THE IMPACT OF MONOPOLY PRICING ON THE LERNER SYMMETRY THEOREM: A COMMENT*

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In an article appearing in this Journal, Edward John Ray [1975] claims to show that, when a domestic producer has monopoly power in the world market, the symmetry between import and export tariffs found by Lerner for the competitive case no longer obtains. This result follows from an even more surprising claim, that in the presence of monopoly "supply functions are no longer homogeneous of degree zero in output prices" [p. 597]. If this result were true, not only the Lerner symmetry theorem but also a large number of results in international trade and general equilibrium theory would need to be qualified under conditions of monopoly pricing.

The purpose of this comment is to show that Ray's result can only obtain if demand functions lack the property of zero homogeneity. First, we prove that, if demand functions are homogeneous, in a nonmonetary economy changes in nominal prices alone cannot have real effects. The Lerner symmetry theorem is a straightforward application of this orthodox result, and therefore must apply independently of whether or not any industry possesses monopoly power. Second, in view of the widespread potential implications of Ray's result and the fact that it has been quoted uncritically elsewhere [Hazari, 1978, pp. 168-75; Auquier and Caves, 1979], we find it important to identify the source of Ray's error explicitly by showing exactly where he introduced the assumption of nonhomogeneity. Finally, we conclude with a brief discussion of the applicability of the Lerner symmetry theorem in the context of a monetary economy.

Following Ray, we assume that the import-competing industry in the home country is monopolistic, and that the country is sufficiently large in world markets to affect the world price of the commodity. Domestic demand for commodity 1 is given by

\[ D_1 = D_1(P_1, P_2, w, r), \]

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where \( P_1 \) denotes the price of commodity 1, \( P_2 \) the price of commodity 2, \( w \) the wage rate, and \( r \) the rental rate on capital, all in nominal terms. As is well-known from demand theory, \( D_1 \) is homogeneous of degree zero in all its arguments [Malinvaud, 1972, p. 34]. By assumption, all production in the rest of the world takes place under competitive conditions. It follows from Ray's own analysis of the competitive case that net imports of commodity 1 from the rest of the world are a function,

\[
M_1(P_1, P_2),
\]

which is homogeneous of degree zero in \( P_1 \) and \( P_2 \). The residual demand facing the monopolistic industry 1 in the home country is given by \( X_1 \), where

\[
X_1(P_1, P_2, w, r) = D_1(\cdot) - M_1(\cdot).
\]

It follows that \( X_1 \) is homogeneous of degree zero in \( P_1, P_2, w, \) and \( r \). We can thus restrict our analysis to the function,

\[
X_1\left(\frac{P_1}{P_2}, \frac{w}{P_2}, \frac{r}{P_2}\right) = X_1\left(\frac{P_1}{P_2}, 1, \frac{w}{P_2}, \frac{r}{P_2}\right),
\]

with no loss of generality.

Ray defines the term \( \eta_1 \) as “the absolute value of the own price elasticity of the residual demand facing domestic producers” [p. 596]. It may be written as

\[
\eta_1(P_1, P_2, w, r) = -\frac{\partial X_1}{\partial P_1} \frac{P_1}{X_1},
\]

where \( P_2, w, \) and \( r \) are held constant.

It is straightforward to show that \( \eta_1 \) is homogeneous of degree zero in \( P_1, P_2, w, \) and \( r \). Define \( S_1 = P_1 X_1 \). Since \( X_1 \) is homogeneous of degree zero in all prices, \( S_1 \) is homogeneous of degree one. Any partial derivative of \( S_1 \) is therefore homogeneous of degree zero [Allen, 1938, p. 317]. Since

\[
\frac{\partial S_1}{\partial P_1} = X_1(1 - \eta_1)
\]

and \( X_1 \) is homogeneous of degree zero, \( \eta_1 \) must be as well.

Consider now the conditions for profit maximization. Equating the marginal revenue products of each factor across sectors implies, as Ray reports [p. 596], that

\[
P_1(1 - (1/\eta_1))[f(k_1) - k_1 f'(k_1)] = P_2[g(k_2) - k_2 g'(k_2)]
\]
(7) \[ P_1(1 - (1/\eta_1))f'(k_1) = P_2g'(k_2), \]

where \(k_1\) and \(k_2\) denote the relevant sectoral capital-labor ratios, and \(f\) and \(g\) denote outputs per worker in industries 1 and 2, respectively. These two equations determine \(k_1\) and \(k_2\) as functions of \(P_1, P_2,\) and \(\eta_1\). Armed with the knowledge that \(\eta_1\) is homogeneous of degree zero in \(P_1, P_2, w,\) and \(r\), it is evident that profit-maximizing values of \(k_1\) and \(k_2\) are as well.\(^1\)

Competition in factor markets implies, as Ray also reports [p. 596], that

(8) \[ w = P_2[g(k_2) - k_2g'(k_2)] \]

(9) \[ r = P_2g'(k_2). \]

Since \(k_1, k_2,\) and \(\eta_1\) are homogeneous of degree zero in \(P_1, P_2, w,\) and \(r,\) conditions (8) and (9) imply that \(w\) and \(r\) are homogeneous of degree one in \(P_1\) and \(P_2.\) Thus, the zero homogeneity of \(\eta_1\) is inconsistent with Ray's claim that, with monopoly, "factor prices are no longer homogeneous of degree one in output prices alone" [p. 597].

Consider now an initial equilibrium that is perturbed by increases in \(P_1\) and \(P_2\) of equal proportion. If \(w\) and \(r\) change in proportion, allowing \(k_1\) and \(k_2\) to remain unchanged, none of the profit-maximizing conditions (6) through (9) is disturbed since \(\eta_1\) is unaffected. Since capital-labor ratios, factor supplies, and full employment are unaffected, so are domestic output levels. Outputs are therefore functions of \(P_1\) and \(P_2\) that are homogeneous of degree zero, just as in the competitive case.

Mathematically, Ray's error is introduced in his equations (18), (20), and (22), where he fails to incorporate the term \(d\eta_1/dP_2,\) implicitly setting it equal to zero. Differentiating (4) appropriately implies that

(10) \[ \frac{d\eta_1}{dP_1} = -\frac{\partial^2 \tilde{X}_1}{\partial P_i^2} \frac{(P_1/P_2)^2}{X_1\eta_1} + 1 + \eta_1 + \sum_{i=1}^{2} \frac{d\eta_1 s_i/P_2}{d\eta_1} \left( \frac{d\eta_1}{P_1 s_i} \right) \]

(11) \[ \frac{d\eta_1}{dP_2} = \frac{\partial^2 \tilde{X}_1}{\partial P_i^2} \frac{(P_1/P_2)^2}{X_1\eta_1} - 1 - \eta_1 \]

\[ + \sum_{i=1}^{2} \frac{d\eta_1 s_i/P_2}{d\eta_1} \left( \frac{d\eta_1}{P_1 s_i} \right), \]

where \(s_1 = w, s_2 = r.\)

1. Consider an initial equilibrium perturbed by increases in \(P_1, P_2, w,\) and \(r\) of equal proportion. Since \(\eta_1\) is unaffected, the values of \(k_1\) and \(k_2\) that satisfy the initial equilibrium also satisfy the new one.
As we have already argued, $w$ and $r$ are homogeneous of degree one in $P_1$ and $P_2$. Thus, (10) and (11) must sum to zero. Ray has introduced nonhomogeneity by allowing the elasticity $\eta_i$ to vary with $P_1$ but not $P_2$. This is inconsistent with demand theory which predicts that $\eta_i$ will not be affected by changes in the nominal price level when relative prices are unchanged. The degree of monopoly power, in other words, is determined by relative rather than absolute prices.

As Ray himself shows, once the zero-homogeneity of domestic production in world prices is established, the Lerner symmetry theorem follows [p. 594]. Thus, the presence of monopoly does not affect this result. More generally, the result that only relative prices matter in nonmonetary models extends to situations in which monopoly elements are present. The number of interesting prices is still one less than the number of commodities.

A final question is to what extent the Lerner symmetry theorem remains applicable in the context of a monetary economy with monopoly pricing. As recent papers by Kaempfer and Tower [1980] and Anderson and Takayama [1980] have shown, import taxes and export taxes continue to be symmetrical in a monetary context whenever they give rise to identical real balance effects. Thus, in a world of flexible exchange rates, the exchange rate depreciation automatically brings about symmetry. Alternatively, if rates are fixed, an import tax is equivalent to an export tax accompanied by a devaluation of the same percentage. These results require only that demand functions be homogeneous of degree zero in all nominal variables, and therefore they too are independent of whether or not any industry possesses monopoly power.

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REFERENCES

