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Discussion

The Effect of Turbulence on Solidification of a Binary Metal Alloy With Electromagnetic Stirring[1]

J. L. Lage.[2] I have read with interest the subject research work published recently by the ASME JOURNAL OF HEAT TRANSFER. In it, two fundamental aspects caught my attention: (1) the inclusion of a Darcy-like term in the momentum equation to model the viscous drag effect imposed by the solid dendrites to the fluid flow within the mushy zone; (2) the use of a $\kappa-\epsilon$ turbulence model to evaluate the eddy viscosity.

My initial comments relate to item (1) above. The momentum Eqs. (7) and (8) of are written in terms of mixture velocity (Bennon and Incropera, 1987). The drag effect of the solid phase is modeled via a Darcy-like term, that is, they treat the mushy zone as a porous medium and assume an isotropic model for the drag effect of the dendrites. Visualization of dendrite growth shows the mushy zone to be highly anisotropic. One would expect this morphologic aspect to play an important role in the flow within this region. My question then is: To what degree is the anisotropic aspect of dendrite growth responsible for the nonaxisymmetric character of the macrosegregation pattern observed experimentally but not revealed by the numerical results of the laminar model assuming symmetric permeability (Prescott et al., 1994)?

My most important comment, item (2) above, concerns the $\kappa-\epsilon$ model used by the authors. Their initiative in taking turbulence effects into consideration is commendable. Their analysis, though, seems to be questionable in one fundamental aspect: The $\kappa-\epsilon$ equations are not built from a physical law as are the momentum equations (Newton's second law of motion). Rather, they are derived from the momentum equations via suitable algebraic manipulations. Therefore, the final form of any $\kappa-\epsilon$ equations is rigidly tied to the form of the momentum equation from which they are derived (e.g., the Jones and Launder, 1972, model equations are consistent only with the momentum equations from which they were derived, namely those used by Jones and Launder). With that in mind, one should immediately identify the absence, for instance, of any buoyancy effect on the κ equation, Eq. (14), of the subject paper. In modeling phase change with strong convective effect, this term, written using Prescott and Incropera's nomenclature as $\rho_1 g \beta_T (\nabla T \cdot \boldsymbol{e}_z)$, should be extremely important (note that $\boldsymbol{e}_z$ is introduced here as the unit vector in the z direction) as indicated by their numerical results in which the temperature gradient within the mushy zone seems very high. It is noteworthy that Prescott and Incropera modified the original κ equation, Eq. (14) into their Eq. (17), including the effect of the Darcy-like term of the momentum equation. They, however, did not recognize that an extra term should also be included in the ϵ equation. Afterall the Darcy-like momentum term should affect both the κ and the ϵ equations. This extra term of the ϵ equation has the interesting general property of being a depletion term for the turbulence dissipation rate equation (Antohe and Lage, 1996), and this term can counterbalance the depletion term brought about by the Darcy-like term into the turbulence kinetic energy equation. The important conclusion is that turbulence can be important within the mushy zone depending upon the relation of these two terms.

The subject work indicates the importance of developing a consistent turbulence model of flow through permeable media, in which the effects of the extra drag terms (viscous and form) present in the momentum equations are taken into consideration. A step toward this objective has been recently taken by Antohe and Lage (1996).

References

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Authors' Closure

Professor Lage correctly points out that the permeability of a columnar dendritic structure is better characterized as anisotropic than isotropic. However, it is generally accepted that an agglomeration of equiaxed dendrites should be modeled with an isotropic permeability. Yoo and Viskanta (1992) examined the effects of anisotropic permeability on numerical predictions of double diffusive convection. It is our opinion that the advantages of using such an anisotropic model can only be reaped if there is access to a reliable directional permeability data base. Unfortunately, such data are, at best, sparse. Three-dimensional simulations, similar to those performed by Neilson and Incropera (1993) (with an isotropic permeability model) would have to be performed to test Professor Lage's hypothesis that anisotropic permeability contributes to the formation of three-dimensional macrosegregation patterns.

The main thrust of Professor Lage's comments was directed at our use of a $k-\epsilon$ model to account for turbulence. Our objective was to determine how turbulence might affect the solidifi-

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cation of a binary alloy and *not* to make a contribution to turbulence modeling. We implemented a model that we believed produced reasonable results, while acknowledging that additional effects (e.g., due to buoyancy and magnetic fields), which might be important, were neglected. We assumed that turbulence is virtually nonexistent in the mushy zone (at least in regions for which solid fractions exceed approximately 2 percent) and, therefore, introduced the Darcy term in the turbulence kinetic energy equation to effect such a result. It was not our intention to account for turbulence transport in the mushy zone. Instead, we accounted for its transport in the fully liquid region, which interacted with the mushy zone. We agree with Professor Lage's assertion that turbulence modeling in alloy solidification systems should be improved, but we would add that any such model developments should be accompanied by experimental validation.

References

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