Journal of Applied Mechanics Discussion

Reexamination of Unsteady Fluid Dynamic Forces on a Two-Dimensional Finite Plate at Small Mach Numbers[1]

C. H. Ellen.[2] Matsuzaki and Ueda[1] have presented a rederivation of expressions in a slightly more restricted form ($M = 0$) than those derived originally in [1]. In stating that the earlier derivation has obtained expressions for the generalized pressures at low frequencies they appear to ignore the fact that precisely the same limits have been applied to obtain their result which, apparently, has a typographical error in equation (37d)[3] and which does not give an explicit form for Q_{nn} [1]. They confirm that Weaver and Unny [2] have introduced a zero-order error by changing the lower limit of integration (otherwise the same result may be derived from their integrals), but do not appear to agree that a similar problem exists with Ishii's results [3]. The point is that Ishii states in his Appendix that $a^R{}_{mn}$ $(M, k = 0) = (1 - M^2)^{-1/2}\, a^R{}_{mn}\ (M = 0, k = 0)$ (in his notation), a result which is incorrect and acknowledged to be incorrect in the discussion of Matsuzaki and Ueda (see also Table 2 of footnote one): this, together with the absence of any reference to singular terms in [3], is the basis of the comments made in [1].

However the purpose of this Discussion is not to go over what would appear to be old and well-known ground but to raise the question of the relevance and the use of this form of two-dimensional fluid dynamics in the study of the panel stability problem. As the frequency of oscillation becomes small the acoustic wavelength increases but must remain small compared with the panel width if the two-dimensional theory is to remain valid. This presents obvious difficulties to any analysis of the nature of panel motion in the neighborhood of a panel divergence condition or at low frequencies when the panel dimensions will violate this theoretical restriction. (In a wind tunnel experiment one might expect conditions to approach two-dimensionality if wall boundary-layer effects can be minimized.) The alternative approach is to examine stability when the acoustic wavelength is large compared with both panel dimensions: use of three-dimensional fluid dynamics generates the generalized pressures given in [1], again at "low frequency" only in the sense that the acoustic wavelengths are large compared with the panel dimensions. With these forms, the neighborhood of a divergence instability, which is, of course, identical for panels in two-dimensional flows and high aspect ratio panels in three-dimensional flows, may be examined with confidence. For a panel (or membrane) in an otherwise rigid surface it has been established generally that, as long as the frequency is not so large that the acoustic wavelengths are small compared with the panel dimensions, the only instability must be a divergence [4, 5]. Therefore the use of an approximate modal approach to examine theoretical oscillatory instabilities appears to serve no purpose unless some distinctly different feature is under examination. (The problem of instability prediction using a truncated convergent series is well known and graphically illustrated by the "supersonic membrane paradox" [6, 7].)

Recent work has identified the importance of the environment of the panel in controlling the type of instability which is likely to occur. When the trailing edge of a panel is free, oscillatory-type instabilities are present but these appear to be suppressed when the trailing edge is fixed [4, 5, 8]. While it has been shown that, when the rigid attachments (upstream and downstream) to the panel are sufficiently large, the divergence instability condition approaches that of a panel in an infinite baffle [5], the theory also suggests that small mass ratio flutter instabilities are possible when the rigid attachment lengths are small in relation to the panel length.

In conclusion, therefore, the scope of this paper by Matsuzaki and Ueda[1] appears limited, both in theory and application, and it would be useful if the proposed additional paper on the stability analysis using the generalized forces obtained in footnote one takes into account the points raised in this Discussion.

References

1 Ellen, C. H., "The Stability of Simply Supported Rectangular Surfaces in Uniform Subsonic Flow," ASME JOURNAL OF APPLIED MECHANICS, Vol. 40, 1973, pp. 68–72.

2 Weaver, D. S., and Unny, T. E., "The Hydroelastic Stability of a Flat Plate," ASME JOURNAL OF APPLIED MECHANICS, Vol. 37, 1970, pp. 823–827.

3 Ishii, T., "Aeroelastic Instabilities of Simply Supported Panels in Subsonic Flow," AIAA Paper No. 65-772, 1965.

4 Ellen, C. H., "The Stability of an Isolated Rectangular Surface Embedded in Uniform Subsonic Flow," ASME JOURNAL OF APPLIED MECHANICS, Vol. 44, 1977, pp. 201–206.

5 Shahrokh, K., and Ellen, C. H., "The Stability of Partially Rigid Two-Dimensional Surfaces in Uniform Incompressible Flow," *Journal of Sound and Vibration*, Vol. 65, No. 3, 1979, pp. 339–351.

6 Bolotin, V. V., *Nonconservative Problems in the Theory of Elastic Stability*, Macmillan, New York, 1963.

7 Ellen, C. H., "Approximate Solutions of the Membrane Flutter Problem," *AIAA Journal*, Vol. 3, No. 6, 1965, pp. 1186–1187.

8 Kornecki, A., Dowell, E. H., and O'Brien, J., "On the Aeroelastic Instability of Two-Dimensional Panels in Uniform Incompressible Flow," *Journal of Sound and Vibration*, Vol. 47, No. 2, 1976, pp. 163–178.

Authors' Closure

The authors would like to thank Dr. Ellen for his comments, especially on the similar applicability of our analysis to his corresponding one.

The main objective of the paper is to reexamine the previous aerodynamic analyses, that is, to clarify issues raised in several papers. Therefore, its scope is the same as those of the analyses reexamined. However, we show that there exists an essential difference between a compressible flow and an artificial incompressible model which is often adopted as a representative case. As a result, we indicated Ishii's error in the incompressible flow analysis which had been overlooked,

[1] By Y. Matsuzaki and T. Ueda, and published in the December, 1980, ASME JOURNAL OF APPLIED MECHANICS, Vol. 47, pp. 720–724.

[2] Associate Manager, Process Engineering, John Lysaght (Australia) Limited, Research & Technology Centre, P.O. Box 77, Port Kembla, 2505, N.S.W. Australia.

[3] A factor of $2/\pi$ appears to be missing from the "c" term.

and proposed a "zero-volume change" vibration mode for dynamic stability analysis.

The expression of equation (37*d*) contains an error as pointed out. However, all the numerical calculation was carried out by using the correctly expressed equation.

Next, let us reexamine Ishii's result for $M \neq 0$ in his Appendix. When $M \neq 0$ and $k = 0$, we obtain from equation (17) of our paper

$$\Phi^*(M \neq 0) = iU\xi/(\beta|\xi|)W^* \exp(-\zeta y)$$

because of $\zeta = \beta|\xi|$, while

$$\Phi^*(M = 0) = iU\xi/|\xi|W^* \exp(-\zeta y)$$

because of $\zeta = |\xi|$ for $M = k = 0$. Comparison between the equations yields $\Phi^*(M \neq 0) = (1 - M^2)^{-1/2}\Phi^*(M = 0)$. Therefore, it does not seem that Ishii's result is incorrect as far as the point just raised is concerned. However, we should repeat that his incompressible aeroforces are invalid for a symmetric natural mode when $k \neq 0$.

As for the proposed paper on the stability analysis, it has already been accepted for publication in this JOURNAL, and will soon be published [9]. In this area, several important problems remain to be resolved. It reexamines again an old and well-known problem, that is, the possibility of postdivergence flutter oscillation which is probably the most controversial point. Since a brief description of this topic is given in [9], we would refrain from repeating it here.

Finally, we'd like to indicate another typographical error: although the explanation in the text is correct, $k = 0$ should read $k = 1$ in the caption of Table 2 of our paper.

Reference

9 Matsuzaki, Y., "Reexamination of Stability of a Two-Dimensional Finite Panel Exposed to an Incompressible Flow," to be published in the ASME JOURNAL OF APPLIED MECHANICS.

A Correct Definition of Elastic and Plastic Deformation and Its Computational Significance[1]

J. Casey[2] and P. M. Naghdi.[3] Our purpose here is to discuss two points raised in a paper of Lubarda and Lee[1] and again in a more recent paper of Lee [1]. These points, which are both related to the use of full invariance requirements in plasticity, in the manner proposed by Green and Naghdi [2] are as follows:

(*a*) Whether it is necessary to demand independent invariance requirements for intermediate stress-free configurations associated with the decomposition of the deformation gradient in the form $\mathbf{F} = \mathbf{F}_e\mathbf{F}_p$; and

(*b*) whether it is possible to specify $\mathbf{F}_e$ to be a symmetric positive definite tensor in the construction of a general theory.[4]

With regard to (*a*), Lubarda and Lee[1] and again Lee [1] demand that only partial invariance requirements be satisfied. Similarly, with reference to (*b*), they assume that without loss in generality the elastic part $\mathbf{F}_e$ of the deformation gradient $\mathbf{F}$ can be chosen to be symmetric. We disagree with the contents of the authors' paper[1] and reference [1] insofar as (*a*) and (*b*) are concerned.

In our recent Brief Note [3], we emphasized the physical grounds for invoking full invariance requirements of the type proposed by Green and Naghdi [2], whenever use is made of intermediate stress-free configurations. Lubarda and Lee (footnote 1, p. 37 after equation (33)) appeal to only a restricted form of invariance requirements, and more recently Lee [1, p. 867] has objected to the use of full invariance requirements because they seem to him "to demand two objectivity checks for the single [current] configuration" and therefore to involve a redundancy. We discuss the two points (*a*) and (*b*) above using the notation of [3].

The invariance requirements in [2, 3] are based on the idea that all configurations, global or local, which differ from any physically possible configuration of a body by a rigid displacement are equivalent. All such configurations are physically indistinguishable and should therefore play an equal role in any theory of material behavior. The invariance requirements insure that these configurations enter the theory in a physically meaningful way. The intermediate stress-free configuration, if it exists, *is* a possible configuration of the body and can, in the words of Lee [1, p. 863], "be achieved physically by . . . *destressing*." It was *precisely because the intermediate stress-free* configuration (whenever it exists) is physically realizable, that in [2, 3] it was assumed that this configuration is subject to *exactly the same* invariance requirements as any other possible configuration of the body. As a consequence, two different orthogonal tensors must appear in the transformation rules for certain fields. For example, when the present configuration and the intermediate stress-free configuration are subjected at time t to arbitrary independent rigid displacements, involving rotations $\mathbf{Q}(t)$ and $\bar{\mathbf{Q}}(t)$, respectively, the tensor $\mathbf{F}_e$ is transformed into

$$\mathbf{F}_e^+ = \mathbf{Q}(t)\mathbf{F}_e\bar{\mathbf{Q}}^T(t). \tag{1}$$

The reason for the appearance of two different rigid body rotation tensors in (1) is simple and may be explained even without appealing to either the stress-free nature of the intermediate configuration, or the elastic-plastic character of the body. To see this, consider any deformable body which is first deformed from a reference configuration κ_0 into a configuration κ_1, and then from κ_1 into the present configuration κ. The deformation gradient $\mathbf{F}$ in the configuration κ can then be expressed as[5]

$$\mathbf{F} = \mathbf{F}_2\mathbf{F}_1 \tag{2}$$

where $\mathbf{F}_1$ is the deformation gradient in the configuration κ_1 and $\mathbf{F}_2$ is the gradient in the configuration κ relative to the configuration κ_1. Since both κ_1 and κ are physically realizable configurations of the body, they are both subject to the usual invariance requirements. If κ_1 is replaced at time t_1 by a configuration κ_1^+ that differs from it by an arbitrary rigid displacement whose rotation tensor is $\mathbf{Q}_1(t_1)$, then $\mathbf{F}_1$ is transformed into

$$\mathbf{F}_1^+ = \mathbf{Q}_1(t_1)\mathbf{F}_1. \tag{3}$$

Similarly, if κ is replaced at the present time t by a configuration κ^+ that differs from κ by an arbitrary rigid displacement whose rotation tensor is $\mathbf{Q}(t)$, $\mathbf{F}$ is transformed into

$$\mathbf{F}^+ = \mathbf{Q}(t)\mathbf{F}. \tag{4}$$

The gradient in the configuration κ^+ relative to κ_1^+ is again given by the chain rule in the form

$$\mathbf{F}_2^+ = \mathbf{F}^+(\mathbf{F}_1^+)^{-1}. \tag{5}$$

From (2)–(5), it follows that

$$\mathbf{F}_2^+ = \mathbf{Q}(t)\mathbf{F}_2\mathbf{Q}_1^T(t_1). \tag{6}$$

Since the rigid displacements by which κ_1 is taken into κ_1^+ and κ is

[1] By V. A. Lubarda and E. H. Lee, and published in the March, 1981, issue of the ASME JOURNAL OF APPLIED MECHANICS, Vol. 48, pp. 35–40.

[2] Department of Mechanical Engineering, University of Houston, Houston, Texas 77004.

[3] Department of Mechanical Engineering, University of California, Berkeley, Calif. 94720.

[4] By a "general theory," we mean one that is at least broad enough to include anisotropic materials and to be valid for all possible motions of the elastic-plastic body.

[5] Since the two configurations κ_1 and κ are global, (2) is obtained by means of the chain rule. A similar form holds if κ_1 is only a local configuration.