Discussion: "Boundary Element Analysis of Multiple Scattering Waves in High Performance Concretes" (Sato, Hirotaka, Kitahara, Michihiro, and Shoji, Tetsuo, 2005, ASME J. Appl. Mech., 72, pp. 165–171)

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This discussion intends to make comments on the recent article of Sato et al. [1] regarding their descriptions of self-consistency conditions for the effective medium in the multiple scattering theory. Equations (13) and (14) in Ref. [1] were obtained originally by Yang and Mal [2] by extending the static generalized self-consistent method (GSCM) with recourse to Waterman and Truell's theory [3]. Recently, the dynamic GSCM was recast [4] based on a simple consideration of wave energy in the model applying the energy theorem for the scattering in absorbing media [5], and also the equivalence between Eqs. (13) and (14) and

$$A_1^*(K_p,\mathbf{e}_1) = B_1^*(K_{sv},\mathbf{e}_1) = 0$$

was shown. In the dynamic GSCM, the self-consistency condition of the effective medium (or the dispersion relation) reduces to vanishing of the wave energy extinction in the model. It should be noted that this can be further generalized to different self-consistent-type formulations of the multiple scattering problem. A simple but fairly general way to show this might be to express the Ewald formula [3] in the self-consistent form

$$K = K + \left(\frac{2\pi n_o}{K}\right)F(\mathbf{e}_1)$$

or therefore

$$F(\mathbf{e}_1) = 0$$

where n_o is the scatter number density and $F(\mathbf{e}_1)$ denotes the forward scattering amplitude defined in the effective medium with consideration of multiple scattering effect. Note that subscript denoting the wave mode is suppressed for brevity.

In the article by Sato et al. [1], several logical and conceptual problems are found (Eqs. (16)–(19)). For example, it is obvious that Eq. (16) is just a sufficient condition to Eqs. (13) and (14), thus the equivalence cannot be so asserted. So it is again with Eq. (19) to Eqs. (17) and (18). Moreover, Eq. (19)

$$A_\alpha^*(K_p,\hat{\mathbf{x}}) = B_\alpha^*(K_{sv},\hat{\mathbf{x}}) = 0 \text{ for } \forall\, \hat{\mathbf{x}} \neq \pm\mathbf{e}_1$$

can be brought up more reasonably from the known physical fact, "the absence of coherence in non-propagation directions," than from Eqs. (17) and (18) which appear to be rather trivial when reminding that the Waterman and Truell formula is meaningful only in the propagation direction. All this awkwardness seems to arise because the dynamic GSCM was formulated in the framework of the Waterman and Truell theory, which is not necessary at all. Even though the paper [4] provides a theoretically simple proof for a physically conceivable fact, the result takes away all those problems in the above caused in the constraint of Waterman and Truell formalism.

References

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