Discussion

C. F. Bonilla.[9] The authors are to be commended on an extremely precise experimental study, permitting, as it does, a re-evaluation of the theory. In computing the logarithmic decrement, the writer wonders, however, whether a linear (or parabolic, if desired) least-squares correlation of the logarithm of each full swing versus the number of the swing (starting at zero for simplicity) would have had advantages.

The following comment does not bear on the pressure coefficient of viscosity, but seems pertinent in view of the atmospheric-pressure viscosities suggested as absolute values:

Vasilesco's viscosity data at and above 0 deg C have been analyzed statistically by the writer and P. L. Walker, H. W. Hoffman, and W. J. Angulo.[10] In addition, the trend with temperature of Vasilesco's viscosity results for nitrogen from 0 to 1100 C was closely matched experimentally. This was considered to corroborate strongly his results on other gases as well. Accordingly, the difference between his and other temperature-coefficient values for air and for nitrogen was surprising. As a check, the following temperature coefficients were computed by differentiating the Sutherland Equations [1] obtained by linear regression of Vasilesco's computed approximate values of Sutherland constant against temperature. These equations agree consistently with his data and of course avoid the large errors possible in a slope computed from two nearby points. These values are all based on Bearden's viscosity for air at 20 C, 181.920 micropoise,[11] which seems the most accurate absolute value available:

Gas	Temperature, deg C	Micropoise /- deg C	Micropoise
Air	21	0.4797	182.3
Nitrogen	21	0.4592	176.2
Argon	20	0.6237	223.2

It is seen that these viscosities are 0.3 to 0.4 per cent higher than those adopted by the authors, the relative agreement, however, thus being within 0.1 per cent. The slopes check well.

A. S. Friedman.[12] In connection with the results of this excellent piece of research, the writer would like to make a few comments concerning some of the particular molecules and the applicability of the Enskog equations.

Botzen[13,14] at Professor Michels' laboratory in Amsterdam, has evaluated the thermal conductivity and the viscosity of nitrogen as a function of pressure on the basis of the Chapman-Enskog model. The agreement with the experimental viscosity data of Michels and Gibson,[15] and with his own experimental thermal-conductivity data, is very good up to pressures of a few hundred atmospheres, but at higher pressures the computed values are significantly lower than experimental. At 25 C and 1000 atm, for example, the Enskog model gives values of viscosity about 9 per cent low.

In the range of the authors' experiments, the agreement of the Enskog theory with the Michels and Gibson experimental viscosity data is within a few tenths of one per cent. Hence it is about 2 per cent lower than the authors' values at about 70 atm. The Enskog equation really gives only the ratio of the viscosity to the viscosity at 1 atm. However, since for nitrogen the 1-atm value of Michels agrees with the value given in the paper, the difference in slope between the authors' curve and theirs is still unexplained.

Michels, Schipper, and Rintoul[16] recently measured the viscosity of hydrogen and deuterium from 25 to 125 C at pressures up to 2000 atm with their capillary viscometer. Extrapolation of their results for hydrogen to 21 C shows their measurements to be in excellent agreement with those of the authors. Michels' value of the viscosity at 1 atm, corrected to 21 C, is 885×10^{-7} poise. The Enskog theory here too agrees up to a couple of hundred atmospheres pressure.

As the authors have indicated, there are no high-pressure data on the viscosity of argon at room temperature available at the present time. Michels, Botzen, and the writer[17] have determined at the van der Waals Laboratory recently, the thermal conductivity of argon at pressures up to 2500 atm. In this case, the limitations of the Enskog theory are more apparent.

The assumptions of the Enskog model—hard elastic spheres, no multiple collisions, and so on—are definitely unrealistic at the higher pressures, and even the agreement at the moderately lower pressures is surprising.

F. G. Keyes.[18] This paper provides the most effective and complete discussion of the use of the oscillating disk as applied to viscosity measurements that the writer has read. The method is particularly suitable for measurements at high temperatures in view of the complications inherent in the capillary-flow method, as has been proved by considerable experience.

The determinations of the pressure coefficient of viscosity for gases are relatively few, and it is in this aspect that the oscillating disk offers great promise of providing really accurate data. Such data are important not alone for their practical utility, but they have a broad bearing on the perfection of our knowledge of intermolecular forces. Therefore the authors are to be commended for having carried out measurements to considerable pressures for two monatomic and two diatomic gases, as well as the very important reference gas, air.

The writer learned through a colleague that the authors have contemplated the use of the oscillating disk for the measurement of densities. Densities at high temperatures are extremely difficult to make, and if the oscillating disk could be used it would be a great boon, since for practical purposes we will continue increasingly to be in need of high-temperature-density data at known pressures and temperatures above 800 F.

The writer would be grateful if the authors will describe the use of the oscillating disk for obtaining density data.

Authors' Closure

The authors wish to express their thanks to Prof. C. F. Bonilla, Dr. A. S. Friedman, and to Prof. F. G. Keyes for their kind re-

[9] Columbia University, New York, N. Y.

[10] "The Viscosity of Steam and of Nitrogen at Atmospheric Pressure and High Temperatures," by C. F. Bonilla, R. D. Brooks, and P. L. Walker, Jr., Proceedings of the General Discussions on Heat Transfer, The Institution of Mechanical Engineers, London, England, 1951.

[11] "A Precision Determination of the Viscosity of Air," by J. A. Bearden, *Physical Review*, vol. 56, 1939, pp. 1023–1040.

[12] Thermodynamics Section, National Bureau of Standards, Washington, D. C.

[13] "The Thermal Conductivity of Gases at High Pressure," by A. Botzen, Dissertation, University of Amsterdam, Holland, 1952.

[14] "The Thermal Conductivity of Nitrogen at Pressures Up to 2500 Atmospheres," by A. Michels and A. Botzen, *Physica*, vol. 19, 1953, p. 585.

[15] The Measurement of the Viscosity of Gases at High Pressures— The Viscosity of Nitrogen to 1000 Atms," by A. Michels and R. O. Gibson, Proceedings of the Royal Society of London, England, series A, vol. 134, 1931, pp. 288–307.

[16] "The Viscosity of Hydrogen and Deuterium at Pressures Up to 2000 Atmospheres," by A. Michels, A. Schipper, and R. H. Rintoul, *Physica*, vol. 19, 1953, p. 1011.

[17] "The Thermal Conductivity of Argon at Pressures Up to 2500 Atmospheres," by A. Michels, A. Botzen, and A. S. Friedman, in preparation, 1953.

[18] Professor, Physical Chemistry, Massachusetts Institute of Technology, Cambridge, Mass. Mem. ASME.

marks and for their useful comments. Taking these in turn as they appear, the following remarks may be made:

From a more detailed examination of accuracy it appears that when the logarithmic decrement is measured with the aid of an optical leverage system and a linear scale the accuracy of the logarithmic decrement deduced from two successive amplitudes is poor and it is doubtful whether a least-square correlation of the logarithm of each full swing versus the number of the swing can do very much to improve it. A method which reduces the measurement of the decrement to that of time seems to offer good hope of increasing the intrinsic accuracy of measurement. The authors are now working along these lines and hope to be able to perform more precise determinations of the logarithmic decrement in the future.

It is gratifying to find that the absolute values of viscosity adopted in the paper do not differ by more than 0.3 to 0.4 per cent from those evaluated by Professor Bonilla. The reason for adopting the simplest possible temperature corrections lies in the fact that the corrections themselves were very small. The largest variation in temperature for any single gas did not exceed about 4 deg C and the actual correction was at most of the order of 0.35 per cent. This is well within an over-all accuracy of 1 per cent claimed in the paper.

Regarding the difference in the slope of the viscosity of nitrogen versus pressure referred to by Dr. A. S. Friedman, the authors agree that measurements with the oscillating-disk viscometer show a tendency of giving results which are somewhat too large, the deviation increasing with pressure. It is suggested that this is due to the imperfections in the theory of the instrument used and, in particular, to the fact that the edge correction computed by Macwood does not seem to be precise enough. Analytical work performed since the presentation of the paper shows that the existence of an edge on a disk tends to increase the velocity gradient near the edge as compared with an infinite disk. Thus

the viscous torque computed by Macwood seems to be too low as borne out by the discrepancy between the absolute and calibrated instrument constant C reported. It seems that this discrepancy increases primarily with viscosity and since a uniform correction was applied, the results at higher pressure are likely to be too high. However, it is believed that the result is still good within the 1 per cent claimed.

The oscillating disk has been used to determine densities and an account of such measurements may be found in the paper by Hollis-Hallett (3) quoted in the paper. It is easy to prove by dimensional analysis that the damping torque on an oscillating (or rotating) disk in a viscous fluid of infinite extent is a function of the similarity parameter

$$m = \frac{\rho}{\rho_0} \frac{\delta}{d_0}$$

Here ρ is the density of the fluid, ρ_0 is the density of the disk, δ is the boundary-layer thickness, and d_0 is the thickness of the disk. Since

$$\delta \sim \sqrt{\nu/\omega_0}$$

where ω_0 is the circular frequency in vacuo, ν is the kinematic viscosity $\nu = \eta/\rho$. Hence

$$m \sim \sqrt{\eta\rho}$$

so that the period of oscillation and the logarithmic decrement depend on $\sqrt{\eta\rho}$ and not on the viscosity alone. These remarks are borne out by the equations given by Hollis-Hallett (3).

One of the authors (J. Kestin) is now engaged on analytical work on the subject under consideration under the sponsorship of the Air Research and Development Command (Contract No. AF 18 [600] — 891) and the first report is due to be published by Brown University in June, 1954.