

the stress and deformation histories to which the element has been subjected. Because of the incremental or flow-type nature of the plasticity law, strain rate appears in the constitutive relation. For application in stress and deformation analysis, total strain rate, and stress and stress rate are needed in the relation. Such a constitutive law is deduced from the kinematic relation expressing the total strain rate in terms of the elastic and plastic components, by substituting from the plasticity law and the time derivative of the usual elasticity law for the plastic and elastic strain rates, respectively. This yields a constitutive relation for the total strain rate in terms of the stress, stress rate, and variables expressing the history of deformation.

For the case when purely elastic deformation could occur in destressing to zero stress, the choice of destressing by pure deformation without rotation, expressed by the symmetric deformation gradient matrix  $V^e$ , constitutes a correct definition of elastic deformation, although not the most general one since arbitrary rigid-body rotation of the unstressed state provides an alternative unstressed state. However, utilizing this particular choice of elastic deformation does permit convenient elimination of the elastic strain-rate term in the kinematic relation for total strain. Any rotation which might have been included in the elastic deformation is automatically embodied in the associated plastic deformation and is determined by the solution process to provide whatever compatibility is needed in the configuration of the body on the general deformation path. Thus a correct, but not the most general, definition of elastic and plastic deformation is achieved.

In order to investigate objectivity of the resulting constitutive equation (to be used in stress and deformation analysis) which contains stress-rate and total strain-rate variables, we need only consider superposed arbitrary rigid rotation on the current state of the body on the general deformation path. This is done in the paper under discussion and the constitutive relation is found to be objective.

Since this applies for any general deformation history, we can choose the deformation path to follow pure elastic destressing with or without rotation from the considered state on the general deformation path, and this establishes objectivity of the unstressed state for rotations independent of that utilized in the objectivity check carried for the body on the general deformation path prior to destressing. Thus we established the full invariance requirement stressed by the discussers on the basis of a simpler analytical development. We thus assert that the statement that we "demand that only partial invariance requirements be satisfied" is incorrect. The fact that it is necessary to employ the constitutive relation containing only the current rate of total strain to substitute into the usual variational principle for determination of the velocity field eliminates the need to simultaneously include the plastic deformation gradient in establishing objectivity for any state. Although the subject paper considers only isotropic elastic and plastic laws, the approach explained in this Closure can be utilized for anisotropic laws by taking into account rotation of the anisotropic characteristics with the deformation of the unstressed state.

Professors Casey and Naghdi maintain that even when the intermediate (unstressed) configuration is not occupied in a given motion, the transformation law involving simultaneous independent rotations of the intermediate and final states must be included in the analysis. In this connection it is interesting to consider the analogous problem of the evaluation of thermal stresses. The theory can be formulated by considering an intermediate state in which the temperature variation occurs, and hence thermal expansion, but the condition of zero stress is maintained. This configuration would comprise an incompatible state of strain and hence a discontinuous map. Superimposing the thermal stresses then produces a compatible final configuration which can be expressed by a differentiable mapping from the initial undisturbed state at uniform temperature. The geometrical characteristics of the intermediate state have much in common with those of the unstressed state in elastic-plastic theory, yet to our knowledge the alleged difficulties often ascribed to the intermediate unstressed state in elastic-plastic theory have never been mentioned in the thermal stress case. Nor have we heard of prescribing an involved objectivity principle including independent rotations of the thermally

expanded configuration and the final thermally stressed configuration. We are not surprised by this omission since it seems natural to us that since the purely thermally expanded configuration does not appear in the final thermoelastic constitutive relation, no advantage would accrue to offset the additional mathematical complexity of including independent rotation of the thermally expanded intermediate state.

We take this opportunity to mention that typographical errors appeared in equations (22), (40), and (61) of the subject paper and they have been corrected in an Erratum which appeared in the September, 1981, issue of the ASME JOURNAL OF APPLIED MECHANICS.

## Basic Transport Equations in Ascending Equiangular Spiral Polar Coordinates<sup>1</sup>

C.-Y. Wang,<sup>2</sup> In this paper,<sup>1</sup> Dr. Ali stated that the torus seems to be the only curved tube which has been analyzed. This is not true. Sinusoidal, once-curved and twice-curved planar tubes (center line lies in a plane) have been studied by Murata, Miyake, and Inaba [1]. Helical tubes were recently investigated by Wang [2].

As soon as the Christoffel symbols for a curvilinear system are obtained, it is fairly easy to produce the field equations through tensors. Murata, et al. [1], developed the coordinate system for the general planar curved tube which would include Ali's work as a special case. The planar coordinates are always orthogonal. Wang [3] developed the coordinate system for the general nonplanar curved tube (arbitrary spatial curvature of the center line) which includes both the helical coordinates and Murata's coordinates as special cases.

It is uncertain that the ascending equiangular spiral polar coordinates are suitable for analytical and numerical work as claimed. This kind of geometry does not lead to an asymptotic state and conditions at both ends highly affect the field properties. Both analytical or numerical attempts would be very difficult.

### References

- 1 Murata, S., Miyake, Y., and Inaba, T., "Laminar Flow in a Curved Pipe With Varying Curvature," *Journal of Fluid Mechanics*, Vol. 73, 1976, pp. 735-752.
- 2 Wang, C.-Y., "The Helical Coordinate System and the Temperature Distribution Inside a Helical Coil," ASME JOURNAL OF APPLIED MECHANICS, Vol. 47, 1980, pp. 951-953.
- 3 Wang, C.-Y., "On the Low Reynolds Number Flow in a Helical Pipe," *Journal of Fluid Mechanics*, Vol. 108, 1981, pp. 185-194.

<sup>1</sup> By S. Ali, and published in the March, 1981, issue of the ASME JOURNAL OF APPLIED MECHANICS, Vol. 48, pp. 190-192.

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## On the Uniqueness and Stability of Endochronic Theory<sup>1</sup>

I. S. Sandler,<sup>2</sup> In the paper,<sup>1</sup> the author presents several conclusions with respect to the uniqueness and stability properties of endochronic theories based on considerations first presented in [1]. Some of the conclusions of the author's paper,<sup>1</sup> however, are incorrect and

<sup>1</sup> By B. J. Hsieh, and published in the December, 1980, issue of the ASME JOURNAL OF APPLIED MECHANICS, Vol. 47, pp. 748-754.

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