Formation Process of the Traveling-Wave State with a Defect in Binary Fluid Convection

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We give results of simulations on the transient evolution from conduction to an extended traveling wave (ETW) state containing a sink defect slightly above the onset of the subcritical instability for a binary fluid mixture with the separation ratio $\psi = -0.47$ confined in a rectangular cell of aspect ratio $L = 46$. The simulation was performed using the two-dimensional full fluid equations of motion. We find that the system starting from a conduction state first experiences a linear stage with independently counterpropagating wave (CPW) packets and then develops a modulated traveling wave (MTW) and a traveling wave (TW) containing transient sink and source defects. It then ultimately evolves into a TW with a stable sink defect similar to those observed in experiments. For our parameters, the system evolves into the nonlinear regime as the lateral concentration profile becomes a trapezoidal, strongly non-harmonic shape in the localized regions. Comparisons with other calculations and experiments are presented.

§1. Introduction

Experiments on convection in a binary fluid mixture in long and narrow cells have revealed a wealth of dynamical features of one-dimensional traveling wave (TW) convection such as a localized traveling wave (LTW) and an extended traveling wave (ETW) with source or sink defects.$^{1-11}$ Various types of dynamical behavior such as the transition from a TW to stationary overturning convection (SOC),$^{12}$ the transition from an LTW to an ETW,$^{13}$ the transition from an ETW to a double localized traveling wave (DLTW),$^{4,5,14}$ the transition from conduction to an LTW or an ETW,$^{1,3,8,11,15}$ and the motion of a source defect caused by heating one of the end walls,$^{16,17}$ have also attracted considerable interest.

The transition to TW convection from conduction at the onset of the oscillatory instability at which the system undergoes an initial subcritical bifurcation has been studied in experiments.$^{1,3,8,11,15}$ In general, the system evolves into a counterpropagating wave (CPW) at the onset of the oscillatory instability,$^{2,6,18-20}$ where the CPW can be either a linear or nonlinear state depending on parameters. With the growth of the convective amplitude, the system can appear in the form of several kinds of distinct patterns until the appearance of a stable TW pattern.$^8$ In our experiment of a slot channel cell with aspect ratio $L = 46$ and separation ratio $\psi = -0.47$, a similar linear stage with a fast TW and transient source or sink structure has been observed.$^{4,5,21,22}$ The simulation by Barten et al. indicates that the
system can end up in an LTW or an ETW state, depending on initial conditions and driving history.\textsuperscript{23,24} Kolodner et al. further suggested that defects can be produced by applying small perturbations to a flow field and by controlling the growth rate of a linear TW through the Rayleigh number.\textsuperscript{11} These indicate that the transition process from the conductive to convective TW state is quite complicated and plays an important role in generating stable patterns, such as an ETW containing defects or an LTW.

In a previous paper\textsuperscript{14} referred to as I below, we numerically detected a DLTW state in the region near the turning (or saddle-node) point on the upper branch of the average convection amplitude curve plotted against the Rayleigh number $R$. To obtain the DLTW state we have taken the ETW state at some higher value of $R$ as the initial state and have started the simulation by lowering $R$ from this value to such a degree that the system comes to remain at the value of $R$ just above the turning point. The purpose of this paper is to report the results of our simulations concerning the manner in which the ETW state taken as the initial state in I has been prepared starting from the conduction state superimposed with a slight amount of convective perturbation.

Using nonlinear perturbation equations\textsuperscript{19,25} not only can we obtain the onset values of convection which are calculated using linear stability analyses,\textsuperscript{26-28} but also we can simulate the growth of the convective amplitude and the transition from the conductive state to the stable convective patterns. In order to further clarify the detailed behavior of the transition, we study the time evolution of convection patterns above the onset of convection in a rectangular cell with $r = 46$ and $\psi = -0.47$ by numerically solving the nonlinear perturbation equations and comparing our results with experiments.

This paper is organized as follows. In §2 we give the equations of motion, the method for solving them and boundary conditions. In §3 we present our numerical results. In §3.1, after determining the onset of the oscillatory instability at which the system undergoes an initial subcritical bifurcation, we determine the quasilinear wave packets with linear growth rates and the oscillatory frequency near the onset of convection in a convectively unstable regime observed in the simulation (Figs. 1 and 2) and two kinds of spatiotemporally modulated traveling wave (MTW) states generated by the interaction and superposition of the TW and its reflected wave (Figs. 3 and 4) with the growth of the amplitude. In §3.2, we first show how the system evolves from the quasilinear wave shown in Fig. 5 to the nonlinear regime with the further growth of the convective amplitude. The main characteristic of this process is that the concentration profile becomes a trapezoidal, strongly nonharmonic shape in the localized region, and the corresponding phase velocity begins to change significantly in space-time, as shown in Figs. 6 ~ 8. We next give the generating process of defects (Fig. 9) due to the competition between TW states with different phase velocities and propagating directions, and finally give the transition process to a stable TW shown in Figs. 10 ~ 12. In §4, we compare our simulation for $r = 46$ with experiments and with our simulation for $r = 12$. The results show that the linear stage and the stable defect state may depend on $r$. This ultimate TW state with a sink is the one we have taken as the initial state in I.
§2. Governing equations and numerical method

We consider a horizontal layer of a binary fluid mixture under a homogeneous gravitational field confined between two horizontal plates heated from below. The system is approximated as consisting of two-dimensional convection in the form of straight rolls by ignoring spatial variation along the roll axes. Let the origin of coordinates be located at the point where the bottom plate intersects the left sidewall at right angles in the experimental cell, and let $x$ and $z$ denote the Cartesian coordinates perpendicular to the roll axis with $z$ directed upward. Let the non-dimensional dynamical variables for convection be denoted by the velocity $u_x$ and $u_z$, the temperature $\theta$ and the combined variable $\eta = \xi + \theta$ instead of the mass concentration $\xi$ of component 1. Using the Oberbeck-Boussinesq approximations and neglecting the Dufour effect, the perturbation equations to the steady thermal conducting state can be written in dimensionless form as

\begin{align*}
\partial_t u_i + u_j \partial_j u_i &= \Delta u_i - \partial_i \left( \frac{D}{\rho_0} \right) + \frac{1}{Pr} \left[ (1 + \psi) \theta - \psi \eta \right] \delta_{i,z}, \quad (i = x, z) \quad (2\cdot1)
\end{align*}

\begin{align*}
\partial_j u_j &= 0, \quad (2\cdot2)
\end{align*}

\begin{align*}
\partial_t \theta + u_j \partial_j \theta &= \frac{1}{Pr} \Delta \theta + Ru_z, \quad (2\cdot3)
\end{align*}

\begin{align*}
\partial_t \eta + u_j \partial_j \eta &= \frac{1}{Pr} \Delta \eta + \frac{L}{Pr} \Delta \eta, \quad (2\cdot4)
\end{align*}

where $\delta_{i,z}$ is Kronecker's delta, all lengths are scaled by the thickness of the fluid layer $d$, the velocities $u_x$ and $u_z$ by $\nu/d$, the time $t$ by $d^2/\nu$, the temperature $\theta$ by $(\nu \gamma_2)/(\alpha g d^3)$, the concentration $\xi$ by $(\nu \gamma_2)/(\alpha g d^3 D)$, and the pressure $p/\rho_0$ by $\nu^2/d^2$. Here, $\nu$ is the kinematic viscosity, $\kappa$ is the thermal diffusivity, $D$ is the concentration diffusivity, $g$ is the acceleration due to gravity, $\alpha$ is the thermal expansion coefficient at constant pressure and concentration, $\alpha = -(\partial \rho(T, p, C_0)/\partial T)/\rho_0$, $\gamma_2 = S_T C_0 (1 - C_0) D$, $S_T$ is the Soret coefficient, $T$ and $C$ are the local temperature and concentration including the conductive part, and $\rho_0$, $C_0$ and $T_0$ are the mean mass density, concentration and temperature, respectively.

This system is characterized by four dimensionless parameters, namely, the Rayleigh number $R = (\alpha g d^3 \Delta T)/(\kappa \nu)$, which is proportional to the temperature difference $\Delta T$, the Prandtl number $Pr = \nu/\kappa$, the Lewis number $L = D/\kappa$, and the separation ratio $\psi = (\beta \gamma_2)/(\alpha D) = S_T C_0 (1 - C_0) \beta/\alpha$ with $\beta = (\partial \rho(T_0, p, C)/\partial C)/\rho_0$, which is the solute expansion coefficient at constant temperature and pressure. The parameter $\psi$ is a measure of the coupling between the temperature and concentration gradient induced by the Soret effect. For convenience, the reduced Rayleigh number $r = R/R_{co}$ is used below, where $R_{co} = 1708$ is the Rayleigh number at the onset of convection for a pure fluid.

For convenience, below we use $u$ and $w$ instead of $u_x$ and $u_z$, respectively. The velocity, the temperature and the concentration respectively obey the rigid (or non-slip), the isothermal and the impermeable boundary conditions at the lower and upper walls of the cell so that $u = w = \partial_z w = \theta = \partial_z \eta = 0$ at $z = 0$ and 1.
While at the lateral walls the velocity and the concentration obey the same boundary condition as at the horizontal walls, the temperature is assumed to obey the adiabatic boundary condition so that $u = \partial_x u = w = \partial_z \theta = \partial_x \eta = 0$ at $x = 0$ and $\Gamma$. The initial conditions were determined by assuming that the flow takes the form of parallel rolls whose wavelength is twice the thickness of the fluid layer and that the initial rolls have small-amplitude envelopes which follow the shape of a Gaussian function. To be more specific, the peak position of the Gaussian function is located at $x = x_0 = 25$. This gives rise to an asymmetry of the convection patterns with respect to the vertical center plane of the cell, as is shown in §3. In simulation, the aspect ratio is chosen as $\Gamma = 46$, and the fluid parameters are $\psi = -0.47$, $Pr = 13.8$ and $L = 0.01$, in conformity with to experiments.\(^5\)

To numerically solve the governing equations, we have used the same procedures as in I: We have used the MAC finite-difference procedure consisting of the staggered mesh system with the uniform spatial resolution $\Delta x = \Delta z = 1/16$, where the central difference formula is used for all spatial derivative terms in Eqs. (2·1) \sim (2·4), and the forward difference is used for the time derivative terms. A time step as small as $\Delta t = 0.0005$ is used for obtaining stable solutions. The pressure equation derived from Eqs. (2·1) \sim (2·4) has been solved using the ICCG method (incomplete Cholesky conjugate gradient method).

§3. Numerical results

3.1. Quasilinear wave and convective instability

In order to obtain the critical Rayleigh number at the onset of convection, we have integrated the nonlinear perturbation equations derived from full hydrodynamic equations under laterally periodic boundary conditions using the finite difference method. For the wavenumber $k = 3.14$, we find that the maximum amplitude in the space $(x, z)$ as a function of time oscillates and increases (resp. decreases) at $r = 1.875$ (resp. 1.865). Experiments using binary fluid mixtures\(^{29}\) and numerical simulations of pure fluid layers\(^{30}\) have all shown that the onset shift of convection due to a finite length of $\Gamma$ becomes small with the increase of $\Gamma$. Therefore, for a large aspect ratio such as $\Gamma = 46$, we can neglect the effect of the onset shift and choose $r_{osc} = 1.875$ as the approximate onset of the oscillatory instability at which the system undergoes an initial subcritical bifurcation to a TW state.

As mentioned above, if a single perturbation of the Gaussian function with a very small amplitude is given, the convection amplitude grows above the onset value of $r$. For $r = 2.1$, the perturbation quickly splits into independent CPW packets, and it rapidly grows with a growth rate proportional to $\varepsilon = (r - r_{osc})/r_{osc}$ as they propagate, as shown in Fig. 1. In order to examine the features of the TW packets, we calculate characteristic parameters such as the growth rate and the phase velocity. Figure 2 displays the maximum amplitude in the space $(x, z)$ as a function of time. When $t > 9$, the maximum amplitudes in both wave packets independently grow exponentially with the linear growth rate until reflection from both end walls occurs.
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Fig. 1. Time evolution from time 0 to 20 in units of $d^2/\nu$ of the temperature field $\theta(x, z = 0.5)$ in the independent CPW packets for a system with $r = 46$ and $\psi = -0.47$ at $r = 2.1$. After a single perturbation of the Gaussian function with a very small amplitude as an initial value is given, the perturbation grows and quickly splits into independent CPW packets with linear growth rates in a convectively unstable regime.

The maximum amplitude in this stage can be expressed as

$$A_{\text{max}} \propto \exp(\gamma_m t). \quad (3.1)$$

Thus the maximum growth rate $\gamma_m$ can be defined as

$$\gamma_m = \frac{d[\ln(A_{\text{max}})]}{dt}. \quad (3.2)$$

For the data shown in Fig. 2, we can obtain $\gamma_m = 0.0775$ over the range of $t$ from 9 to 52, where $\gamma_m$ is scaled by $\nu/d^2$. For the oscillatory frequency, we find that in our numerical simulation (denoted below by the suffix nn) it maintains a constant value of $\omega_{\text{nn}} = 1.40$ when $t \leq 52$, while an approximate calculation using linear stability theory (denoted by the suffix lt) for TW$^2$ shows $\omega_{\text{lt}} = 1.36$. Furthermore, the phase velocity is determined as $U_{\text{nn}} = 0.444$, $U_{\text{lt}} = 0.431$, where $U$ and $\omega$ are scaled by $\nu/d$ and $\nu/d^2$, respectively. This shows that our numerical simulation using the nonlinear perturbation equations agrees well with the approximate solution of the linear theory and with the simulation result $\omega_{\text{ln}} = 1.37$ using the linearized equations of motion (denoted by the suffix ln).$^{27}$ Based on the above characteristics, a propagating disturbance similar to those observed in experiments$^{29}$ is properly referred to as a linear or quasilinear wave packet.

In a reference frame moving at the group velocity $(x - x_0)/t$, the initial disturbance starting from $x = x_0$ grows with $\gamma_m = 0.0775$ and its width disperses gradually with time. Therefore, it is possible to find the fronts ($x_1$ and $x_2$) of a
disturbance for which the growth rate $\gamma$ returns to zero, where we assume $x_2 > x_1$. If the propagating velocities at the two fronts satisfy the conditions $(x_1 - x_0)/t > 0$ and $(x_2 - x_0)/t > 0$ for the right-propagating wave packet, the disturbance propagates away quickly enough relative to its growth and spreading so that the point of origin ($x_0 = 25$) settles back to its undisturbed state, which implies the disturbance at $x_0 = 25$ decays to zero. Therefore, we say the system is convectively unstable for some Rayleigh number $R_c$ at $(x_1 - x_0)/t > 0$ and $\gamma > 0$. In contrast, we say the system becomes absolutely unstable for the other Rayleigh number $Ra$ at $(x_1 - x_0)/t < 0$, namely, for any fixed $x$, the disturbance will lead to exponential growth in time even though the disturbance itself may propagate away as a wave packet. From the temporal growth rate evaluated at a fixed position ($x_0 = 25$) shown in Fig. 1 and the maximum growth rate $\gamma_0$, we find that the system is convectively unstable at the early linear stage of disturbance propagation at $r = 2.1$.

After the time $t$ in Fig. 2 exceeds 52, the reflection at the end walls sets in and the maximum amplitude as a function of time no longer follows the exponential growth and begins to oscillate again with time. Figure 3 illustrates a kind of spatiotemporal MTW which is generated by the superposition of the TW and its reflected wave from the end walls, where the TW packet which is located near the right end wall has reversed its direction of propagation and begins to move toward the central region due to the reflection effect, while the reflection effect at the left end wall has already occurred, but the TW located there is still confined near the wall. The difference in the motion between the left- and the right-going TW arises from the assumed asymmetry of the Gaussian envelop of the initial state. In addition, the convection amplitude is also significantly modulated with time.

When a pair of independent CPW packets reflected from the two end walls meets at the central region of the cell and interact with each other, another type
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Fig. 3. Time evolution from time 51.25 to 60 \([d^2/\nu]\) of the temperature field \(\theta(x, z = 0.5)\) in the MTW state which originates from the superposition of both TW and reflected waves from the walls at \(r = 2.1\), \(R = 46\) and \(t > 52\).

Fig. 4. Time evolution from time 106.25 to 115 \([d^2/\nu]\) of the temperature field \(\theta(x, z = 0.5)\) in the MTW state generated by the interaction between the left-going TW reflected from the right end wall and the right-going TW reflected from the left end wall and confined near the central region for a system with \(R = 46\) and \(\psi = -0.47\) at \(r = 2.1\).

of MTW confined near the central region is generated, as shown in Fig. 4. In this case, convection occupies the central region and conduction dominates the two side regions. Figure 5 displays the spatiotemporal structure of the temperature field in the stages of a quasilinear wave and MTW states, where each curve represents the time evolution of the nodal points or the spatial points at which the lateral profile of
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Fig. 5. The spatiotemporal structure of the temperature field from time 0 to 105 \(d^2/v\) for a system with \(\Gamma = 46\) and \(\psi = -0.47\) at \(r = 2.1\). Each solid line represents the time evolution of the nodal points or the spatial points at which the temperature field \(\theta(x, z = 0.5)\) vanishes; i.e., its phase becomes 0 or \(\pi\). The region between the two solid lines represents the size and the propagating direction of convective rolls. The phase velocity is almost constant.

The temperature field \(\theta(x, z = 0.5)\) at the midheight of a binary fluid layer vanishes; i.e., the points at which the phase becomes 0 or \(\pi\). The part between two solid lines represents the size and the traveling direction of the convective rolls. The phase velocities of rolls are almost constant.

3.2. Spatiotemporal defect

Figure 6 displays the time evolution of the concentration field \(\xi(x, z = 0.5)\) with one sink and two source defects, where we can find that the lateral concentration profile becomes trapezoidal with the lapse of time, a strongly non-harmonic shape in the two localized regions near \(x = 14\) and \(x = 27\). In the corresponding positions, the vertical velocity and temperature profiles also assume the maximum convection amplitude in space, but they retain their harmonic shapes, as shown in Fig. 7. This characteristic of the concentration profiles clearly indicates that for a cell with a large value of \(\Gamma\) and for a large negative value of \(\psi\), the system first evolves into a nonlinear localized state.

Figure 8 displays the spatiotemporal structure of the temperature field at \(r = 2.1\) which appears after that shown in Fig. 5. Corresponding to the appearance of the strongly non-harmonic shape in the concentration field \(\xi(x, z = 0.5)\) at
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Fig. 6. Time evolution from time $116.25$ to $135$ [$d^2/\nu$] of the concentration field $\xi(x, z = 0.5)$ for a system with $\Gamma = 46$ and $\psi = -0.47$ at $t = 2.1$, where the lateral concentration profile comes to assume a trapezoidal, strongly non-harmonic shape in the two localized regions near $x = 14$ and $27$.

Fig. 7. The lateral profiles of the convective field at the midheight of the fluid layer for a system with $\Gamma = 46$ and $\psi = -0.47$ at $t = 2.1$. Near $x = 14$ and $27$, the lateral concentration profile assumes a trapezoidal, non-harmonic shape, but the vertical velocity and temperature profiles maintain harmonic shapes. (a) Vertical component of the velocity $w$, (b) perturbation temperature $\theta$, and (c) perturbation concentration $\zeta = \eta - \theta$. 
Fig. 8. Time evolution from time 105 to 195 \([d^2/\nu]\) of the temperature field where each solid line represents the phase of TW as shown in Fig. 5. The sink defect is located near the central region, and the two source defects are migrating toward the left and right end walls. When \(t > 175\), the regions near the two ends are again dominated by the conductive state.

For \(t = 120\) shown in Fig. 7, the effect of the nonlinearity also begins to lead to the change of the traveling speeds of the rolls in space-time. With the migration of the source defects toward the two end walls, the TW is further modulated as a result of successive reflection, splitting and superposition, so that the waves starting from the two source defects and moving toward the end walls quickly decay, and the system again falls in the conduction state near the two ends. However, the TW convection with the sink defects still dominates the central region.

Figure 9 displays the generating process of the defects, where the two dashed lines on the left represent the formation of a source defect due to competition between the traveling waves with different propagation directions and phase velocities, while the two dashed lines on the right show the generation of a sink defect which originates from the invasion of a fast TW from the right side.

Figure 10 displays the spatiotemporal structure of the temperature field at \(r = 2.1\) following that shown in Fig. 8. At this stage, convection and conduction coexist in the cell. A fast TW on the right rapidly spreads. As the fast TW similar to that on the right invades from the left of the slow TW at \(t = 245\), the source defects have all disappeared, but the sink defects remain. Since the sink defects which occur on the right (resp. left) side of the slow TW migrate toward the central region at a speed \(U = 0.1\) (resp. 0.01) almost equal to the phase velocity of the adjacent slow TW, “space-time grain boundaries” formed by these sinks materialize at the front boundaries of the fast TW and divide the pattern in Fig. 10 into different
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Fig. 9. The formation of defects in the time evolution from time 186.25 to 195 $[d^2/\nu]$ of the temperature field $\theta(x, z = 0.5)$ for a system with $\Gamma = 46$ and $\psi = -0.47$ at $r = 2.1$. The two dashed lines on the left show the formation of a source defect where one convective roll splits into three rolls, and the two dashed lines on the right show the generation of a sink defect where three convection rolls become one roll.

Fig. 10. Time evolution from time 195 to 285 $[d^2/\nu]$ of the temperature field where each solid line represents the phase of TW as shown in Fig. 5. The region dominated by the slow TW with a long wavelength is shrinking, whereas those by the fast TW with a short wavelength on both sides are expanding.
Fig. 11. Time evolution from time 315 to 415 \(d^2/\nu\) of the temperature field for a system with \(\Gamma = 46\) and \(\psi = -0.47\) at \(r = 2.1\), where each solid line represents the phase of TW as shown in Fig. 5. The source defect has disappeared, and convection has dominated the entire cell. However, the sink defects still remain and form the three “space-time grain boundaries”.

When the fast traveling waves spread to both end walls, a TW containing only sink defects comes to fill the whole cell and forms three “space-time grain boundaries”, as shown in Fig. 11. In this stage, the distribution of wavenumber is remarkably non-uniform due to the coexistence of the fast TW with short wavelength (\(\lambda = 1.66\)) and the slow TW with long wavelength (\(\lambda = 2.5\)). However, the wavenumber adjustment in space occurs successively, and the wavenumber tends to be uniform in such a manner that generation of the sink defects causes continuous annihilation of a pair of rolls with a long wavelength in the central region and the resultant invasion of the fast TW (\(U = 0.18 \sim 0.22\)) from both sides of the slow TW (\(U = 0 \sim 0.1\)) leads to diminution of the central region with a long wavelength, as shown in Fig. 11.

With the lapse of a long period of time, the long wavelength region is destroyed, and the system evolves into a TW state containing a single sink defect, as shown in Fig. 12. The spatial position of the sink defect is fixed because the TW on both sides of the defect maintains an almost constant wavelength and phase velocity (\(U = 0.13\)). The generation of two rolls at the two end walls and the destruction of a pair of rolls on the interface of the CPW occur alternatingly. As a result, the total number of the rolls oscillates between \(n\) and \(n + 2\). In our experiments using a slot channel cell with \(\Gamma = 46\), similar TW states containing only sink or source defect
states have also been observed for at least two cases: (i) for 8 wt-% ethanol in water with a temperature difference $\Delta T = 10.22^\circ C$ and (ii) for 11.3 wt-% butanol in water at the mean temperature $T = 16.05 \sim 15.98^\circ C$ with $\Delta T = 9.031 \sim 9.197^\circ C$. 22)

§4. Summary and discussion

In this paper we have given simulation results on the transient evolution from a conduction to an ETW state containing a sink defect slightly above the onset of the subcritical instability for a binary fluid mixture with the separation ratio $\psi = -0.47$ confined in a rectangular cell of aspect ratio $\Gamma = 46$. The simulation was performed using the two-dimensional full fluid equations of motion. We have found that the system starting from a conduction state first experiences a linear stage with CPW packets which oscillate with a linear frequency and grow exponentially with a linear growth rate. The system then develops MTW and TW containing transient sink and source defects, and ultimately evolves into a TW with a stable sink defect similar to those observed in experiments. For a cell with large aspect ratio, we have found that the system evolves into the nonlinear regime as the lateral concentration profile becomes a trapezoidal, strongly non-harmonic shape in the localized regions.

In this paper, we first determined the value of the Rayleigh number at the onset of convection. We next obtained the quasilinear TW with the maximum linear growth rate $\gamma_m = 0.0775$ and the linear oscillatory frequency $\omega_{mn} = 1.40$ in a convectively unstable regime at $r = 2.1$ for a rectangular cell with $\Gamma = 46$ and $\psi = -0.47$. This value of $\omega_{mn}$ agrees well with the approximate solution 29) and the numerical result. 27) The experiment for $\psi = -0.57$ 29) exhibits a linear growth stage similar to that of our simulation, where after a short oscillatory period, the system at $r = 1.0009$ $r_{osc}$ quickly evolves into the linear stage with $\gamma = 0.000116$ [s$^{-1}$] and $\omega_e = 0.0199$ [s$^{-1}$],
in agreement with the approximate solution $\omega_{tt} = 0.020 [s^{-1}]$. The observed result for $\psi = -0.28$ has also shown $\omega_{t} = 0.0111 [s^{-1}]$ in the exponential growth stage, in agreement with the approximate solution $\omega_{tt} = 0.0115 [s^{-1}]$. This shows that the simulation can reproduce the behavior of the linear stage well and can accurately predict the value of the linear frequency. In contrast, the maximum growth rate $\gamma_{m}$ depends on $\varepsilon$ so that it cannot be represented by a single numerical constant.

On the other hand, Kolodner et al. employed a technique of launching pulses into a flow field for studying the stability of convection pulses. They found that after a disturbance wave packet is applied to a flow field in an annular cell, it quickly splits into a pair of independent CPW packets exhibiting linear behavior. Their method for applying the disturbance to a flow field and the obtained linear behavior are all similar to those in our simulation for $\Gamma = 46$. However, we have also carried out similar calculations at several values of $r$ for another set of parameters, such as $\Gamma = 12$ and $\psi = -0.11$, and obtained only the CPW which is observed in a number of experiments in rectangular cells. It turns out that if $\Gamma$ decreases below a certain value, reflections from the two end walls quickly appear before the disturbance wave packet decomposes into two independent wave packets. The superposition and interaction between the TW and the reflected wave lead to the oscillation of the maximum amplitude in the space $(x, z)$ with time. Thus, in a cell with smaller $\Gamma$ the system immediately generates the CPW or the source defect instead of the two independent CPW packets with a linear growth rate.

Bensimon et al. first observed the spatiotemporal defect states in their experiments, and found that competition between the transient source and sink defects which are boundaries of the fast TW and the slow TW leads to the “space-time migration of the grain boundaries”. For a suitably assigned value of $r$, the system ultimately evolves into the TW with a sink defect in an annular cell or the TW with a source defect in an annular cell with an inserted separating wall (whose role is to make this system similar to a rectangular cell). Kolodner et al. further studied the defect features and the method of its generation and obtained the TW with sink and source defects in an annular cell by the procedure of launching pulses, controlling the growth of independent linear wave packets by properly adjusting $r$. Our simulation reveals that for $\Gamma = 46$ and $\psi = -0.47$ the system eventually evolves into a TW with a sink defect due to the competition between the source and sink defects which originate from the split, reflection and superposition of the TW. In contrast, for $\Gamma = 12$ and $\psi = -0.11$ the system evolves into the LTW without source or sink defects, similar to the experimental results, after the transient source defects disappear. Therefore, it seems clear that the two independent CPW packets exhibiting linear behavior which can only prevail at large values of $\Gamma$ play a very important role in forming the spatiotemporal defects. A source or a sink defect may appear in either an annular or a long rectangular cell. However, their stability depends strongly on the value of $\Gamma$ and how they are produced experimentally.

Some elementary processes, such as CPW and the appearance of a roll state with a source defect, which appeared in the course of our system evolution were already obtained numerically using various types of the amplitude equations. We stress here, however, that in the present simulation we pursued the whole tran-
sient process of spatiotemporal evolution of the convection patterns starting from a linearly unstable roll state to the ultimate roll state containing a sink defect at a fixed value of $r$ slightly above the onset of the subcritical instability $r_{osc}$ using the full hydrodynamic equations of motion.

Kaplan and Steinberg discussed the mechanism operative in pinning and slippage of a source defect associated with the CPW on the basis of their experimental observations. It is believed that the same reasoning also applies to our simulation results in which the source defect is pinned during the initial CPW stage (Fig. 5) whereas its slippage is observed to occur at the later stage of evolution (Fig. 8). However, at present we cannot show the definite reasons why the defect behaves differently at the different stages of evolution.

Kaplan and Steinberg have introduced two distinctive mechanisms governing the transition process from the LTW to the ETW. This implies that in longer cells the instability of the LTW is related to convective amplification of spontaneous fluctuations, while in shorter cells it occurs due to a linear front instability associated with the convective to absolute instability transition. In our results, displayed in Figs. 8 and 10, which show the invasion of the LTW into the conduction state, however, it appears that the front propagation does not exhibit behavior which is peculiar to the one or the other of the above-mentioned instabilities.

If we compare more closely the ultimately attained state given in Fig. 12 with the experimentally observed one for the long slot channel with $\Gamma = 46$ given in Fig. 1 of Ref. 5), we notice the following difference. While the TW state in Fig. 12 contains only a single sink defect, that obtained in the experiment contains two adjacent source defects. Furthermore, the TW states on both sides of the defect region have opposite and coincident propagation directions for the present simulation and the previous experimental results, respectively. Since these states were taken as the initial states by lowering $R$ for producing the DLTW near the turning point, the above-mentioned difference between the two cases leads to the difference between the patterns of the DLTW in the sense that a pair of the TW states in the DLTW have opposite (resp. coincident) traveling directions in the simulation (resp. experimental) results.

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**References**