Neutrino Oscillation and Ortho-Para Mixing Model for Family Mixing

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Neutrino oscillations are discussed based on the ortho-para mixing model for family mixing. The model naturally leads to a neutrino mixing matrix of the Acker-Pakvasa type. In the model CP violations can be concretely discussed. Using reported data on neutrino oscillations, the rephasing invariant CP violation parameter \( J_{CP} \) is predicted to be \( \sim 4 \times 10^{-3} \), which is roughly one hundred times that in the quark mixing.

§1. Introduction

Preon model is an idea along the line of atomic view. Experiments have shown that leptons and quarks are very complicated objects. It seems difficult to construct a beautiful theory of Nature by regarding them as fundamental matter. If we take a preon model, namely compositeness of leptons and quarks,\(^1\) there is a possibility that we can attribute their complicated properties to their compositeness. Hence, without recourse to complicated matter such as leptons and quarks, we might be able to construct a theory at a short distance based on simpler and more beautiful matter. This will be a virtue of the preon model. Many of the complicated properties of leptons and quarks are related with the family problem. One of the tasks of the preon model of fundamental importance will be to settle the family problem on the basis of simple and beautiful matter.

We have developed concretely a preon model by introducing preons with charge \( e/2 \) and the preonic charge which is identified with the magnetic charge.\(^2\)\(^-\)\(^8\) The model has some interesting features. In particular, it predicts a stable weakly interacting massive particle (WIMP), which behaves as cold dark matter in the Universe,\(^4\) and does naturally the existence of \( CP \) violations.\(^5\)\(^,\)\(^6\) Recently, it has been shown\(^8\) that the model has an advantage in explaining the unique features of the precise \( Z \) decay data, e.g., large \( R_l \) and large \( R_b \).

On the basis of our preon model, we have proposed ortho-para mixing model\(^5\) for family mixing and understood rather successfully the Cabibbo-Kobayashi-Maskawa matrix, the relations among quark masses, and \( CP \) violation effects. In this paper we study neutrino mixings based on the ortho-para mixing model. The validity of treating neutrinos in the same footing as quarks is not self-evident, since neutrinos are special in that they have surprisingly small masses. However, neutrinos behave almost completely in the same manner as charged leptons and quarks in weak interactions which occur through exchanges of bound states of preons and so are governed by preon dynamics. This strongly suggests that preon dynamics sees neutrinos as
particles with the same properties as other fermions. Encouraged by this, we apply
the ortho-para mixing model also to neutrinos.

Recently, Acker and Pakvasa have shown in their interesting paper\(^9\) that it is
possible to account for all three experimental indications for neutrino oscillations
with just three neutrino flavors and given concretely a mixing matrix. It will be
shown that our model naturally leads to the neutrino mixing matrix of the Acker­
Pakvasa type. In the phenomenological analysis of Acker and Pakvasa, \(CP\) violation
effects were not discussed concretely. One of the unique features of the ortho-para
mixing model is that it enables us to study \(CP\) violations in a definite way. In this
paper we discuss \(CP\) violation effects concretely.

§2. Neutrino mixing matrix in ortho-para mixing model

(a) Formulation

The interaction lagrangian of leptons and quarks with \(W\) boson is described,
using their weak interaction eigenstates, \(f^W\) and \(g^W\), as follows:

\[
L_W \sim h \sum g_i^W O_{ij} f_i^W W^\mu + \text{h.c.},
\]

where \(i\) is the family index, \(h\) the coupling constant and \(O_{ij} = \gamma_\mu (1 - \gamma_5)\).

\(f_i^W\) are related to mass eigenstates \(f_i^m\) through a unitary matrix \(U_f\) as

\[
f_i^W = \sum (U_f)_{ij} f_j^m.
\]

Hence we have

\[
L_W \sim \sum \bar{g}_i^m (U_g^\dagger U_f)_{ij} f_j^m W + \text{h.c.},
\]

where coupling constants \(h\) and \(O_{ij}\) are suppressed. \(U_g^\dagger U_f\) is the so-called Cabibbo-
Kobayashi-Maskawa matrix. The flavor eigenstate \(f_i\) is defined through

\[
L_W \sim \sum \bar{g}_i^m f_i W + \text{h.c.}
\]

Thus we have

\[
f = U_g^\dagger U_f f^m.
\]

In the ortho-para mixing model for family mixing,\(^5\) matrix \(U_f\) has the property,

\[
U_f^\dagger M_{op} U_f = \text{diag}(m_1, m_2, m_3, m_4),
\]

where

\[
M_{op} = \begin{pmatrix}
0, & 0, & 0, & M_1 \\
0, & 0, & 0, & M_2 \\
0, & 0, & m, & M_3 \\
M_1^*, & M_2^*, & M_3^*, & M
\end{pmatrix},
\]

with \(M \gg m, |M_i|\). \(M\) is of the order of preon dynamics mass scale, \(\sim 1000\) TeV.
\(m_i\) (\(i = 1, 2\) and \(3\)) are the masses of \(i\)-th family ortho-fermion, namely usual leptons
and quarks, and \(m_4\) is the para-fermion mass. We have that, in the leading order of
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\[ M^{-1}, m_1 = 0, m_2 = (|M_1|^2 + |M_2|^2) / M, m_3 = m \text{ and } m_4 = M. \]

In our model, the mass patterns of leptons and quarks are naturally hierarchical, namely,

\[ m_1 \ll m_2 \ll m_3. \tag{7} \]

The \( M_i \) are the mixing masses between ortho- and para-fermions due to the breaking of \( SU(6)_{wc} \) symmetry and are naturally complex.\(^5\),\(^6\) \( CP \) violations take place through the phases of \( M_i \). The CKM matrix is the \((3 \times 3)\) block of \( U_g^1 U_f \). CKM matrices are unitary in a good approximation because \( M \) is much greater than other mass parameters.

(b) Neutrino mixing matrix

Flavor eigenstates of neutrino, \( \nu_a \) (\( a = e, \mu \) and \( \tau \)) are related to the mass eigenstates \( \nu^m \) through neutrino mixing matrix \( U \),

\[ \nu_a = \sum U_{ai} \nu^m_i \quad \text{and} \quad \bar{\nu}_a = \sum U_{ai}^* \bar{\nu}_i^m. \tag{8} \]

For neutrinos produced through the vertex \( l - \nu - W \) (\( l = e, \mu \) and \( \tau \)), the matrix \( U \) is \( 3 \times 3 \) block of \( U_g^1 U_\nu = \text{CKM matrix for } (l\nu) \) system. Hereafter, \( m_i \) (\( i = 1, 2 \) and 3) denotes the mass of \( i \)th family neutrino. The masses of charged leptons are expressed as \( m_e, m_\mu \) and \( m_\tau \). In our model we have \( m_1 \ll m_2 \ll m_3 \) with \( m_1 \simeq 0 \).

In our model, \( CP \) violations are \( O(M^{-2}) \) effects and therefore it is necessary to calculate matrix elements of \( U \) (\( U_{ai} \)) to the extent of \( O(M^{-2}) \). \( U_{ai} \) are given, using mass parameters for neutrino \( (m, M_1, M) \) and for charged lepton \( (m', M_1', M') \), as follows:

\[ U_{e1} = \cos \theta, \]
\[ U_{e2} = \sin \theta(1 - \alpha^2 / 2), \]
\[ U_{e3} = -\alpha \sin \theta(1 + x \alpha - \alpha / x), \]
\[ U_{\mu_1} = -\sin \theta(1 - \beta^2 / 2), \]
\[ U_{\mu_2} = \cos \theta(1 - \alpha^2 / 2 - \beta^2 / 2 + \alpha \beta e^{i\phi}), \]
\[ U_{\mu_3} = -\alpha \cos \theta(1 + x \alpha - \alpha / x) + \beta(1 + y \beta - \beta / y)e^{i\phi}, \]
\[ U_{\tau_1} = \beta \sin \theta(1 + y \beta - \beta / y), \]
\[ U_{\tau_2} = -\beta \cos \theta(1 + y \beta - \beta / y) + \alpha(1 + x \alpha - \alpha / x)e^{i\phi} \]
\[ \text{and} \quad U_{\tau_3} = (1 - \alpha^2 / 2 - \beta^2 / 2)e^{i\phi} + \alpha \beta \cos \theta, \tag{9} \]

where, using \( A = (|M_1|^2 + |M_2|^2)^{1/2} \) and \( A' = (|M_1'|^2 + |M_2'|^2)^{1/2} \), \( \alpha = A|M_3|/(M m) \), \( \beta = A'|M_3'|/(M' m') \), \( x = |M_3|/A \) and \( y = |M_3'|/A' \). In Eq. (9), terms not having \( m \) or \( m' \) in their denominators are neglected because they are negligibly small, and a phase convention is taken in which the first column and first row are real. In order for Eq. (9) to be valid, the conditions, \( m \gg A^2 / M \) and \( m' \gg A'^2 / M' \), are necessary besides the conditions mentioned above. These conditions are equivalent to that \( m_3 \gg m_2 \) and \( m_\tau \gg m_\mu \). \( \theta \) is determined by the ratios of \( M_1 / M_2 \) and \( M'_1 / M'_2 \) alone, namely independent of the large parameter \( M \), and so \( \cos \theta \) and \( \sin \theta \) are parameters.
of $O(1)$. $\alpha$ and $\beta$ are small parameters of $O(M^{-1})$. The Jarlskog $CP$ violation parameter, $J_{CP}$, is, in the leading order of $M^{-1}$,
\[ J_{CP} \equiv \text{Im}(U_{11} U_{22} U_{12}^* U_{12}^*) = -\alpha \beta \cos \theta \sin^2 \theta \sin H. \] (10)
Since both $A$ and $M_3$ are ortho-para mixing masses, it is expected that their values are similar, namely $x, y \sim 1$. In the CKM matrix for $(ud)$ system ($= V$), taking $x, y = 1$ leads to $V_{cb} \sim m_s/m_b$, in agreement with the data. The neutrino oscillation data are not so precise at the present stage. Therefore, we take an approximation that $x, y = 1$. Thus we obtain
\[
(U_{ai}) = \begin{pmatrix}
\cos \theta, & \sin \theta(1 - \alpha^2/2), & -\alpha \sin \theta \\
-\sin \theta(1 - \beta^2/2), & \cos \theta(1 - \alpha^2/2 - \beta^2/2) + \alpha \beta e^{iH}, & -\alpha \cos \theta + \beta e^{iH} \\
\beta \sin \theta, & -\beta \cos \theta + \alpha e^{iH}, & (1 - \alpha^2/2 - \beta^2/2)e^{iH} + \alpha \beta \cos \theta
\end{pmatrix},
\] (11)
where $\alpha = m_2/m_3$ and $\beta = m_\mu/m_\tau = 0.06$. The unique features of $U$ are that $|U_{rr}| \ll 1, |U_{ai}|$ are $O(1)$ for $i = 1, 2$ and other elements are much less than unity. These features are just those obtained phenomenologically by Acker and Pakvasa. The key points of our model are whether the value of $\alpha$ determined from mixing angles is consistent with the one done from neutrino masses and whether something definite can be predicted for $CP$ violation effects.

§3. Confrontation with the data

The neutrino survival and transition probabilities are given by
\[
P_{ab} = \sum U_{bi} \exp(-iE_i t) U_{ai}^* \text{ and } P_{ab} = \sum U_{bi}^* \exp(-iE_i t) U_{ai},
\] (12)
where $P_{ab}(P_{ab})$ is the $\nu_a \rightarrow \nu_b (\bar{\nu}_a \rightarrow \bar{\nu}_b)$ transition probability. Defining that $\Delta_{ij} \equiv (E_i - E_j)t = (m_i^2 - m_j^2)L/2E$ and $\delta m_{ij}^2 \equiv (m_i^2 - m_j^2)$, we have
\[
P_{ab} = \sum U_{bi} U_{bj}^* U_{ai}^* U_{aj} \exp(-i\Delta_{ij}), \text{ and}
\]
\[
P_{\bar{a}\bar{b}} = \sum U_{\bar{b}i} U_{\bar{b}j} U_{\bar{a}i} U_{\bar{a}j} \exp(-i\Delta_{ij}) \text{ with } \Delta_{ij} = \delta m_{ij}^2 L/2E.
\] (13)
(a) Atmospheric neutrino anomaly

In our model the transition probability of $\mu$ neutrino to $\tau$ neutrino $P_{\mu\tau}$ is much less than unity because of the smallness of $\alpha$ and $\beta$. Hence the observed small values of the flux ratio $R = \nu_\mu/\nu_\tau$ \cite{10,11} must be explained by $\nu_\mu - \nu_\tau$ oscillations arising from the first $2 \times 2$ block of $U$ matrix. The Kamiokande collaboration showed that, in the case of two flavor $\nu_\mu - \nu_e$ oscillations, $\sin^2 2\theta = 0.6 - 1.0$ and $\delta m_{21}^2 = 0.007 - 0.09$ eV$^2$ with best fit values $(\sin^2 2\theta, \delta m_{21}^2) = (1.0, 0.018 \text{ eV}^2)$, \cite{11} which implies that $\theta$ is near $45^\circ$ and $m_2$ is $O(0.1 \text{ eV})$ (note that $m_1 \ll m_2$ in our model). Acker and Pakvasa have argued \cite{9} that for this value of $\theta$ and the value of $\delta m_{21}^2$ in this range the solar neutrino anomaly \cite{12} can be explained by the $\nu_e \rightarrow \nu_\mu$ vacuum transition.
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(b) LSND experiments on $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ transition

The value of $m_2$ of $O(0.1\ eV)$ implies $\Delta_{21} \simeq 0$ in accelerator experiments. Hence the LSND $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ events appear through indirect transitions as proposed by Babu, Pati and Wilczek.\textsuperscript{14} Taking $\Delta_{31} = \Delta_{32} = \Delta = m_3^2 L/2E$, \[ P_{\mu e} = 4|U_{e3}|^2|U_{\mu 3}|^2\sin^2(\Delta/2). \] (14)

Babu et al. showed\textsuperscript{14} that a combination of LSND experiments with other constraints (Bugey, CDHS, Fermilab E531 and CHARM-II\textsuperscript{15}) almost uniquely determines that $m_2^2 \sim 2\ eV^2$ and $4|U_{e3}|^2|U_{\mu 3}|^2 = 1.2 - 1.5 \times 10^{-3}$. Here we take that $m_3 = 1.4\ eV$. Acker and Pakvasa have given the allowed range, $|U_{e3}| = 0.129 - 0.141$ and $|U_{\mu 3}| = 0.123 - 0.134$,\textsuperscript{9} which are used in this paper. In our model we have \[ |U_{e3}| = \alpha \sin \theta \quad \text{and} \quad |U_{\mu 3}| = (\alpha^2 \cos^2 \theta + \beta^2 - 2\alpha\beta \cos \theta \cos H)^{1/2}. \] (15)

Since $m_2 < 0.3\ eV$ from the atmospheric neutrino anomaly,\textsuperscript{11} $\alpha$ is strongly constrained, $\alpha = m_2/m_3 < 0.3/1.4 = 0.21$, which results in that $\sin \theta > 0.65$ from $U_{e3} > 0.129$. From solar neutrino anomaly, $\sin \theta$ is required to be not far from 0.7. Following Acker and Pakvasa, we take $\sin \theta \leq 0.764$. As $\sin \theta$ is smaller, a larger $\alpha$ becomes necessary from $|U_{e3}|$, which results in a larger $\delta m_{21}^2$. The atmospheric neutrino data\textsuperscript{11} favor $\delta m_{21}^2$ considerably smaller than $0.1\ eV^2$. Therefore, we study here the following two cases.

1. Case 1: $\sin \theta = 0.707$

From $U_{e3}$ we obtain that $\alpha = 0.18 - 0.20$. Here we take $\alpha = 0.19$. A small positive value of $\cos H$ seems to be necessary. By taking $\cos H = 0.23$, we obtain that $|U_{e3}| = 0.134$ and $|U_{\mu 3}| = 0.134$ and so $4|U_{e3}|^2|U_{\mu 3}|^2 = 1.3 \times 10^{-3}$. In this case, $m_2 = 0.27\ eV$ and so $\delta m_{21}^2 = 0.073\ eV^2$, which lies near the edge of the region allowed by the atmospheric neutrino anomaly. However, this should not be taken seriously since, if $m_3$ is taken to be $1.2\ eV$, $\delta m_{21}^2$ reduces to $0.05\ eV^2$.\textsuperscript{9}

2. Case 2: $\sin \theta = 0.764$

We can take $\alpha = 0.17$, which implies $m_2 = 0.24\ eV$. Taking $\cos H = -0.18$, we obtain that $|U_{e3}| = 0.130$ and $|U_{\mu 3}| = 0.134$, so that $4|U_{e3}|^2|U_{\mu 3}|^2 = 1.2 \times 10^{-3}$.

As seen from these case studies, $\cos H$ is required to be near zero. In fact, if $\cos H = \pm 1$, $|U_{\mu 3}| = (\alpha \cos \theta + \beta)$ or $(\alpha \cos \theta - \beta)$, which is significantly away from the allowed region. In Case 2, $U$ matrix is as follows:

\[ U = \begin{pmatrix}
0.645, & 0.753, & -0.130 \\
-0.763, & 0.633 \pm 0.010i, & -0.120 \pm 0.059i \\
0.046, & -0.069 \pm 0.167i, & -0.170 \pm 0.968i
\end{pmatrix}, \] (16)

where $\pm$ is the sign of $\sin H$. Equation (16) implies that, in short base line experiments with $L \sim 30\ m$ where $\Delta_{21} \sim 0$, $P_{\mu \tau} = 0.07\sin^2(\Delta/2)$.

The sum of $m_i$ is $\sim 1.7\ eV$. The so-called mixed dark matter model seems to favor $\sum m_i \sim 4\ eV$.\textsuperscript{16} Introducing cosmological constant may settle the problem. As for the interpretation of LSND experiments, however, some controversy seems to exist.\textsuperscript{17} In order to obtain a conclusive answer to whether $m_3$ of $4\ eV$ is excluded or not, it may be necessary to accumulate more data.
§4. CP violations in neutrino oscillations

From Eq. (13) we obtain for the differences due to CP violations between neutrino flavor conversion probabilities,\(^9\),\(^18\)

\[ \Delta P = P_{\mu e} - P_{\mu \bar{e}} = P_{\tau e} - P_{\tau \bar{e}} = P_{\mu \tau} - P_{\mu \bar{\tau}} = -4J_{CP}(\sin \Delta_{21} + \sin \Delta_{13} + \sin \Delta_{32}), \number^{(17)} \]

where \( J_{CP} = \text{Im}(U_{11}U_{22}U_{12}^*U_{21}^*) \). As shown in the above, in our model, the combination of atmospheric neutrino experiments and LSND experiments lead almost uniquely to \( \alpha \sim 0.17, \theta \sim 45^\circ \) and \( \sin H \sim \pm 1 \). Hence it is predicted that \( J_{CP} = -\alpha \beta \cos \theta \sin^2 \theta \sin H \sim \pm 4 \times 10^{-3} \), which is strikingly large, roughly one hundred times that in quark mixing. It will not be difficult to observe CP violation effects in neutrino oscillations in experiments suitably designed. In short base line experiments in which \( \Delta_{31} \sim 0 \) and \( \Delta_{13} \sim -\Delta_{32} \), CP violation effects disappear. In long base line experiments with \( L \sim 100 \text{ km} \) where \( \Delta_{31} \Delta_{32} \gg 1 \) and \( \Delta_{21} = O(1) \), we have that \( |\Delta P| \sim 0.016|\sin \Delta_{21}| \). In Case 2, we obtain from Eq. (16), for long base line experiments,

\[ P_{\mu \tau} = 0.033 + 0.003 \cos \Delta_{21} - 0.0075 \eta \sin \Delta_{21}, \]
\[ P_{\mu \bar{\tau}} = 0.033 + 0.003 \cos \Delta_{21} + 0.0075 \eta \sin \Delta_{21}, \]
\[ P_{\mu e} = 0.470 - 0.469 \cos \Delta_{21} + 0.0075 \eta \sin \Delta_{21}, \]
\[ P_{\mu \bar{e}} = 0.470 - 0.469 \cos \Delta_{21} - 0.0075 \eta \sin \Delta_{21}, \] with
\[ \eta = \sin H/|\sin H|. \number^{(18)} \]

Unfortunately, at the present stage, our model is not developed enough to predict the sign of \( \sin H \). Future experiments will explain about it. Since \( J_{CP} \) is proportional to \( m_3^{-1} \), even if we take \( m_3 = 4 \text{ eV} \), \( J_{CP} \) is not small, \( \sim 1.3 \times 10^{-3} \), which may be measurable. Future long base line experiments will be able to give an important information of the value of \( m_3 \).

§5. Remarks

In a phenomenological analysis, masses, \( m_i \), and elements of matrix \( U \) are not directly related. Therefore, to some extent, \( U \) are determined independently of \( m_i \). However, in our model, matrix elements of \( U \) are directly related to the values of \( m_i \). It is not a trivial thing that, with these severe constraints, we can determine successfully masses, \( m_2 \) and \( m_3 \), and the matrix \( U \). It is also striking that CP violation effects are almost uniquely determined except for the sign of \( \sin H \), if both the data on atmospheric neutrino anomaly and on the \( \bar{\nu}_\mu \to \bar{\nu}_e \) conversion by LSND are taken at face value. Future long base line experiments will rather easily test our results.

The neutrino mixing matrix \( U \) determined has typical features of the ortho-para mixing model, say \( \sin \theta \) of \( O(1) \) and small values of \( U_{e3}, U_{\mu 3} \). However, \( U \) appears to have features different from the quark CKM matrix \( V \), e.g., \( U_{e3} \gg V_{ub} \). In our model
the “large” $U_{e3}$ stems from large $\sin \theta (~0.7)$ and “large” $m_2/m_3 (~0.17)$ and the small $V_{ub}$ does from small $\sin \theta c (~0.2)$ and small $m_s/m_b (~0.03)$. However, it is reasonable to consider that both $\sin \theta$ and $\sin \theta c$ are parameters of $O(1)$. Comparing with the corresponding quantity in charged leptons, $m_\mu/m_\tau (~0.06)$, we can say that both $m_2/m_3$ and $m_s/m_b$ are similar to $m_\mu/m_\tau$. It is not absurd to say that both $U$ and $V$ have typical features of the ortho-para mixing model.

From $m_2 \sim A^2/M \sim 0.2$ eV, we obtain, for $M \sim 10^6$ GeV, that the ortho-para mixing mass $A$ is roughly 10 MeV in neutrinos. The value of $A$ in charged leptons is roughly 300 GeV since $m_\mu \sim 100$ MeV $\sim A^2/M$. A reason why ortho-para mixing masses in neutrinos are so small compared with those in other fermions is not clear at present. It may be one of the most important keys to clarify a preon dynamics. It might be intimately related to the neutrality of neutrinos. If, due to the neutrality of neutrinos, the ortho-para mixing masses vanish in the leading order of $SU(6)_{we}$ breaking and they come from higher orders of it, the smallness of them could be understood. This problem remains to be studied.

Finally we make a bold speculation on an origin of the mass of the third family neutrino. In the ortho-para mixing model, three mass parameters exist, preon dynamics mass scale, $M$, ortho-para mixing masses, $M_i$, and $m$. The parameters $M$ and $M_i$ are related through the relation $m_2 \sim A^2/M$. However, the parameter $m$ is an isolated one given from outside. In addition, it is extraordinarily smaller than $|M_i|$. We have $A/m \sim 10^7$ since $m \sim m_3 \sim 1$ eV. These facts suggest that $m$ and $M_i$ have a different origin from each other. Perhaps, while ortho-para mixing masses are controlled by preon dynamics, $m$ originates from a physics at a deeper layer of nature, e.g., subpreon physics or Planck scale Physics. Note a relation that $m \sim 1$ eV $\sim (10^5$ GeV$)^2/M_{Pl}$ and the fact that $10^5$ GeV does not differ greatly from the preon dynamics mass scale, $\sim 10^6$ GeV. This might suggest that $m$ (namely, the mass of the third family neutrino) is a revelation at the preon level of the Planck scale physics. The possibility that the masses of the third family quarks and charged lepton have a subpreonc origin was already mentioned.

References


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