMP has been developed. A complete report on the instrumentation is given by Turgut et al. (1987).

An electronic block diagram of the HR–BSMP system is shown in Fig. 2. The seismometer package consists of three orthogonally mounted seismometers (Teledyne-Geotech Model S-750) and two tiltmeters (Sperry Model 02383-01). The frequency response of the S-750s is flat between 0.01 and 100 Hz. However, the low frequency limit of the S-750s turns out to be about 30 s for our application, due to internal noise levels. A new differential pressure sensor was specifically designed for the measurement of small amplitude, low frequency (periods of 1–125 s) wave-induced pressures in up to 200 m water depth. The seismometer package is connected to the pressure sensor by a 2 m, 12-conductor umbilical cable. A 32-conductor, electro-mechanical cable connects two HR–BSMP sensor units to an accompanying ship. The shipboard data recording system consisted of band pass filter amplifiers followed by a multi-channel FM data recorder.

As shown in Fig. 3, the HR–BSMP can be deployed either in a buried or plate-mounted configuration. Burial of the seismometer housing is achieved by use of a hydraulic jet burial system developed by the authors' group (Turgut et al. 1987). Basically, the burial system consists of a three-pronged burial bracket that holds the seismometer housing. The pressure sensor housing is mounted outside the burial dome, and remains on the seabed during the data collection. An electro-magnetic release mechanism, controlled from the ship, uncouples the two housings from the burial dome when the unit reaches the seabed. Seawater is pumped at high pressure through the prongs of the burial bracket, causing water jets at the tips of the prongs to liquify.
the sediments and excavate the sediments from underneath the seismometer housing. The burial bracket and seismometer housing sink under their own weight into the sediments. After burial, the dome and bracket are hauled back up to the ship so as not to interfere with the seabed motions and pressures.

After burial or deployment, the BSMP units are allowed to 'settle' into the sediments for a period of several hours before the start of data recording. The BSMP-to-instrument coupling is a very important aspect of the seismometer deployment. It should be noted that seabed-to-BSMP resonance effects are not anticipated due to the low frequencies of the water wave-induced bottom motions. Trevorrow et al. (1989) discusses the sediment coupling characteristics of the HR–BSMP.

A total of five HR–BSMP arrays were deployed at five sites on the New Jersey Shelf in water depths ranging from 12 to 135 m, using the R/V Atlantic Twin in 1987 August. Two of these sites were selected at the 1976 Atlantic Margin Coring (AMCOR) Project (Hathaway et al. 1976, 1979) boreholes AMCOR 6009 and 6010 on the outer continental shelf off the coast of New Jersey.

4 OBSERVATIONS OF WAVE FIELDS

The AMCOR 6010 and 6009 sites are located approximately 120 km off the coast of New Jersey. At the AMCOR 6010 site (N 39°03', W 73°07', water depth 70 m), one BSMP (303) was buried and another (302) was deployed on a plate. The separation between the two BSMP was approximately 30 m and the array axis is oriented N 220°. Waves were travelling roughly from the NE.

The method of data analysis is basically the same as in previous experiments (Trevorrow et al. 1988a). A small portion of the BSMP raw data set of an analogue recording of the three seismometer acceleration and pressure transducer signals from BSMP 303 is shown in Fig. 4. Typically, several (as many as seven) instruments were deployed and recorded simultaneously, for durations as long as 15 hr (usually 8 hr). The basic goal of the data processing algorithms is spectral averaging. This reduces the acceleration and pressure time-series into a form useful to the shear modulus inversion algorithm, and drastically reduces the effects of ambient, random noise. The pressure and three-component acceleration time-series are digitized at 4 Hz and divided into 1024 s (17 min) segments. Each segment is converted into power spectra, cross-spectra and horizontal and vertical admittance spectra. Seabed admittance is defined as the ratio of seabed displacement to water wave amplitude at the surface. The power, cross and admittance spectra from each time segment are averaged over all the segments in the entire data set.

The power spectra, coherence and the phase lag between the pressure signals from the two BSMPs are shown in Fig. 5. Notice that the two power spectra are almost identical. For a wide band of wave periods between 8 s to at least 200 s, the coherence between the two pressure signals is very high. For the same wave period band, the phase varies continuously from about 90° to 10°. Note, that the wavelength of a surface gravity water wave with period of 8 s is 100 m. The phase lag of 90° corresponds to a quarter of a wavelength, which is about 25 m. As the incident ocean swell propagates nearly parallel to the BSMP array axis, this confirms that the pressure signals are due to freely

![Figure 2. Electronic block diagram of the High Resolution-Bottom Shear Modulus Profiler (HR–BSMP) system.](image-url)
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(a) Buried Configuration

(b) Burial System

(c) Plate-mounted Configuration

Figure 3. HR-BSMP seabed deployment configurations. (a) Buried in the sediment, (b) hydraulic jet seismometer burial system, (c) plate-mounted.

propagating gravity water waves. More sophisticated analysis of directional spectra of wave fields using a six-point array made at a different site substantiates this observation (Goodman et al. 1989).

It is most significant to note that the low frequency pressure signals as low as 5 mHz are due to travelling gravity waves. Pressure measurements in the deep ocean (e.g. Webb & Cox 1984) showed that even in the deep ocean the bottom pressure at frequencies lower than 30 mHz is clearly due to the surface gravity water waves. A 5 mHz travelling water wave has a wavelength of 5300 m in 70 m of water depth and a wavelength of 40 000 m in 4000 m of water depth. The depth of penetration of the BSMP inversion method is approximately one-half of the longest wavelength used in the calculation. Therefore, if the seabed motion due to the long water waves can be accurately measured, the BSMP method can be used to extract the shear modulus and porosity structures of the seafloor as much as about 2.5 km into the continental shelf and about 20 km into the deep ocean floor. Long-period seismometers (e.g. Gurlap S-3) would solve this problem.

The power spectra of the vertical acceleration, the coherence and the phase lag between the vertical acceleration and the pressure, and the admittance spectra from BSMP 303 are shown in Fig. 6. The coherence is fairly high in the microseismic band (periods less than 3 s) and nearly perfect in the surface wave band (periods between 8 and 20 s). Inside the water wave band (8–20 s) the
The admittance magnitude and phase become smooth and relatively noise free. The admittance spectral components in this high-coherence range are used in the inversion process to extract the shear modulus profile. The phase spectra approaches 180°, indicating a very nearly lossless, elastic seabed response to the water wave pressures. For the long wave band (with periods between 20 and 200 s), although there is a small coherence between the vertical acceleration and pressure, the coherence is not as high as that for the two pressure signals because of the limited sensitivity of seismometers at this frequency range. Through the use of the more sensitive seismometers discussed earlier, the seabed admittance spectra may be accurately measured at these ultra low frequencies. Although not shown here, coherence is lost significantly for the plate-mounted BSMP. This indicates that the BSMP-seabed coupling is important at this water depth, and is significantly improved by burial. Complete reports on the spectral analysis and BSMP-coupling are given by Trevorrow et al. (1988a, 1989).

5 COMPARISONS OF BSMP INVERSIONS WITH BOREHOLE DATA

In 1976, the US Geological Survey conducted a 60-day expedition called the Atlantic Margin Coring (AMCOR) Project in order to obtain core samples by drilling beneath the floor of the continental shelf and slope of the eastern USA aboard the D/V GLOMAR CONCEPTION (Hathaway et al. 1976, 1979; Richards 1977). The coring...
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Figure 6. Seismic and pressure data from AMCOR 6010 site, HR-BSMP 303: (a) vertical acceleration power spectrum, (b) coherence spectra between pressure and vertical admittance, (c) vertical admittance spectrum, (d) vertical admittance phase spectrum.

Figure 7. Comparison of inverted shear modulus profile from HR-BSMP 303 with reference profile calculated from measured borehole porosity profile of Fig. 6. Inset shows comparison of vertical admittance data (with error bound) and calculated values from the inverted profile. RMS admittance mismatch error is 8.0 per cent. Eigenvalue expansion limit is 4.

Figure 8. Stratigraphic column and porosity profile measured from cores and density log of AMCOR 6010 borehole (reproduced from Hathaway et al. 1976, 1979 and Richards 1977). Porosity profile converted from BSMP inverted shear modulus profile of Fig. 7 is also plotted for comparison.
penetrated as much as 310 m below the seafloor at 19 sites in water depths ranging from 20 to 300 m. Porosity, among other properties, was measured from the cores collected. Borehole logs, including density logs and resistivity logs, were also taken. The internal friction angles $\phi_0$ of the sediment cores are also measured. The average value of $\phi_0$ is approximately 30° ($K_0 = 0.5$) which is used in calculations of the overburden pressure in equation (6). We compare the BSMP inversion results with the AMCOR boring data at AMCOR 6010 and 6009 sites in Figs 7 through 10.

The validity of such comparisons depends on the accuracy of the AMCOR porosity data (Figs 8 and 10) and the uniqueness of the shear modulus profile results (Figs 7 and 9) from the BSMP inversion scheme. As to the AMCOR porosity profiles, we had no control over the measurements made in 1976. However, because the porosity profiles measured from cores are consistent with the borehole density logs (Richards 1977), the depth variations in the AMCOR porosity data must be real and not due to the measurement uncertainty. The credibility of the shear modulus profile calculated by the BSMP inversion must be assessed from the uniqueness of the inversion results obtained from the measured admittance data used in the BSMP inversion. Therefore, we briefly discuss the uniqueness of the BSMP inversion on the measured admittance data in the following.

The geophysical inverse methods used to perform this calculation are outlined in Trevorrow et al. (1988b), and will be only briefly summarized here. The input data to this inversion scheme are 20 discrete spectral components of vertical admittance, taken for example from Fig. 6(c). The 20 data points (e.g. Fig. 7, inset) are chosen on the basis of highest coherence and lowest uncertainty, which in this case limits the good data to the wave period band from 7 to 20 s.

Since the relation between admittance and shear modulus (the forward problem) is non-linear, we use the usual linearization-iteration procedure, with the inverse of the differential matrix calculated using a truncated singular value decomposition (SVD). Not surprisingly, this method suffers from the usual constraints of resolution and uncertainty. Since the porosity calculation follows directly from the shear modulus profile, the porosity profile must have similar resolution and uncertainty characteristics.

Resolution is the ability of the method to perceive fine structures (variations less than a few meters) in the shear modulus profile. The use of the truncated SVD expansion is at the heart of this concept. The lower-order SVD eigensolutions give more smoothed and averaged shear modulus profiles, whilst the finer details in the profile are contained in the more unstable, higher-order eigensolutions. It is impossible to extract a stable, convergent solution using the full SVD expansion. Also, the uncertainty bounds on the shear modulus profile increase dramatically with retaining more terms in the expansion. The necessity of truncation, therefore, results in an unavoidable loss of resolving power. With these BSMP data sets it is typical to retain only four terms in the SVD expansion, which enables the method to resolve sediment structures a few meters in thickness.

Also, it has been found that resolution is degraded at depths greater than one-half of the longest wavelength available in the admittance data set. This limits the maximum penetration of the method. In the current example (Fig. 7, inset) the longest wavelength (20 s in 70 m of water) is 462 m, thus limiting the penetration to 200 m.
different final results both satisfying the data) does not seem to be a problem with this particular inversion algorithm. Uniqueness was tested by performing the inversion with several different shapes and magnitudes of initial guess. They were all found to converge to within the quoted error bounds on the final shear modulus profiles, to be shown in Figs 7 and 9. Smooth, parabolic initial profiles are used to avoid imposing any arbitrary structures on the final results. Although such testing is not rigorous and can never be exhaustive, we feel satisfied that non-uniqueness is minimal in this inversion algorithm.

The inverted shear modulus profile within the top 200 m of sediment at the AMCOR 6010 site is shown in Fig. 7. The error bound shown is $\pm 1$ SD, and comes from the combination of admittance data uncertainty and the use of the truncated SVD expansion. The first four eigenvalues are retained in the final result. The admittance data used as input to the inversion are compared with the computed admittance values in the inset of Fig. 7. An error bound of $\pm 1$ SD is shown on the admittance data. Using the conversion scheme outlined in section 2, the AMCOR borehole porosity data (shown in Fig. 8) is converted to a shear modulus profile and compared to the inverted result. The inverted BSMP result appears to be an averaged version of the alternating thin layers of sand and clay. None-the-less, the overall structures extracted by the BSMP method agree very well with the direct borehole profiles.

The borehole porosity profile and the stratigraphic column of AMCOR 6010 site for the topmost 200 m of the seabed are shown in Fig. 8. 'The section consists of 310 m of Miocene through Pleistocene dark gray silty clays and gray to olive sands, usually fine graded' (Hathaway et al. 1976). The inverted BSMP shear modulus profile is converted into a porosity profile and compared with the borehole porosity data. This is essentially the same comparison as Fig. 7, but serves to demonstrate that the methods outlined in Section 2 can work in both directions. The error bound on the inverted porosity profiles comes from the use of equations 2 and 5 to calculate the variation in porosity due to the uncertainty bounds on shear modulus shown in Fig. 7. As in Fig. 7, there is good overall agreement in the two results, bearing in mind the 'depth-averaged' characteristic of the inversion results.

Similar comparisons between the BSMP inverted profiles, the borehole profiles and the stratigraphic column of the borehole at AMCOR 6009 (N 38°51', W 73°36', water depth 58 m) site are shown in Figs 9 and 10. 'The section contains 300 m of Miocene through Pleistocene sands and silty clays, usually dark gray or dark olive gray in color. Shell fragments and probably glauconite grains are common in the lower part of the sections.' (Hathaway et al. 1976). Excellent overall agreements between the BSMP profiles and borehole profiles are shown for this site also.

6 CONCLUSIONS

The BSMP inversion method using surface gravity wave-induced bottom motion can accurately determine the magnitude and depth structures of the sediment shear modulus and porosity. Good agreement is obtained between experimentally measured, inverted shear modulus and porosity profiles and borehole data. Empirical relations between the void ratio and shear modulus are used to convert shear modulus profile to a porosity profile or vice versa. The BSMP inversion penetrated as much as 200 m below the seafloor with a good depth resolution, in water depth ranging up to 135 m.

Long-period surface gravity waves with period up to 200 s have been observed by a pressure sensor array. With improved seismometers, the potential penetration depth of the BSMP inversion therefore is about 2 km into the continental shelf and 20 km into the deep ocean floor.

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