The Mechanism of the Anomalous Energy Shift between s-States in Mirror Nuclei with a Halo Structure

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A mechanism of the anomalous energy shift between s-states in mirror nuclei is investigated by using the complex scaling method (CSM). The s-states are described by a resonant core+p system and by a loosely bound core+n system, where the latter reproduces a neutron halo structure. To understand the mechanism of the anomalous energy shift appearing in the resonant s-state of the core+p system, we pick out the complex expectation values of the Coulomb potential and kinetic energy operators from the Hamiltonian for resonant states, using the procedure recently developed in the framework of the CSM. We find that the kinetic energy reduction is large, in addition to the Coulomb energy reduction in the ordinary Thomas-Ehrman shift. As a result, a large energy shift is reasonably explained by the combination of two mechanisms of the kinetic energy reduction and the Coulomb energy reduction.

§1. Introduction

Recently, much interest has been concentrated on the study of the neutron- and proton-drip-line nuclei. The problem of anomalous s-states in the light drip-line nuclei is one of the most interesting subjects in studies on unstable nuclei and/or nuclear synthesis. For example, we can point out a virtual s-wave problem of the $^9\text{Li}$-n subsystem in $^{11}\text{Li}$, and an inversion problem of the $1/2^+$ ($s_{1/2}$) ground and the $1/2^-$ ($p_{1/2}$) excited states in $^{11}\text{Be}$. The anomalous properties of the s-wave in p-shell nuclei are the key in understanding the so-called halo structure. Although many studies on this problem already exist, finding the mechanism to explain the s-states in the same energy region as the p-states is still an open problem. On the other hand, in connection with the energy shift of s-wave states, there is another interesting problem. This concerns the Thomas-Ehrman effect, which might be a useful tool for the study of the s-wave halo structure through spectroscopic analyses of mirror nuclei.

The Thomas-Ehrman shifts are systematically observed in the mirror states of the s-wave, as shown in Fig. 1. The energy differences ($\Delta E_{s-d}$) between $1/2^+$ and $5/2^+$ levels, corresponding to the s- and d-wave states, respectively, in the core+p system are enhanced from those ($\Delta E_{s-d}^n$) in the mirror core+n system. These enhancements of the differences are thought to be caused by the lowering of the s-orbit energy; namely, the Thomas-Ehrman shifts can be understood based on the Coulomb energy reduction of the s-wave. The repulsive Coulomb energy is reduced for a spatially extended s-wave nucleon in comparison with the case of a d-wave nucleon because of the lack of a centrifugal potential. About a half century ago,
this idea was proposed for normal nuclei existing along the stable line. Recently, in close connection with the problem of the "halo structure" in nuclei near the drip line, interest in the Thomas-Ehrman shift has been revived. If an s-wave neutron is weakly bound with respect to the core+n threshold, the Thomas-Ehrman shift is considered to become much larger in the mirror core+p system in comparison with the deeply bound case.\(^8\)

In the previous works,\(^3,5\) we mentioned that largely enhanced Thomas-Ehrman shifts were calculated for mirror nuclei \(^{10}\)Li-\(^{10}\)N and \(^{11}\)Be-\(^{11}\)N. The nuclei of \(^{10}\)Li and \(^{11}\)Be have been studied as typical neutron-rich systems for the last several years. As mentioned above, the understanding of low-lying s-states in these nuclei is the main point of the matter. We showed that the energy shift of s-states in the mirror nuclei of \(^{10}\)N and \(^{11}\)N causes a large enhancement of the Thomas-Ehrman shift due to the very loosely (barely) bound nature of the neutron. In those papers,\(^3,5\) we pointed out that the kinetic energy reduction due to the extended wave function of loosely bound s-states is also important in connection with the "halo structure", though the Thomas-Ehrman effect is essentially caused by the reduction of the Coulomb energy as explained above. The purpose of this paper is to clarify the two mechanisms in loosely bound core+N systems: (i) the original Thomas-Ehrman shift (the Coulomb energy reduction), (ii) the kinetic energy reduction in the core+p systems. Such a development of an understanding regarding the s-wave energy shift, based on the above two mechanisms, is very interesting in the investigation of the mechanism bringing about the halo structure.

In order to study the resonant mechanism of the drip-line nuclei, the complex scaling method (CSM)\(^9\) has been developed and applied to many nuclei,\(^10-12\) Recently, Homma, Myo and one of the present authors (Katô)\(^13\) discussed the calculation of the resonant matrix elements by using the complex scaled wave functions. As an example, they showed that the complex expectation value of \(\langle r^2 \rangle\) is easily calculated.
calculated and exactly equal to that of the convergent factor method. Based on this recent development of the CSM, we can pick out the complex expectation values of operators such as the Coulomb potential and the kinetic energy for resonant states.

In this paper, we investigate the resonant properties of the s-wave energy shift which comes from the kinetic energy reduction in addition to the original Thomas-Ehrman effect due to the Coulomb energy reduction. In order to divide the kinetic energy reduction and the Coulomb energy reduction, we use the complex expectation values of resonant states by using the new method which is developed in Ref. 13). First, in §2, we describe the present model and method, and show its reliability with a simple potential model. Second, in §3, we apply it to a core+N model of weakly- and loosely-bound and resonant systems, whose relative angular momenta are those of s- and d-waves. Lastly, a summary and conclusion are given.

§2. Method

2.1. The complex scaling method

We describe the practical prescription of how to solve bound and resonant states by using the CSM. In the CSM, we define the following transformation $U(\theta)$ of spatial coordinates and their conjugate momenta:

$$U(\theta) : \begin{align*} r &\rightarrow r \cdot \exp(i\theta), \\ p &\rightarrow p \cdot \exp(-i\theta). \end{align*} \quad (2.1)$$

Here $\theta$ is a scaling parameter of a real number. By using this transformation, we rewrite the Shrödinger equation $H\Phi = E\Phi$ as

$$H_\theta \Phi_\theta = E\Phi_\theta, \quad (2.2)$$

where $H_\theta = U(\theta)HU^{-1}(\theta)$ and $\Phi_\theta = U(\theta)\Phi$.

According to the ABC theorem, we can obtain resonance energies ($E_r$) and widths ($\Gamma$) as complex eigenvalues of the complex scaled Hamiltonian (non-hermitian) $H_\theta$ with the complex scaled wave functions $\Phi_\theta$; resonant eigenvalues should be complex numbers ($E_r - i\Gamma/2$) independent of the scaling parameter $\theta(> \frac{1}{2}\tan(\Gamma/(2E_r)))$. It is also a very promising property of the CSM that the Hamiltonian $H(\theta)$ gives the same bound-state (real and negative) eigenvalues as those of the original (non-scaled) Hamiltonian, independently of $\theta$. All other eigenvalues of $H_\theta$ — except those of bound and resonant solutions — depend on $\theta$, and this dependence is regularly proportional to $\exp(-2i\theta)$.

2.2. The complex expectation value

We now briefly explain the complex expectation value of an operator with bound and resonant solutions obtained by using the CSM. The complex expectation value is usually defined by introducing a convergent factor $\exp(-\alpha r^2)$ as

$$\langle \tilde{\Phi}(k)|\hat{O}|\Phi(k)\rangle = \lim_{\alpha \to 0} \int dr \Phi^*_1(-k^*, r)\hat{O}\Phi_2(k, r)e^{-\alpha r^2}. \quad (2.3)$$
By using Cauchy’s theorem, this complex expectation value of the convergent factor method is equal to that of the CSM as

\[(\Phi_{\theta}(k)|\hat{O}_{\theta}|\Phi_{\theta}(k)) \equiv \int dr \Phi(k, re^{i\theta})\hat{O}_{\theta}\Phi(k, re^{i\theta})\]

\[= \lim_{a \to 0} \int dr \Phi^*(-k^*, r)\hat{O}\Phi(k, r)e^{-a|r|^2}, \tag{2.5}\]

where the round brackets represent the so-called c-products. Then, we can easily calculate the complex expectation values by using the complex scaled wave function and determine those which have no \(\theta\)-dependence (\(\theta > \frac{1}{2}\tan(\Gamma/(2E_r))\)). Numerically, this \(\theta\)-independence is confirmed for some operators, such as \(r^2\), as discussed in a recent paper.13)

2.3. An interpretation of the complex expectation value of operators in the Hamiltonian

The complex expectation value of the Hamiltonian with the obtained complex eigenfunction of bound and resonant states should be equal to the obtained complex energy eigenvalue:

\[(\Phi_{\theta}|H_{\theta}|\Phi_{\theta}) = E(\Phi_{\theta}|\Phi_{\theta}). \tag{2.6}\]

Then, the sum of the complex expectation values of the kinetic energy operator and the potential operator in the Hamiltonian is equal to the complex energy eigenvalue:

\[E = (\Phi_{\theta}|T_{\theta}|\Phi_{\theta}) + (\Phi_{\theta}|V_{\theta}|\Phi_{\theta}). \tag{2.7}\]

Here \((\Phi_{\theta}|\Phi_{\theta})\) is normalized to a unity by using the \(L^2\) property of the complex scaled eigenfunction. In the above equation, the complex scaled eigenfunction and the complex scaled operators have \(\theta\) dependence, respectively. Then, we may naively expect that the individual part of the complex expectation value has a \(\theta\)-dependence. However, as mentioned in the previous subsection, \((\Phi_{\theta}|\hat{O}_{\theta}|\Phi_{\theta})\) does not have any \(\theta\) dependence. Therefore, we can uniquely divide the complex expectation value of the Hamiltonian into kinetic energy and potential energy parts, which are \(\theta\)-independent when \(\theta > \frac{1}{2}\tan(\Gamma/(2E_r))\).

When we write the complex energy eigenvalue as \(E = \mathcal{R}(E) + i\mathcal{I}(E)\), we can easily interpret \(\mathcal{R}(E)\) as a resonance energy \(E_r\) and \(-\mathcal{I}(E)\) as a decay width \(\Gamma/2\) from the relation,

\[e^{-iEt} = e^{-i\mathcal{R}(E)E}e^{i\mathcal{I}(E)t} = e^{-iE_r t}e^{-\Gamma/2t}. \tag{2.8}\]

Since, \(\mathcal{R}(E)\) is \(\mathcal{R}(\langle T \rangle) + \mathcal{R}(\langle V \rangle)\), and \(\mathcal{I}(E)\) is \(\mathcal{I}(\langle T \rangle) + \mathcal{I}(\langle V \rangle)\), we can write as

\[E_r = \mathcal{R}(\langle T \rangle) + \mathcal{R}(\langle V \rangle) \equiv T_r + V_r, \tag{2.9}\]

\[\frac{\Gamma}{2} = -\mathcal{I}(\langle T \rangle) - \mathcal{I}(\langle V \rangle) \equiv \frac{\Gamma^T}{2} + \frac{\Gamma^V}{2}, \tag{2.10}\]

where \(\langle O \rangle\) represents \((\Phi_{\theta}|O_{\theta}|\Phi_{\theta})\). The physical meaning of \(T_r\) should be the energy of the kinetic part of resonant states, and \(V_r\) should be that of the potential part.
According to an interpretation, the imaginary part of the complex expectation value is the uncertainty of the observation itself corresponding to the real part. As far as the imaginary parts of the complex expectation values for the kinetic energy and potential operators are concerned, their sum is related to the decay width \( \Gamma \), which can be observed. We divide \( \Gamma \) into \( \Gamma^T \) and \( \Gamma^V \), which are the imaginary parts of the complex expectation values of \( T \) and \( V \), respectively. However, it is worthwhile mentioning that we sometimes have negative values (\( \Gamma^0 < 0 \)) in the expectation values. Then, the individual part of \( \Gamma^T \) and \( \Gamma^V \) does not have the meaning of the partial width for each operator and it might only represent “an ability to decay”.

2.4. The complex expectation value in a schematic potential model

In this subsection, as an example, we calculate the complex expectation values of the kinetic energy and potential energy operators for a simple potential model, which has been thoroughly studied. We will show not only that their sum is equal to the energy eigenvalue but also that each expectation value is \( \theta \) independent. These points were discussed in the previous subsection. Later, by using those properties, we will divide the potential expectation value into nuclear and Coulomb parts, and discuss the \( s \)-wave resonant properties coming from the Coulomb potential.

The Hamiltonian operator is assumed as follows:

\[
H = -\frac{\hbar^2}{2\mu} \nabla + V(r), \quad \left( \frac{\hbar^2}{\mu} = 1 \right),
\]

\[
V(r) = V_a(r) + V_r(r) = -8.0e^{-0.16r^2} + 4.0e^{-0.04r^2},
\]

where \( V_a(r) \) is an attractive and short range potential and \( V_r(r) \) is a repulsive and long range potential. Therefore \( V(r) \) has a potential barrier as shown in Fig. 2.

We calculate the complex expectation values \( \langle T \rangle \), \( \langle V_a \rangle \) and \( \langle V_r \rangle \) in the case of the \( 0_2^+ \) state (\( E_r = 1.63 \) MeV, \( \Gamma = 0.246 \) MeV). The \( \theta \)-dependences are shown in Fig. 3. The left figure (a) shows real parts of the complex expectation values, and the right figure (b) shows the imaginary parts. As seen in Fig. 3, the complex expectation values have no \( \theta \)-dependence for both real and imaginary parts, and their sum is calculated as

\[
\langle H \rangle = \langle T \rangle + \langle V_a \rangle + \langle V_r \rangle = (1.070610 - 0.068053i) + (-2.192036 + 0.100461i) + (2.753707 - 0.044702i) = 1.632281 - 0.122946i \text{ (MeV)}.
\]

This value is exactly the same as the obtained eigenvalue \( E_{0_2^+} \).
§3. Analyses of the $s$-wave energy shift

In this section, for the mirror state of the “weakly” bound $s$-state in the core+$n$ system ($S_n=1$ MeV), we show that the $s$-wave energy shift is dominated by the Thomas-Ehrman effect (the Coulomb energy reduction). On the other hand, for the mirror state of the “loosely” bound $s$-state in the core+$n$ system ($S_n=0.1$ MeV), we point out that in addition to the Thomas-Ehrman effect the kinetic energy reduction is also important. Here, it is worthwhile mentioning that we distinguish the “loosely” bound state from the “weakly” bound state; “loosely bound” means that the separation energy of a neutron is in the order of keV, and “weakly bound” means this is in the order of MeV. Though this may not be the exact definition of “weakly bound” and “loosely bound”, it is sufficient for the present discussion.

3.1. The Thomas-Ehrman effect on the $s$-wave energy shift for “weakly” bound states

In this subsection, we investigate the Thomas-Ehrman effect in the $s$-wave energy shift for the mirror states of the “weakly” bound $s$-wave neutron with halo structure. In order to study this, we use the Hamiltonian

$$H = -\frac{\hbar^2}{2\mu} \nabla + V_l^N(r) + V_C(r), \quad (3.1)$$

$$V_s^N(r) = V_s \exp \left( -\frac{r^2}{b_s^2} \right), \quad (3.2)$$

$$V_d^N(r) = V_d \exp \left( -\frac{r^2}{b_d^2} \right), \quad (3.3)$$

$$V_C(r) = Z \frac{e^2}{r} \text{erf}(sr), \quad (3.4)$$

where $V_l^N(r)$ is a nuclear potential whose $(V_s, b_s)$ or $(V_d, b_d)$ is determined by the separation energies $S_n$ of a nucleon in $s$- or $d$-orbit, respectively. $V_C(r)$ is a Coulomb potential with the charge $Z$ of the core nucleus.
In order to make the $s$- and $d$-wave weakly bound states ($S_n=1$ MeV) and the $s$-wave halo, we choose the parameters $\mu=9/10M$, $b_s=b_d=1.2(10)^{1/3}$ fm, $V_s=-73.51$ MeV, $V_d=-96.55$ MeV and $s=0.5$ fm$^{-1}$. By using these parameters, we obtain a root mean square radius $R_{\text{rms}}=5.79$ fm for the $s$-state and $R_{\text{rms}}=3.69$ fm for the $d$-state.

In Fig. 4, we show the behavior of resonant poles for $s$- and $d$-waves, increasing the charge $Z$ from 0 to 9 in order to see the energy difference due to the Coulomb interaction. As mentioned in the previous section, the real part of the resonant pole is the resonant energy $E_r$ and the imaginary part is half of the decay width $\Gamma$. Then, the difference between the real parts of $s$- and $d$-wave resonant poles implies an energy shift which may correspond to the Thomas-Ehrman shift in the usual bound states as

$$\Delta E_{s-d} = \Delta E_{s-d}(Z) + \Delta E_{s-d}(Z = 0)$$

$$= \mathcal{R}(E_d(Z) - E_s(Z)) + \mathcal{R}(E_d(0) - E_s(0))$$

$$\simeq \mathcal{R}(E_d(Z) - E_s(Z)),$$

because $E_d(0) \approx E_s(0)$. In the present calculations, both $E_d(0)$ and $E_s(0)$ are $-1$ MeV.

In the case $Z=9$, as is seen from Fig. 4, the difference is as large as $\Delta E_{s-d} = 1.26$ MeV. According to the Thomas-Ehrman mechanism, this energy shift is estimated as

$$\Delta E_{s-d} = \mathcal{R}(E_d(Z) - E_s(Z))$$

$$\approx \mathcal{R}(|d|V_C^*|d) - |s|V_C^*|s|)$$

$$\equiv \Delta V_{s-d}^C.$$

If we use $R_{\text{rms}}$ values in the case $Z=0$ as mentioned above, $R_{\text{rms}}^d=5.79$ fm and $R_{\text{rms}}^s=3.69$ fm, we can roughly evaluate the Coulomb potential energy:

$$\Delta V_{s-d}^C = Z e^2 \left( \frac{1}{R_{\text{rms}}^d} - \frac{1}{R_{\text{rms}}^s} \right)$$

$$= 9(1.44)(0.271 - 0.173) = 1.27 \text{ MeV}.$$

The estimated value is very near to the calculated energy shift $\Delta E_{s-d}=1.26$ MeV, and this result confirms the Thomas-Ehrman mechanism due to the Coulomb interaction.

In order to see the Coulomb energy difference $\Delta V^C$ more accurately, we calculate the complex expectation value of the Coulomb potential, $\langle V^C \rangle$, together with the
Table I. The obtained complex expectation values of $s$-states (left) and $d$-states (right) with complex energy eigenvalues.

<table>
<thead>
<tr>
<th>$Z$</th>
<th>$(V_s^c)$ (MeV)</th>
<th>$(T_s) + (V_s^N)$ (MeV)</th>
<th>$E_s$ (MeV)</th>
<th>$(V_d^c)$ (MeV)</th>
<th>$(T_d) + (V_d^N)$ (MeV)</th>
<th>$E_d$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00</td>
<td>-1.00</td>
<td>0.00</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-1.00</td>
</tr>
<tr>
<td>1</td>
<td>0.33</td>
<td>-0.99</td>
<td>0.45</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-0.55</td>
</tr>
<tr>
<td>2</td>
<td>0.63</td>
<td>-0.97</td>
<td>0.89</td>
<td>-0.99</td>
<td>-0.10</td>
<td>-0.10</td>
</tr>
<tr>
<td>3</td>
<td>0.88</td>
<td>-0.91</td>
<td>1.31 - 0.00i</td>
<td>-0.97 + 0.00i</td>
<td>0.34 - 0.00i</td>
<td>0.34 - 0.00i</td>
</tr>
<tr>
<td>4</td>
<td>1.03 - 0.16i</td>
<td>-0.79 + 0.15i</td>
<td>0.24 - 0.01i</td>
<td>1.72 - 0.03i</td>
<td>-0.95 + 0.02i</td>
<td>0.77 - 0.00i</td>
</tr>
<tr>
<td>5</td>
<td>1.35 - 0.37i</td>
<td>-0.85 + 0.30i</td>
<td>0.50 - 0.07i</td>
<td>2.14 - 0.06i</td>
<td>-0.94 + 0.05i</td>
<td>1.20 - 0.01i</td>
</tr>
<tr>
<td>6</td>
<td>1.68 - 0.55i</td>
<td>-0.91 + 0.40i</td>
<td>0.77 - 0.16i</td>
<td>2.55 - 0.10i</td>
<td>-0.93 + 0.08i</td>
<td>1.62 - 0.03i</td>
</tr>
<tr>
<td>7</td>
<td>2.01 - 0.72i</td>
<td>-0.96 + 0.47i</td>
<td>1.05 - 0.25i</td>
<td>2.97 - 0.15i</td>
<td>-0.92 + 0.10i</td>
<td>2.05 - 0.05i</td>
</tr>
<tr>
<td>8</td>
<td>2.37 - 0.90i</td>
<td>-1.02 + 0.54i</td>
<td>1.34 - 0.36i</td>
<td>3.39 - 0.21i</td>
<td>-0.91 + 0.14i</td>
<td>2.48 - 0.07i</td>
</tr>
<tr>
<td>9</td>
<td>2.73 - 1.07i</td>
<td>-1.08 + 0.59i</td>
<td>1.64 - 0.48i</td>
<td>3.80 - 0.27i</td>
<td>-0.91 + 0.17i</td>
<td>2.90 - 0.10i</td>
</tr>
</tbody>
</table>

The sum of those of the kinetic energy and the nuclear potential $\langle T \rangle + \langle V^N \rangle$. As shown in Table I, we confirm that $\Delta E_{s,d} \approx \Delta V_{s,d}^C$. For the $\langle T \rangle + \langle V^N \rangle$ part, the kinetic energy and the nuclear potential energy cancel each other in both $s$- and $d$-states. Then, the energy difference coming from $\langle T \rangle + \langle V^N \rangle$ is not much larger than $\langle V^C \rangle$ in the $s$-wave energy shift. Next, we discuss the minor deviation of $\Delta E_{s,d}$ from the simple Thomas-Ehrman mechanism arising in the “weakly” bound system (this deviation cannot be neglected in the “loosely” bound system which is investigated in the next subsection). As is seen in Table I, the real part of the complex expectation value of the Coulomb potential increases with increasing charge $Z$. Their rate of increase is smaller than that of a linear dependence on $Z$, because wave functions of a resonant nucleon can be spatially extended differently from a usual bound nucleon and thus reduce the Coulomb energies. On the other hand, as is seen from Table I, the decreases of the kinetic energy and the nuclear potential energy almost cancel each other, and then $\langle T \rangle + \langle V^N \rangle$ is approximately constant ($\sim -1$ MeV in the present calculation). Since we fit $S_n=1$ MeV for the core+$n$ system, if $\langle T \rangle + \langle V^N \rangle$ is smaller than $-1$ MeV, as in the $s$-wave core+$p$ case of $Z=8$ and 9, it is implied that the kinetic energy decrease is larger than the potential energy increase in the core+$p$ system. This kinetic energy reduction mechanism for the core+$p$ system ($Z=8, 9$) is expected to be caused by the Coulomb barrier top effect on the $s$-wave (as will be discussed in the next subsection), though it is very small in this weakly bound case.

3.2. The Coulomb barrier top effect on the $s$-wave energy shift for a “loosely” bound system

We discuss another mechanism of the energy shift for an $s$-wave nucleon only arising in the case of “loosely” bound states, where the large part of the energy shift comes from the kinetic energy reduction in addition to the usual Thomas-Ehrman shift in the case of weakly and usual bound states. In the case of the usual Thomas-Ehrman shift, as discussed in the previous subsection, the energy difference between $s$- and $d$-wave proton and neutron is approximated as

$$\Delta E_{s,d} \approx R(\langle d|V^C|d \rangle - \langle s|V^C|s \rangle) \quad (3\cdot13)$$

$$\equiv \Delta V_{s,d}^C. \quad (3\cdot14)$$
However, in the case of loosely bound states, we have $\Delta E_{s-d} > \Delta V_{s-d}^C$. To explain the enhancement of $\Delta E_{s-d}$, we need to look for another cause in addition to the Coulomb energy shift. Such a correction in $\Delta E_{s-d}$ is described by the energy shift of the $s$-wave resonance energy, which is expressed as

$$\mathcal{R}(E_p^s) = \mathcal{R}(E_n^s) + \mathcal{R}((s|V^C|s)) + \Delta E_p^s. \quad (3.15)$$

When a neutron is in a very loosely bound or unbound (virtual) state with respect to the core+$n$ threshold, i.e. when $E_n^s \sim 0$, we have

$$\mathcal{R}(E_p^s) \approx \mathcal{R}((s|V^C|s)) + \Delta E_p^s. \quad (3.16)$$

However, in this situation, the resonance energy of the $s$-wave proton is obtained around the Coulomb barrier top ($\mathcal{R}(E_p^s) \approx V_B^C$), and then we have

$$\Delta E_p^s \approx V_B^C - \mathcal{R}((s|V^C|s)) < 0. \quad (3.17)$$

From Eq. (3.8), the energy shift is described as

$$\Delta E_{s-d} \approx \mathcal{R}(E_d^p) - V_B^C \quad \approx \mathcal{R}((d|V^C|d) - (s|V^C|s)) - \Delta E_p^s. \quad (3.18)$$

In Fig. 5, in order to explain this schematically, we show the energy shift $\Delta E_{s-d}$ (solid lines) of the $s$-states measured from $d$-states in the core+$p$ systems ($Z=6$), which are mirror for weakly and loosely bound core+n systems. The weakly bound system with $S_n=1$ MeV is described by the core+$n$ potential given in Eqs. (3.1)~(3.4) with the parameters used in the calculations of the previous subsection. On the other hand, the loosely bound system with $S_n=0.1$ MeV is given by the same potential as in the weakly bound case but with different values of the depth parameter: $V_s=-64.91$ MeV and $V_d=-93.13$ MeV. For the core+$p$ systems, we employ the Coulomb potential assuming $Z=6$ in addition to the same nuclear potential for the core+$n$ system. We also represent the real part of the complex expectation value for the Coulomb potential by dotted lines. As is seen from Fig. 5, a larger energy shift is obtained for the mirror state of a loosely bound state in the core+n system. We cannot understand such an energy shift as a Coulomb energy shift alone. We can explain the additional energy shift ($\sim \Delta E_p^s$) by considering the reduction of the kinetic energy in $\langle T \rangle$ and $\langle V^N \rangle$. On the other hand, for the mirror state of the weakly bound state, the energy shift is dominantly caused by the Coulomb energy difference between the $s$- and $d$-states. In Table II, we give the complex expectation values explicitly. We can also understand from this table, in the loosely bound case, that the large energy reduction of $\langle T \rangle$ is not canceled out by the increase of $\langle V^N \rangle$. This situation is different from the weakly bound case.

Finally, in order to see the barrier top effect occurring for the $s$-wave and not for the $d$-wave, in Fig. 6 we display the resonance energy and the real part of the complex expectation values of the Coulomb potential for the $s$- and $d$-waves of the loosely bound core+$p$ system together with the core+$p$ potential form. For the $d$-state, the obtained resonance energy (2.41 MeV) and the Coulomb energy (2.49 MeV)
Table II. The obtained complex expectation values of loosely bound $s$-states (left) and weakly bound $s$-states (right) in the core+$N$ systems.

<table>
<thead>
<tr>
<th>$Z$ (case)</th>
<th>loosely bound case</th>
<th>weakly bound case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(\langle T \rangle)$ (MeV)</td>
<td>$(\langle V_N \rangle)$ (MeV)</td>
</tr>
<tr>
<td></td>
<td>$(\langle T \rangle)$ (MeV)</td>
<td>$(\langle V_N \rangle)$ (MeV)</td>
</tr>
<tr>
<td>0 (core+n)</td>
<td>3.42</td>
<td>-3.52</td>
</tr>
<tr>
<td>6 (core+p)</td>
<td>0.92-6.01i</td>
<td>-1.32+6.26i</td>
</tr>
</tbody>
</table>

are almost the same, except for the energy difference which comes from $\mathcal{R}\langle T \rangle + \mathcal{R}\langle V_N \rangle = -0.08$ MeV (we fit $E = \mathcal{R}\langle T \rangle + \mathcal{R}\langle V_N \rangle = -0.1$ MeV in the core+n system). On the other hand, for the $s$-state, the situation is drastically changed, because the Coulomb energy increase (1.62 MeV) of the core+p system from the core+n system is larger than the Coulomb barrier height (1.21 MeV). In this case, the resonant wave function extends spatially thereby reducing the kinetic energy. This makes the resonance energy (1.22 MeV) of the $s$-wave approach the top of the Coulomb barrier (1.21 MeV), though the decay width $\Gamma$ becomes large.

§4. Summary and conclusion

In this paper, the $s$-wave energy shifts arising from the kinetic energy reduction in addition to the Thomas-Ehrman effect on core+$N$ systems were investigated by using the complex scaling method (CSM). In the present analysis, we discussed the expectation values of the Coulomb potential and kinetic energy operators for the resonant states. For the weakly bound systems, we studied the Thomas-Ehrman effects on the $s$-wave energy shift, where the calculated energy shift $\Delta E_{s,d}$ is almost the same as the Coulomb energy shift $\Delta V_{s,d}^C$, except for a slight deviation which comes from the sum of the kinetic energy and nuclear potential parts $(\langle T \rangle + (\langle V_N \rangle))$. Such an $s$-wave energy shift can be understood as the difference of the Coulomb matrix elements for different particle orbits on the usual way. In this study, we explicitly showed that their differences were caused from the spatially extended wave function of an $s$-wave halo (skin) state. For the loosely bound systems, on the other hand, we found the additional $s$-wave energy shift which is explained by the Coulomb barrier top effect. This effect arises on the condition that the Coulomb matrix element of the $s$-wave $\mathcal{R}\langle V_s^C \rangle$ is larger than the Coulomb barrier top energy $V_B^C$, though this condition is not satisfied for the $d$-wave due to the centrifugal
potential. On this situation, the resonant wave function of the core+p system is spatially extended than core+n system reducing the kinetic energy.

In conclusion, larger energy shifts of the s-states will occur in the mirror nuclei of the loosely bound or barely bound core+n systems in comparison with weakly bound (or deeply bound) systems. These are caused by both of the Coulomb energy reduction and the kinetic energy reduction for the spatially extended wave functions of the s-wave. Through a study of the energy shifts of the s-states in the core+p systems, we can expect to obtain further information on the halo structure closely connected to s-wave neutrons, because the energy shifts reflect the extended wave functions. As far as the new method calculating the complex expectation values is concerned (which is discussed in Ref. 13), it is very useful to analyze resonant properties.

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