Discussion: "Applicability and Limitations of Simplified Elastic Shell Equations for Carbon Nanotubes" (Wang, C. Y., Ru, C. Q., and Mioduchowski, A., 2004, ASME J. Appl. Mech., 71, pp. 622–631)

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I wish to point out that there are equations for the vibration ([1], pp. 259–261) and buckling [2] of elastically isotropic circular cylindrical shells that are as accurate as, but much simpler than, the so-called Exact Flügge Equations (Model III) that the authors use as their standard of comparison for the two sets of approximate equations they analyze, namely, the (simplified) Donnell Equations (Model I) and the Simplified Flügge Equations (Model II). (I use the adjective "so-called" because there is no set of two-dimensional shell equations that is "exact.") On pp. 225–230 of [1] Niordson presents one possible derivation of the Morley-Koiter equations in terms of midsurface displacements in which the two equations of tangential equilibrium (or motion) are *identical* to the simplified Donnell equations—that is, the first two of the authors' Flügge equations (3) with the coefficients of the small parameter $(1-\nu^2)(D/EhR^2)$ set to zero—whereas the equation of normal equilibrium (or motion) may be obtained from the third Flügge equation by replacing the coefficient of $(1-\nu^2)(D/EhR^2)$ in brackets by $2R^2\nabla^2 w + w$, where $\nabla^2 = \partial^2/\partial x^2 + R^{-2}\partial^2/\partial\theta^2$.

A simplified set of buckling equations for an elastically isotropic circular cylindrical shell under uniform axial, torsional, and internal pressure loads may be found in [2] where, as may be seen there from Eqs. (3.25)–(3.29), the equations for buckling of a simply supported cylinder under a uniform axial load or a uniform internal pressure are considerably simpler than the analogous Flügge equations yet free of the defects of the simplified Donnell equations. (A notable feature of these equations is that Poisson's ratio ν appears only in the combined parameter D/EhR^2.)

It is also important to point out that these simple, accurate equations have been shown *rigorously* [3,4] to be as accurate as the Flügge equations for *any* problem that can be formulated as a variational principle using the Rayleigh quotient. The key is the demonstration that the modified strain-energy density that leads to the Morley-Koiter equations (and their analog for buckling) differs from the strain-energy density of the Flügge equations by terms of relative order h/R—terms that are of the same order as the intrinsic errors in the Flügge equations.

References

[1] Niordson, F. I., 1985, *Shell Theory*, North-Holland, Amsterdam.
[2] Danielson, D. A., and Simmonds, J. G., 1969, "Accurate Buckling Equations for Arbitrary and Cylindrical Elastic Shells," Int. J. Eng. Sci., **7**, pp. 459–468.
[3] Danielson, D. A., and Simmonds, J. G., 1971, "A Proof of the Accuracy of a Set of Simplified Buckling Equations for Circular Cylindrical Shells," *Developments in Theoretical and Applied Mechanics*, Vol. 5, University of North Carolina Press, pp. 1015–1028.
[4] Simmonds, J. G., 1974, "Comments on the Paper 'On an Accurate Theory for Circular Cylindrical Shells' by S. Cheng," ASME J. Appl. Mech., **41**, pp. 541–542.

